Acta Mathematica Academiae Paedagogicae Nyíregyháziensis
25 (2009), 119–127
www.emis.de/journals
ISSN 1786-0091

ON THE EXISTENCE OF UNIQUE COMMON FIXED POINTS FOR CERTAIN CLASSES OF WEAKLY COMPATIBLE MAPS IN NORMED LINEAR SPACE

GBENGA AKINBO AND OLUSEGUN OWOJORI

Abstract. In this work, we obtain some common fixed points and coincidence points results for weakly compatible selfmaps A, S and B, T of a normed linear space, satisfying certain contractive conditions of integral type. Our results generalize those of Pathak et al [16], Jungck [6] and others.

1. Introduction and Preliminaries

In 1976, Jungck [6] used commuting mapping concept as a tool to generalize the Banach fixed point theorem. This was followed by variety of extensions, generalizations and their applications, giving rise to different notions such as weak commutativity (S. Sessa [17]), compatibility, compatibility of types (A), (B), (C) and (P) (see [1], [3], [14], [15], [16], etc). The concept of R-weakly commuting pairs, i.e., the pair (f, g) of maps satisfying $d(fgx, gfx) \leq Rd(fx, gx),\ x \in X,\ R > 0$, where X is a metric space, was introduced by Pant [14]. In 1998, Jungck and Rhoades [9] defined two maps f and g of a metric space to be weakly compatible if and only if they commute at their coincidence points. Since then the study of common fixed points for contractive-type maps has been centered on this notion of weak compatibility. For more on the relationship between compatibility and its weaker forms, see Djoudi and Aliouche [3], P. P. Murthy [12].

Recently, Pathak et al. [16], in 2006, obtained some existence and uniqueness results for a class of weakly compatible, parametrically $\varphi(\epsilon, \delta; a)$-contraction mappings in metric space.

Throughout this paper, we shall always refer to $\mathbf{R}_+$ as the set of nonnegative real numbers.

2000 *Mathematics Subject Classification.* 47H10, 54H25.

Key words and phrases. common fixed points, coincidence points, parametrically $\varphi(\epsilon, \delta; a)$-contraction mappings, weak compatible mappings.

Definition 1.1 (Pathak et al. [16]). Let A, B, S, T be selfmappings of a metric space (X, d) such that $AX \subseteq TX$ and $BX \subseteq SX$. Define a function $\delta\colon (0,\infty) \to (0,\infty)$ such that $\delta(\epsilon) > \epsilon$ for all $\epsilon > 0$. The pair (A, B) is said to be parametrically $\varphi(\epsilon, \delta; a)$-contraction with respect to the pair (S, T) if for some $a \in (\frac{1}{2}, 1]$ and for all $x, y \in X$, the following are satisfied:

$$ad(Ax, By) + (1-a)d(By, Ty) \leq \varphi(ad(Sx, Ty) + (1-a)d(Ax, Sx)) \tag{1.1}$$

where $\varphi\colon \mathbf{R}_+ \to \mathbf{R}_+$ is such that

(a) φ is continuous;
(b) $\varphi(t) < t$ for all $t > 0$;
(c) $\epsilon \leq d(By, Ty) < \delta(\epsilon)$ implies $\varphi(d(Ax, Sx)) < \epsilon$;
(d) $\varphi(0) = 0$.

Definition 1.2 (Jungck and Rhoades [9]). A pair of mappings (A, S) is called weakly compatible if they commute at their coincidence points. (A coincidence point of A and S is any point u satisfying $Au = Su$.)

The following result was obtained by Pathak et al. [16].

Theorem 1.1 (Pathak et al. [16]). *Let S and T be selfmaps of a metric space (X, d) and the pair (A, B) is parametrically $\varphi(\epsilon, \delta; a)$-contraction with respect to the mappings (S, T). Let TX be complete, then there exist $u, v, w \in X$ such that $Au = Su = w = Bv = Tv$.*

Furthermore, if the pair (A, S) and (B, T) are weakly compatible, then w is the unique common fixed point of the mappings A, B, S and T.

In proving Theorem 1.1, the following iteration procedure was used.

