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A CLASS OF ANALYTIC FUNCTIONS BASED ON AN EXTENSION OF AL-OBOUDI OPERATOR

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Abstract. In this paper we introduce the classes $TS_p^\lambda(\alpha,\beta)$ and $TV^\lambda(\alpha,\beta)$, $\alpha \in [-1,1)$, $\lambda \geq 0$, $\beta \geq 0$ of analytic functions with negative coefficients. The classes are motivated by the study of Acu and Owa (2006). We obtain a coefficient characterization, growth and distortion theorems and a convolution result for functions in these classes.

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1. Introduction

Let H be the set of functions regular in the unit disc $\Delta = \{z : |z| < 1\}$. Let $A = \{f(z) \in H/f(0) = f'(0) - 1 = 0\}$ and $S = \{f(z) \in A : f(z)$ is univalent in $\Delta\}$, where

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1}$$

For $0 \leq \alpha < 1$ let $S^*(\alpha)$ and $K(\alpha)$ denote the subfamilies of S consisting of functions starlike of order α and convex of order α, respectively. For convenience, we write $S^*(0) = S^*$ and $K(0) = K$ motivated by geometric considerations, Goodman [4], [5], introduced the classes UCV and UST of uniformly convex and uniformly starlike functions. Ma and Minda [6] and Ronning [8] gave a one-variable analytic characterization for UCV, namely, a function $f(z)$ of the form (1) is UCV if and only if $Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \left|\frac{zf''(z)}{f'(z)}\right|, \quad z \in \Delta$. Goodman [4] showed that the classical Alexander result, namely, $f(z) \in K$ if and only if $zf'(z) \in S^*$, does not hold between the classes UCV and UST.

Ronning (see [8], [9]) introduced the class S_p consisting of parabolic starlike functions $g = zf'(z)$, $f \in UCV$ and the class $S_p(\alpha)$ of functions of the form (1) for which $Re\left\{\frac{zf'(z)}{f(z)} - \alpha\right\} \geq \left\{\frac{zf'(z)}{f(z)} - 1\right\}$, $\alpha \in [-1, 1)$, $z \in \Delta$. Ronning [9] also defined the class $UCV(\alpha)$, of uniformly convex functions $f(z)$ of order α for which $zf' \in S_p(\alpha)$.

Geometrically, $S_p(\alpha)$ is the family of functions $f(z)$ for which $\frac{zf'(z)}{f(z)}$ takes values that lie inside the parabola $\Omega = \{w : Re(w - \alpha) > |w - 1|\}$, which is symmetrical about the real axis and whose vertex is $w = \frac{1+\alpha}{2}$.

The subfamily T of S consists of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n,\ a_n \geq 0, \text{for} n = 2, 3, \cdots, z \in \Delta. \tag{2}$$

Silverman [10] investigated functions in the classes $T^*(\alpha) = T \cap S^*(\alpha)$ and $C(\alpha) = T \cap K(\alpha)$. Subramanian et al. [11] introduced the classes $TS_p(\alpha)$ and $TV(\alpha)$, $\alpha \in [-1, 1)$ as follows: A function $f(z)$ of the form (2) is in $TS_p(\alpha)$, $\alpha \in [-1, 1)$ if $Re\left\{\frac{zf'(z)}{f(z)} - \alpha\right\} \geq \left|\frac{zf'(z)}{f(z)} - 1\right|$ and is in $TV(\alpha)$ if $zf' \in TS_p(\alpha)$.

Let $n \in N$ and $\lambda \geq 0$. Denote by D_λ^n the Al-Oboudi operator (see [3]) defined by $D_\lambda^n : A \to A$,

$$\begin{aligned} D_\lambda^0 f(z) &= f(z) \\ D_\lambda^1 f(z) &= (1-\lambda)f(z) + \lambda z f'(z) = D_\lambda f(z) \\ D_\lambda^n f(z) &= D_\lambda(D_\lambda^{n-1} f(z)) \end{aligned}$$

Note that for $f(z)$ given by (1), $D_\lambda^n = z + \sum_{j=2}^{\infty}[1 + (j-1)\lambda]^n a_j z^j$. When $\lambda = 1$, D_λ^n is the Sălăgean differential operator (see [7]) $D^n : A \to A$, $n \in N$, defined as:

$$\begin{aligned} D^0 f(z) &= f(z) \\ D^1 f(z) &= Df(z) = zf'(z) \\ D^n f(z) &= D(D^{n-1} f(z)) \end{aligned}$$

Acu and Owa [2] considered the operator D_λ^β extending the Al-oboudi operator D_λ^n and using this operator Acu [1] introduced and studied the classes $TL_\beta(\alpha)$ and $T^c L_\beta(\alpha)$.

Definition 1 [2] *Let* $\beta, \lambda \in R$, $\beta \geq 0$, $\lambda \geq 0$ *and* $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$. *We denote* *by* D_λ^β *the linear operator defined by* $D_\lambda^\beta : A \to A$,

$$D_\lambda^\beta f(z) = z + \sum_{j=2}^{\infty} [1 + (j-1)\lambda]^\beta a_j z^j.$$

Remark 1 *If* $f \in T$, $f(z) = z - \sum_{j=2}^{\infty} a_j z^j$, $a_j \geq 0$, $j = 2, 3, \ldots$, $z \in \Delta$ *then* $D_\lambda^\beta f(z) = z - \sum_{j=2}^{\infty} [1 + (j-1)\lambda]^\beta a_j z^j$.

Definition 2 [1] *Let* $f(z) \in T$ *with* $f(z)$ *given by* (2). *Then for* $\alpha \in [0, 1)$, $\lambda \geq 0$, $\beta \geq 0$, $f(z)$ *is in the class* $TL_\beta(\alpha)$ *if* $Re \frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta f(z)} > \alpha$, *and is in the class* $T^c L_\beta(\alpha)$ *if* $Re \frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1} f(z)} > \alpha$.

In this paper, using the operator D_λ^β, we introduce the classes $TS_p^\lambda(\alpha, \beta)$ and $TV^\lambda(\alpha, \beta)$ and obtain coefficient characterization for these classes when the functions have negative coefficients. This leads to extremal properties. We also obtain growth and distortion theorems and convolution result for functions in these classes.

2. Main Results

Definition 3 *Let* $f \in T$, $f(z) = z - \sum_{j=2}^{\infty} a_j z^j$, $a_j \geq 0$, $j = 2, 3, \ldots$, $z \in \Delta$.
We say that $f(z)$ *is in the class* $TS_p^\lambda(\alpha, \beta)$ *if*

$$Re \left\{ \frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta f(z)} - \alpha \right\} \geq \left| \frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta f(z)} - 1 \right|, \quad \alpha \in [-1, 1), \lambda \geq 0, \beta \geq 0.$$

We say that $f(z)$ *is in the class* $TV^\lambda(\alpha, \beta)$ *if*

$$Re \left\{ \frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1} f(z)} - \alpha \right\} \geq \left| \frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1} f(z)} - 1 \right|, \quad \alpha \in [-1, 1), \lambda \geq 0, \beta \geq 0.$$

Remark 2 $f(z) \in TS_p^\lambda(\alpha, \beta)$ *if and only if* $\frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta f(z)} \prec q_\alpha(z)$ *and* $f(z) \in TV^\lambda(\alpha, \beta)$ *if and only if* $\frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1} f(z)} \prec q_\alpha(z)$ *where* $q_\alpha(z) = 1 + \frac{2(1-\alpha)}{\pi^2} \left(log \frac{1+\sqrt{z}}{1-\sqrt{z}} \right)^2$ *maps* Δ *onto the parabolic region* $\Omega = \{w : Re(w - \alpha) > |w - 1|\}$ *which lies inside the sector* $-\pi/4 < arg w < \pi/4$.