Definition 1.3. Let A, B, S and T be selfmaps of a metric space X satisfying

$$AX \subseteq TX \text{ and } BX \subseteq SX. \tag{1.2}$$

Then for any $x_0 \in X$ there exists a point $x_1 \in X$ such that $y_0 = Ax_0 = Tx_1$ and for this point x_1, we can choose a point $x_2 \in X$ such that $y_1 = Bx_1 = Sx_2$ and so on. In general, we can define a sequence $\{y_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1} \text{ and } y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}, \quad n = 0, 1, 2, \ldots \tag{1.3}$$

This is called (S, T)-iteration on X.

Remark 1.1. Observe that if we choose $a = 1, A = B$ and $S = T = I$, where I is the identity mapping, then A reduces to a φ-contraction and (1.3) reduces to the Picard iteration.

If, in addition, we set $\varphi(t) = t$, A is a nonexpansive mapping and (1.3) becomes the Krasnoselskij iteration, see [10].

This sequence has been proved to converge to the unique common fixed point of A, B, S and T by several authors under various conditions (See Jungck [6], Chugh and Kumar [2], Pathak et al. [15, 16], Babu and Prasad [1], Djoudi and Aliouche [3]).

In this paper we do away with condition (c) of Definition 1.1 and employ the iteration process (1.3).

2. Main Results

We now present our main results in this paper.

Theorem 2.1. *Let A, B, S and T be selfmaps of a normed linear space X with $AX \subseteq TX$ and $BX \subseteq SX$ satisfying the following condition.*

(2.1)
$$h\|Ax - By\|^p + (1-h)\|By - Ty\|^p \le \varphi(h\|Sx - Ty\|^p + (1-h)\|Ax - Sx\|^p),$$

where, $p > 0$, $h \in (\frac{1}{2}, 1]$ and $\varphi\colon \mathbf{R}_+ \to \mathbf{R}_+$ is such that:

(a) *φ is continuous;*
(b) *$\varphi(t) < t$ for all $t > 0$;*

Let SX or TX be a complete subspace of X and the pairs (A, S) and (B, T) be weakly compatible, then A, B, S and T have a unique common fixed point.

We shall require the following Lemmas in the proof of Theorem 2.1. Our method of proof is almost the same as that of Pathak et al. [10].

Lemma 2.1. *Let the mappings A, B, S and T be as in Theorem 2.1. Then the (S,T)-iteration defined on X is a Cauchy sequence.*

Proof. Since $AX \subseteq TX$ *and* $BX \subseteq SX$, we can define the (S,T)-iteration on $x_0 \in X$ as in (1.3). Therefore, choosing $k = 2n, q = 2m-1$, k and q are of different parities, and we have

$$\begin{aligned}
&h\|Ax_{2n} - Bx_{2m-1}\|^p + (1-h)\|Bx_{2m-1} - Tx_{2m-1}\|^p \\
&\qquad = h\|y_{2n+1} - y_{2m}\|^p + (1-h)\|y_{2m} - y_{2m-1}\|^p \\
&\qquad\qquad = h\|y_{k+1} - y_{q+1}\|^p + (1-h)\|y_{q+1} - y_q\|^p
\end{aligned}$$

and

$$\begin{aligned}
&h\|Sx_{2n} - Tx_{2m-1}\|^p + (1-h)\|Sx_{2n} - Ax_{2n}\|^p \\
&\qquad = h\|y_{2n} - y_{2m-1}\|^p + (1-h)\|y_{2n} - y_{2n+1}\|^p \\
&\qquad\qquad = h\|y_k - y_q\|^p + (1-h)\|y_k - y_{q+1}\|^p
\end{aligned}$$

Hence, from (2.1),

$$\begin{aligned}
(2.2)\quad & h\|y_{k+1} - y_{q+1}\|^p + (1-h)\|y_{q+1} - y_q\|^p \\
&\qquad \le \varphi(h\|y_k - y_q\|^p + (1-h)\|y_k - y_{q+1}\|^p)
\end{aligned}$$

Now, let x_0 be an arbitrary point in X. Then from (2.2) and (2.1)(b), choosing $k = 2n, q = 2m-1$,

$$\begin{aligned}
&h\|y_{2n+1} - y_{2n}\|^p + (1-h)\|y_{2n} - y_{2n-1}\|^p \\
&\qquad \le \varphi(h\|y_{2n} - y_{2n-1}\|^p + (1-h)\|y_{2n} - y_{2n+1}\|^p)
\end{aligned}$$

$$< h \left\| y_{2n} - y_{2m-1} \right\|^p + (1-h) \left\| y_{2n} - y_{2n+1} \right\|^p$$