Theorem 1 *Let $\alpha \in [-1,1)$, $\lambda \geq 0$, $\beta \geq 0$. The function $f(z) \in T$ of the form* (1) *is in the class $TS_p^\lambda(\alpha,\beta)$ if and only if*

$$\sum_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]a_j < 1-\alpha \tag{3}$$

and is in the class $TV^\lambda(\alpha,\beta)$ if and only if

$$\sum_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta+1}[2(j-1)\lambda+1-\alpha]a_j < 1-\alpha \tag{4}$$

Proof. Let $f \in TS_p^\lambda(\alpha,\beta)$ with $\alpha \in [-1,1)$, $\lambda \geq 0$, $\beta \geq 0$. We have

$$Re\left\{\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}f(z)}-\alpha\right\} \geq \left|\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}f(z)}-1\right|.$$

If we take $z \in [0,1)$, $\beta \geq 0$, $\lambda \geq 0$, we have

$$\frac{1-\sum\limits_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta+1}a_j z^{j-1}}{1-\sum\limits_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta}a_j z^{j-1}}-\alpha \geq 1-\frac{1-\sum\limits_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta+1}a_j z^{j-1}}{1-\sum\limits_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta}a_j z^{j-1}}$$

This yields

$$\sum[1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]a_j z^{i-1} \leq 1-\alpha.$$

Letting $z \to 1^-$ along the real axis we have

$$\sum_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]a_j < 1-\alpha.$$

Conversely, let us take $f(z) \in T$ for which the relation (3) hold.

If suffices to show that

$$\left|\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}f(z)}-1\right|-Re\left\{\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}f(z)}-1\right\} \leq 1-\alpha, \quad z \in \Delta.$$

We have

$$\begin{aligned}\left|\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}f(z)}-1\right|-Re\left\{\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}f(z)}-1\right\} &\leq 2\left|\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}f(z)}-1\right| \\ &= 2\left|\frac{D_\lambda^{\beta+1}f(z)-D_\lambda^{\beta}f(z)}{D_\lambda^{\beta}f(z)}\right| \\ &\leq \frac{2\sum_{j=2}^{\infty}(j-1)\lambda[1+(j-1)\lambda]^{\beta}|a_j||z^{j-1}|}{1-\sum_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta}|a_j||z^{j-1}|} \\ &\leq \frac{2\sum_{j=2}^{\infty}(j-1)\lambda[1+(j-1)\lambda]^{\beta}|a_j|}{1-\sum_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta}|a_j|}\end{aligned}$$

This last expression is bounded above by $1-\alpha$ if

$$\sum_{j=2}^{\infty}[1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]a_j \leq 1-\alpha.$$

The proof of the second part of the theorem is similar and so is omitted.

Theorem 2 *For* $-1 \leq \alpha < 1$, $\beta \geq 0$, $\lambda \geq 0$, $TS_p^{\lambda}(\alpha,\beta) = TL_{\beta}\left(\frac{1+\alpha}{2}\right)$ *and* $TV^{\lambda}(\alpha,\beta) = TL_{\beta}^{c}\left(\frac{1+\alpha}{2}\right)$.

Proof. This result is a consequence of theorem 2.1 and in the necessary and sufficient coefficient condition [1] for the classes $TL_{\beta}(\alpha)$ and $T^{c}L_{\beta}(\alpha)$.

Remark 3 *When* $\lambda = 1$ *and* $\beta = 0$, *we have* $TS_p^{\lambda}(\alpha,\beta) = TS_p(\alpha) = T^{*}(\frac{1+\alpha}{2})$ *and* $TV^{\lambda}(\alpha,\beta) = TV(\alpha) = C(\frac{1+\alpha}{2})$ *[11].*

The following inclusion result can be seen using the conditions (3) and (4).

Theorem 3 $TS_p^{\lambda}(\alpha,\beta+1) \subseteq TS_p^{\lambda}(\alpha,\beta)$, $TV^{\lambda}(\alpha,\beta+1) \subseteq TV^{\lambda}(\alpha,\beta)$ *where* $\beta \geq 0$, $\alpha \in [-1,1)$ *and* $\lambda \geq 0$.