That is,

$$(2h-1) \left\| y_{2n+1} - y_{2n} \right\|^p < (2h-1) \left\| y_{2n} - y_{2n-1} \right\|^p$$

Since $h \in (\frac{1}{2}, 1]$, we have

$$\left\| y_{2n+1} - y_{2n} \right\|^p < \left\| y_{2n} - y_{2n-1} \right\|^p.$$

Similarly for $p = 2n+1$ *and* $q = 2n$, we have

$$\left\| y_{2n+2} - y_{2n+1} \right\|^p < \left\| y_{2n+1} - y_{2n} \right\|^p.$$

Since $p > 0$, then $\{\|y_n - y_{n+1}\|\}_{n=0}^{\infty}$ is a decreasing sequence which converges to its greatest lower bound, say, $t \geq 0$.

Suppose $t > 0$, from (2.1), for $x = x_{2n}, y = x_{2n-1}$, we obtain

$$\begin{aligned} &h \left\| Ax_{2n} - Bx_{2n-1} \right\|^p + (1-h) \left\| Bx_{2n-1} - Tx_{2n-1} \right\|^p \\ &\qquad \leq \varphi(h \left\| Sx_{2n} - Tx_{2n-1} \right\|^p + (1-h) \left\| Ax_{2n} - Sx_{2n} \right\|^p) \end{aligned}$$

That is,

$$\begin{aligned} &h \left\| y_{2n} - y_{2n-1} \right\|^p + (1-h) \left\| y_{2n-1} - y_{2n-2} \right\|^p \\ &\qquad \leq \varphi(h \left\| y_{2n} - y_{2n-2} \right\|^p + (1-h) \left\| y_{2n} - y_{2n-1} \right\|^p) \end{aligned}$$

Letting $n \to \infty$,we have $t^p \leq \varphi(t^p) < t^p$. This is a contradiction. Therefore $t = 0$. Hence,

$$\lim_{n\to\infty} \|y_n - y_{n+1}\| = 0. \tag{2.3}$$

We now show that the sequence $\{y_n\}$ defined by (1.3) is Cauchy. By virtue of (2.3) it suffices to show that the subsequence $\{y_{2n}\}$ of $\{y_n\}$ is Cauchy. Suppose not. Then there exist $\epsilon > 0$ such that $\|y_{2n_i} - y_{2m_i}\| \to \epsilon$, as $i \to \infty$. Also, as in Djoudi and Nisse [4],

$$\left\| y_{2n_i+1} - y_{2m_i} \right\|, \left\| y_{2n_i} - y_{2m_i-1} \right\| \to \epsilon, \text{ as } i \to \infty. \tag{2.4}$$

Therefore,

$$\begin{aligned} &h \left\| Ax_{2n_i} - Bx_{2m_i-1} \right\|^p + (1-h) \left\| Bx_{2m_i-1} - Tx_{2m_i-1} \right\|^p \\ &\qquad \leq \varphi(h \left\| Sx_{2n_i} - Tx_{2m_i-1} \right\|^p + (1-h) \left\| Ax_{2n_i} - Sx_{2n_i} \right\|^p) \end{aligned}$$

so that

$$\begin{aligned} &h \left\| y_{2n_i+1} - y_{2m_i} \right\|^p + (1-h) \left\| y_{2m_i} - y_{2m_i-1} \right\|^p \\ &\qquad \leq \varphi(h \left\| y_{2n_i} - y_{2m_i-1} \right\|^p + (1-h) \left\| y_{2n_i+1} - y_{2n_i} \right\|^p). \end{aligned}$$

Letting $i \to \infty$, by (2.3) and (2.4),

$$h\epsilon^p \leq \varphi(h\epsilon^p) < h\epsilon^p,$$

which is also a contradiction. Therefore, $\{y_n\}$ is Cauchy. $\square$

Lemma 2.2. *Let $\varphi\colon \mathbf{R}_+ \longrightarrow \mathbf{R}_+$ be a continuous function satisfying $\varphi(t) < t$ for all $t > 0$. Then, $\varphi(0) = 0$. Hence, $\varphi(t) \leq t$ for all $t \geq 0$.*

We are now in a convenient position to prove Theorem 2.1.

Proof of Theorem 2.1. Since the subsequence $\{y_{2n}\}$ of $\{y_n\}$ which is in SX is Cauchy, and SX is complete, then $\{y_{2n}\}$ converges to a point $x^* = Su \in SX$ for some $u \in X$.

If we substitute $x = u$, $y = x_{2n-1}$ into (2.1),

$$h\,\|Au - Bx_{2n-1}\|^p + (1-h)\,\|Bx_{2n-1} - Tx_{2n-1}\|^p \leq \varphi(h\,\|Su - Tx_{2n-1}\|^p + (1-h)\,\|Au - Su\|^p)$$

or,

$$h\,\|Au - y_{2n}\|^p + (1-h)\,\|y_{2n} - y_{2n-1}\|^p \leq \varphi(h\,\|x^* - y_{2n-1}\|^p + (1-h)\,\|Au - x^*\|^p)$$

Letting $n \to \infty$,

$$h\,\|Au - x^*\|^p + (1-h)\,\|x^* - x^*\|^p \leq \varphi(h\,\|x^* - x^*\|^p + (1-h)\,\|Au - x^*\|^p)$$

or,

$$h\,\|Au - x^*\|^p \leq \varphi((1-h)\,\|Au - x^*\|^p) \leq (1-h)\,\|Au - x^*\|^p\,,$$

by Lemma 2.2. This yields

$$(2h-1)\,\|Au - x^*\|^p \leq 0.$$

But since $h \in (1/2, 1]$, $\|Au - x^*\|^p = 0$. Hence, $Au = x^* = Su$.

Since $AX \subseteq TX$, there exists some $v \in X$ such that $Au = Tv$, so that $x^* = Su = Au = Tv$. If we now put $x = u$, $y = v$ into (2.1), we obtain

$$h\,\|Au - Bv\|^p + (1-h)\,\|Bv - Tv\|^p \leq \varphi(h\,\|Su - Tv\|^p + (1-h)\,\|Au - Su\|^p),$$

yielding

$$h\,\|x^* - Bv\|^p + (1-h)\,\|Bv - x^*\|^p \leq \varphi(h\,\|x^* - x^*\|^p + (1-h)\,\|x^* - x^*\|^p).$$

That is,

$$\|x^* - Bv\|^p = 0.$$

Hence,

$$Su = Au = x^* = Bv = Tv. \tag{2.5}$$

Since the pair (A, S) is weakly compatible, that is, they commute at their coincidence point u, then

$$AAu = ASu = SAu. \tag{2.6}$$

Now, substituting $x = Au$, $y = v$ into (2.1), by (2.5) and (2.6) we obtain

$$h\,\|AAu - x^*\|^p \leq \varphi(h\,\|AAu - x^*\|^p) \leq h\,\|AAu - x^*\|^p$$

Thus, $AAu = x^* = Au$, that is, $Au = u$. This, together with the first equality in (2.6), yields $Su = u$. Therefore, $u \in X$ is a common fixed point of A and S.

Considering that B and T are also weakly compatible, by a similar process it is easy to see that $v \in X$ is a common fixed point of B and T, using (2.1) and (2.5).

It is obvious that $u = v$. Indeed, putting $x = u$, $y = v$ back in (2.1), we see that $h\|u-v\|^p \leq \varphi(h\|u-v\|^p)$, that is $\|u-v\| = 0$. Consequently, $u = v = x^*$ is a common fixed point of A, B, S and T.

Finally for uniqueness, let if possible, x' be another common fixed point of A, B, S and T such that $x^* \neq x'$. Then $\|x^* - x'\| > 0$, and

$$\begin{aligned}
\|x^* - x'\|^p &= h\|x^* - x'\|^p + (1-h)\|x^* - x'\|^p \\
&= h\|Ax^* - Bx'\|^p + (1-h)\|Bx^* - Tx'\|^p \\
&\leq \varphi(h\|Sx^* - Tx'\|^p + (1-h)\|Ax^* - Sx'\|^p) \\
&= \varphi(h\|x^* - x'\|^p + (1-h)\|x^* - x'\|^p) \\
&= \varphi(\|x^* - x'\|^p) \\
&< \|x^* - x'\|^p .
\end{aligned}$$

This is a contradiction. Hence, x^* is the unique common fixed point of A, B, S and T. This completes the proof. □

Theorem 2.1. is a special case of Theorem 2.2. below. For the latter reduces to the former when $\psi(t) = 1$.