Theorem 4 *a) If* $f_1(z) = z$ *and* $f_j(z) = z - \frac{1-\alpha}{[1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]} z^j$, *then* $f \in TS_p^{\lambda}(\alpha,\beta)$ *if and only if it can be expressed in the form* $f(z) = \sum_{j=1}^{\infty} \lambda_j f_j(z)$, *where* $\lambda_j \geq 0$ *and* $\sum_{j=1}^{\infty} \lambda_j = 1$.

b) If $f_1(z) = z$ *and* $f_j(z) = z - \frac{1-\alpha}{[1+(j-1)\lambda]^{\beta+1}[2(j-1)\lambda+1-\alpha]} z^j$, *then* $f \in TV^{\lambda}(\alpha,\beta)$ *if and only if it can be expressed in the form* $f(z) = \sum_{j=1}^{\infty} \lambda_j f_j(z)$, *where* $\lambda_j \geq 0$ *and* $\sum_{j=1}^{\infty} \lambda_j = 1$.

Proof. Let $f(z) = \sum_{j=1}^{\infty} \lambda_j f_j(z)$, $\lambda_j \geq 0$, $j = 1, 2, \ldots$, with $\sum_{j=1}^{\infty} \lambda_j = 1$.

We have

$$\begin{aligned} f(z) &= \sum_{j=1}^{\infty} \lambda_j f_j(z) \\ &= \lambda_1 z + \sum_{j=2}^{\infty} \lambda_j \left(z - \frac{1-\alpha}{[1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]} z^j \right) \\ &= \sum_{j=1}^{\infty} \lambda_j z - \sum_{j=2}^{\infty} \lambda_j \frac{1-\alpha}{[1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]} z^j \\ &= z - \sum_{j=2}^{\infty} \lambda_j \frac{1-\alpha}{[1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]} z^j \end{aligned}$$

Since

$$\begin{aligned} \sum_{j=2}^{\infty} [1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]\lambda_j \frac{1-\alpha}{[1+(j-1)\lambda]^{\beta}[2(j-1)\lambda+1-\alpha]} \\ &= (1-\alpha) \sum_{j=2}^{\infty} \lambda_j \\ &= (1-\alpha)(1-\lambda_1) \\ &< 1-\alpha \end{aligned}$$

the condition (3) for $f(z) = \sum_{j=1}^{\infty} \lambda_j f_j(z)$ is satisfied. Thus $f(z) \in TS_p^{\lambda}(\alpha,\beta)$.

Conversely, we suppose that $f(z) \in TS_p^\lambda(\alpha, \beta)$, $f(z) = z - \sum_{j=2}^{\infty} a_j z^j$, $a_j \geq 0$ and we take $\lambda_j = \frac{[1 + (j-1)\lambda]^\beta [2(j-1)\lambda + 1 - \alpha]}{1 - \alpha} a_j \geq 0$, $j = 2, 3, \dots$, with $\lambda_1 = 1 - \sum_{j=2}^{\infty} \lambda_j$ so that $f(z) = \sum_{j=1}^{\infty} \lambda_j f_j$.

Using the condition (3), we obtain

$$\sum_{j=2}^{\infty} \lambda_j = \frac{1}{1-\alpha} \sum_{j=2}^{\infty} [1 + (j-1)\lambda]^\beta [2(j-1)\lambda + 1 - \alpha] a_j < \frac{1}{1-\alpha}(1 - \alpha) = 1$$

so that $1 - \lambda_1 < 1$ or $\lambda_1 > 0$. This completes our proof.

The proof of the second part of the theorem is similar.

Corollary 1 *The extreme points of* $TS_p(\alpha, \beta)$ *are* $f_1(z) = z$ *and* $f_j(z) = z - \frac{1-\alpha}{[1+(j-1)\lambda]^\beta [2(j-1)\lambda+1-\alpha]} z^j$, $j = 2, 3, \dots$ *and the extreme points of* $TV^\lambda(\alpha, \beta)$ *are* $f_1(z) = z$ *and* $f_j(z) = z - \frac{1-\alpha}{[1+(j-1)\lambda]^{\beta+1} [2(j-1)\lambda+1-\alpha]} z^j$, $j = 2, 3, \dots$.

We can obtain growth and distortion results as in the following corollary.