Theorem 2.2. *Let A, B, S and T be selfmaps of a normed linear space X such that*

$$AX \subseteq TX, \;\; BX \subseteq SX,$$

and

$$(2.7) \quad h\left(\int_0^{\|Ax-By\|}\psi(t)dt\right)^p + (1-h)\left(\int_0^{\|By-Ty\|}\psi(t)dt\right)^p \leq \varphi[h\left(\int_0^{\|Sx-Ty\|}\psi(t)dt\right)^p + (1-h)\left(\int_0^{\|Ax-Sx\|}\psi(t)dt\right)^p],$$

where $p > 0$, $h \in (\frac{1}{2}, 1]$,φ is as in Theorem 2.1.1 and $\psi\colon \mathbf{R}_+ \to \mathbf{R}_+$ is a Lebesgue integrable mapping which is summable nonnegative and such that

$$(2.8) \qquad \int_0^{\epsilon}\psi(t)dt > 0 \text{ for each } \epsilon > 0.$$

Suppose that one of SX and TX is complete and the pairs (A, S) and (B, T) are weakly compatible. Then A, B, S and T have a unique common fixed point in X.

We first state the following useful lemma before proving Theorem 2.2.

Lemma 2.3. *Let A, B, S and T be selfmaps of a normed linear space X satisfying (2.7) for all x, y in X, where $0 < h \leq 1$, $p \geq 1$ and ψ satisfies (2.8). Then, the sequence $\{y_n\}$ defined by (1.3) is Cauchy in X.*

Proof of Theorem 2.2. By Lemma 2.3. the subsequence

$$\{y_{2n-1}\} = \{Tx_{2n-1}\} \subseteq TX$$

is a Cauchy sequence. Since TX is complete, it converges to a point $z = Tu$ for some $u \in X$. Hence, subsequences $\{Ax_{2n-2}\}$, $\{Bx_{2n-1}\}$, $\{Sx_{2n}\}$ also converge to z.

If $Bu \neq z$, using (2.7), we get

$$h\left(\int_0^{\|Ax_{2n}-Bu\|}\psi(t)dt\right)^p + (1-h)\left(\int_0^{\|Bu-Tu\|}\psi(t)dt\right)^p$$
$$\leq \varphi[h\left(\int_0^{\|Sx_{2n}-Tu\|}\psi(t)dt\right)^p + (1-h)\left(\int_0^{\|Ax_{2n}-Sx_{2n}\|}\psi(t)dt\right)^p].$$

Letting $n \to \infty$, we have

$$\left(\int_0^{\|z-Bu\|}\psi(t)dt\right)^p \leq \varphi(0) = 0,$$

which contradicts (2.8).

Therefore

$$\int_0^{\|z-Bu\|}\psi(t)dt = 0,$$

and (2.8) implies that $z = Bu = Tu$. Since $BX \subseteq SX$, there exists $v \in X$ such that $z = Bu = Sv$. If $z \neq Av$, using (2.7) we have

$$h\left(\int_0^{\|Av-Bu\|}\psi(t)dt\right)^p + (1-h)\left(\int_0^{\|Bu-Tu\|}\psi(t)dt\right)^p$$
$$\leq \varphi[h\left(\int_0^{\|Sv-Tu\|}\psi(t)dt\right)^p + (1-h)\left(\int_0^{\|Av-Sv\|}\psi(t)dt\right)^p],$$

that is,

$$h\left(\int_0^{\|Av-z\|}\psi(t)dt\right)^p \leq \varphi\left[(1-h)\left(\int_0^{\|Av-z\|}\psi(t)dt\right)^p\right]$$
$$\leq (1-h)\left(\int_0^{\|Av-z\|}\psi(t)dt\right)^p,$$

which implies that

$$(2h-1)\left(\int_0^{\|Av-z\|}\psi(t)dt\right)^p < 0.$$

This is also a contradiction. Therefore, $Tu = Bu = z = Av = Sv$.

Since the pair (B,T) is weakly compatible, we have $Bz = BTu = TBu = Tz$, and putting $x = v,\ y = z$ into (2.7), we obtain

$$h\left(\int_0^{\|z-Bz\|}\psi(t)dt\right)^p \le \varphi\left(h\left(\int_0^{\|z-Bz\|}\psi(t)dt\right)^p\right),$$

yielding $z = Bz = Tz$.

Similarly, we can show that $z = Az = Sz$ from the weak compatibility of (A,S) and (2.7).

Hence, z is a common fixed point of A, B, S and T.

The uniqueness of z follows from (2.7). □

Corollary 2.1. *Let A and S be selfmaps of a normed linear space X satisfying $AX \subseteq SX$ and*

$$h\left(\int_0^{\|Ax-Ay\|}\psi(t)dt\right)^p + (1-h)\left(\int_0^{\|Ay-Sy\|}\psi(t)dt\right)^p$$
$$\le \varphi[h\left(\int_0^{\|Sx-Sy\|}\psi(t)dt\right)^p + (1-h)\left(\int_0^{\|Ax-Sx\|}\psi(t)dt\right)^p],$$

for all x, y in X, $0 < h \le 1$, $p \ge 1$ and ψ satisfies (2.8). Suppose SX is complete and (A,S) is weakly compatible. Then, A and S have a unique common fixed point in X.

Proof. Put $B = A$ and $T = S$ in Theorem 2.2. □

References

[1] G. V. R. Babu and K. N. V. V. Vara Prasad. Some common fixed point theorems of two pairs of weakly compatible maps. *J. Appl. Funct. Differ. Equ. JAFDE*, 1(1):9–22, 2006.

[2] R. Chugh and S. Kumar. Common fixed points for weakly compatible maps. *Proc. Indian Acad. Sci. Math. Sci.*, 111(2):241–247, 2001.

[3] A. Djoudi and A. Aliouche. Common fixed point theorems of Gregus type for weakly compatible mappings satisfying contractive conditions of integral type. *J. Math. Anal. Appl.*, 329(1):31–45, 2007.

[4] A. Djoudi and L. Nisse. Greguš type fixed points for weakly compatible maps. *Bull. Belg. Math. Soc. Simon Stevin*, 10(3):369–378, 2003.

[5] C. O. Imoru, G. Akinbo, and A. O. Bosede. On the fixed points for weak compatible type and parametrically $\phi(\epsilon, \delta; a)$-contraction mappings. *Math. Sci. Res. J.*, 10(10):259–267, 2006.

[6] G. Jungck. Commuting mappings and fixed points. *Amer. Math. Monthly*, 83(4):261–263, 1976.

[7] G. Jungck. Coincidence and fixed points for compatible and relatively nonexpansive maps. *Internat. J. Math. Math. Sci.*, 16(1):95–100, 1993.

[8] G. Jungck, P. P. Murthy, and Y. J. Cho. Compatible mappings of type (A) and common fixed points. *Math. Japon.*, 38(2):381–390, 1993.

[9] G. Jungck and B. E. Rhoades. Fixed points for set valued functions without continuity. *Indian J. Pure Appl. Math.*, 29(3):227–238, 1998.

[10] M. A. Krasnoselski. Two remarks on the method of successive approximations. *Uspekhi Math. Nauk (N.S.)*, 10(1):123–127, 1955.

[11] A. Meir and E. Keeler. A theorem on contraction mappings. *J. Math. Anal. Appl.*, 28:326–329, 1969.

[12] P. P. Murthy. Important tools and possible applications of metric fixed point theory. In *Proceedings of the Third World Congress of Nonlinear Analysts, Part 5 (Catania, 2000)*, volume 47, pages 3479–3490, 2001.

[13] R. P. Pant. Common fixed points of noncommuting mappings. *J. Math. Anal. Appl.*, 188(2):436–440, 1994.

[14] R. P. Pant. Common fixed points of four mappings. *Bull. Calcutta Math. Soc.*, 90(4):281–286, 1998.

[15] H. K. Pathak, M. S. Khan, and S. M. Kang. Fixed and coincidence points for contractions and parametrically nonexpansive mappings. *Math. Sci. Res. J.*, 8(1):27–35, 2004.

[16] H. K. Pathak, R. K. Verma, S. M. Kang, and M. S. Khan. Fixed points for weak compatible type and parametrically $\phi(\epsilon, \delta; a)$-contraction mappings. *Int. J. Pure Appl. Math.*, 26(2):247–263, 2006.

[17] S. Sessa. On a weak commutativity condition of mappings in fixed point considerations. *Publ. Inst. Math. (Beograd) (N.S.)*, 32(46):149–153, 1982.

Received February 15, 2008.

Department of Mathematics,
Obafemi Awolowo University,
Ile-Ife, Nigeria