Corollary 2
a) If $f \in TS_p^\lambda(\alpha, \beta)$, $\alpha \in [-1, 1)$ *then*

$$\begin{aligned} r - \frac{1-\alpha}{(1+\lambda)^\beta (2\lambda + 1 - \alpha)} r^2 \leq |f(z)| &\leq r + \frac{1-\alpha}{(1+\lambda)^\beta (2\lambda + 1 - \alpha)} r^2 \\ 1 - \frac{2(1-\alpha)}{(1+\lambda)^\beta (2\lambda + 1 - \alpha)} r \leq |f'(z)| &\leq 1 + \frac{2(1-\alpha)}{(1+\lambda)^\beta (2\lambda + 1 - \alpha)} r, \quad |z| = r \end{aligned}$$

b) If $f \in TV^\lambda(\alpha, \beta)$, $\alpha \in [-1, 1)$ *then*

$$\begin{aligned} r - \frac{1-\alpha}{(1+\lambda)^{\beta+1} (2\lambda + 1 - \alpha)} r^2 \leq |f(z)| &\leq r + \frac{1-\alpha}{(1+\lambda)^{\beta+1} (2\lambda + 1 - \alpha)} r^2 \\ 1 - \frac{2(1-\alpha)}{(1+\lambda)^{\beta+1} (2\lambda + 1 - \alpha)} r \leq |f'(z)| &\leq 1 + \frac{2(1-\alpha)}{(1+\lambda)^{\beta+1} (2\lambda + 1 - \alpha)} r, \quad |z| = r \end{aligned}$$

The results are best possible.

3. Hadamard Product

Definition 4 *For two functions* $f(z), g(z) \in T$, $f(z) = z - \sum_{j=2}^{\infty} a_j z^j$ $(a_j \geq 0, j = 2, 3, \dots)$ *and*
$g(z) = z - \sum_{j=2}^{\infty} b_j z^j$ $(b_j \geq 0, j = 2, 3, \dots)$, *the modified Hadamard product* $f * g$ *is defined by*
$(f * g)(z) = z - \sum_{j=2}^{\infty} a_j b_j z^j$.

Theorem 5 *If* $f(z) = z - \sum_{j=2}^{\infty} a_j z^j \in TS_p^{\lambda}(\alpha, \beta)$ $(a_j \geq 0, j = 2, 3, \dots)$ *and* $g(z) \in T$ *with* $g(z) = z - \sum_{j=2}^{\infty} b_j z^j \in TS_p^{\lambda}(\alpha, \beta)$, $b_j \geq 0, j = 2, 3, \dots$, $\alpha \in [-1, 1)$, $\lambda \geq 0$, $\beta \geq 0$ *then* $f(z) * g(z) \in TS_p^{\lambda}(\alpha, \beta)$.

A similar result holds for $TV^{\lambda}(\alpha, \beta)$.

Proof. We have

$$\sum_{j=1}^{\infty} [1 + (j-1)\lambda]^{\beta} [2(j-1)\lambda + 1 - \alpha] a_j < 1 - \alpha$$

$$\sum_{j=1}^{\infty} [1 + (j-1)\lambda]^{\beta} [2(j-1)\lambda + 1 - \alpha] b_j < 1 - \alpha$$

We know that $f(z) * g(z) = z - \sum_{j=2}^{\infty} a_j b_j z^j$.

From $g(z) \in T$, by using theorem, we have $\sum_{j=2}^{\infty} j b_j \leq 1$ we notice that $b_j < 1$, $j = 2, 3, \dots$.

Thus

$$\sum_{j=2}^{\infty} [1+(j-1)\lambda]^{\beta} [2(j-1)\lambda+1-\alpha] a_j b_j < \sum_{j=2}^{\infty} [1+(j-1)\lambda]^{\beta} [2(j-1)\lambda+1-\alpha] a_j < 1-\alpha.$$

This means that $f(z) * g(z) \in TS_p^{\lambda}(\alpha, \beta)$, $\beta \geq 0$, $\lambda \geq 0$ and $\alpha \in [-1, 1)$.

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