A GENERALIZATION OF THE STABILITY OF EQUILIBRIUM IN A REPEATED GAME

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Abstract. This paper investigates some new theoretical aspects concerning repeated games, which are considered fundamental in dealing with conflict situations. The determination of equilibrium and the study of its stability become more complicated when dealing with repeated conflict situations. The repetition of a game can create new opportunities for solving conflict situations if we consider the past behavior of each player. This paper provides a generalization of the concept of repeated game. In this way, a repeated game with heterogeneous structure is defined. An important issue is that the known theoretical results concerning the repeated game are valid for the repeated game with heterogeneous structure. This new type of game is much closer to the reality of the repeated conflict situations. By modifying the structure of the game, we understand the insertion or the elimination of strategies in/ from the set of strategies, the modifications of the winning functions, or both of the above alternatives. The equilibrium state of a conflict situation cannot hold, in general, for a long period of time. That is why it is necessary to divide this period in intervals of time, taking into consideration the changes in the structural elements of the constituent game.

Keywords: repeated game, equilibrium, stability.

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1.Introduction

This paper concerns some new theoretical aspects of repeated games, which are considered fundamental in dealing with conflict situations. Our objective is to perform a quasi-formalization of the concept of repeated game.

The equilibrium is a notion that characterizes all conflict situations. Although it can be easily defined, it is not always easily discernable. The determination of equilibrium and the study of its stability become more complicated when dealing with repeated conflict situations.

Within the following paragraph the most important theoretical results concerning repeated games will be reviewed. Friedman (1971) shows the existence of payoff vectors, for which players have a higher interest than in the case of the Nash equilibrium. The folk theorem for infinitely repeated games with no discount ($\delta = 1$) was successfully studied by Aumann and Shapeley (1976) and Rubinstein (1979). A complete description of the set of payoff combinations for repeated games was achieved by Abreu (1983), Benoit and Krishna (1985) and Fudenberg and Maskin (1986). A detailed overview of the above results can be found in Van Damme (1991) and Sabourin (1989).

The repeated conflict situations are analyzed within a finite or infinite time horizon. We are mostly interested in the finite horizon repeated conflict situations, considering the finite existence of the players. In order to investigate the consistency of the model, characterized by an increasing number of repetitions, we will approach the infinite horizon conflict situations as well. Repeating a game substantially increases the set of possible strategies because actions, that is, strategies of the stage game, can be made conditional on the observed behavior of the players in previous stages of the game. In this way, some strategy combinations of the stage game that do not represent a Nash equilibrium can be implemented as Nash equilibriums within the repeated game.

Within our approach, a repeated game is not considered a finite or infinite repetition of the same game, entitled the stage game. The stage game can change its structure and, in this way, can become a new game. Most of the repeated conflict situations change the conditions in which they are repeated out of objective reasons. This led us to defining the concept of a repeated game within which the stage game changes its structure in time. Under these conditions, a repeated game does no longer display a homogeneous structure. The heterogeneity becomes a new characteristic of a repeated game.

Two examples, well known in the literature and presented here, will allow us to make a few comments.

2. The quasi-formal specification of a repeated game with heterogeneous structure

It is well known that a repeated game is a special case of a game in extensive form, in which the tree of game consists of the reiteration of the same game, entitled the stage game (Eichberger, 1993). We consider a game in strategic form, $\Gamma = \{I, S_{i\in I}, P_{i\in I}\}$, the stage game of a T times repeated game, where I represents the players, $S_{i\in I}$ is the set of strategies and $P_{i\in I}$ is the set of payoff functions. Theoretically, T can be finite or infinite. However, in the economic practice, a conflict situation cannot repeat infinitely. Following the definition of the repeated game, the stage game, Γ , is repeated a number of finite/ infinite times. This definition looses its justification if during the period $(0, T)$ the structure of the stage game is affected. By modifying the structure of the game, we understand the insertion or the elimination of strategies in/ from the set $S_{i\in I}$, the modification of the winning functions $P_{i\in I}$, or both of the alternatives. Therefore, the following definition must be considered:

Definition 1. *The game with modified structure,*$\Gamma_1 = \{I, S^1_{i\in I}, P^1_{i\in I}\}$ *, is a game resulting from modifying the structure of the game* $\Gamma = \{I, S_{i\in I}, P_{i\in I}\}$*, where* $S^1_{i\in I}$ *is the actualized set of the strategies, while* $P^1_{i\in I}$ *is the set of the modified payoff functions, obtained from* $P_{i\in I}$*.*

A first remark concerning the quasi-formal description of a repeated game with heterogeneous structure states that the set of stages, T, is divided into subsets, as it follows: a subset T_h will include the t consecutive stages in which the structure of the stage game is not modified. If H subsets result by dividing T, then $T = \cup_{h=1}^{H} T_h$. By using an adequate notation, we can assume that T is made up of the first T natural numbers (T- finite) or that it is the set of natural numbers, (T – infinite). If the initial stage game,Γ, related to a conflict situation, remains unmodified for a given number of stages, then T_1will include those first stages. The game Γ, repeated T_1 times, represents the first constitutive element of a repeated game with heterogeneous structure. This game, repeated T_1 times, is denoted with Γ^{T_1} and represents the first constituent game of a repeated game with heterogeneous structure. We assume that after T_1 stages, the game Γ will turn into the game with modified structure, Γ_1, which is also repeated T_2 times. The game Γ_1 is the second constituent game of the repeated game with heterogeneous structure and is denoted with Γ^{T_2}. In general, we can define the set T_h and the constituent game Γ^{T_h} . We must emphasize the fact that by modifying the structure of a game, as presented above, the essence of the analyzed conflict is not affected.

Definition 2. *A repeated game with heterogeneous structure, denoted by*

Γ_e, *relative to a game* Γ, *can be defined as a finite sequence of constituent games:*

$$\Gamma_e = \left\{ \Gamma^{T_1}, \Gamma^{T_2}, ..., \Gamma^{T_H} \right\}.$$

REMARK 1. *The constituent game,* Γ^{T_h}, *is made starting from the previous constituent game,* $\Gamma^{T_{h-1}}$.

OBSERVATION 2. *A finite number of constituent games of the game* Γ_e *must be imposed, out of practical reasons. This does not affect the case when* T *is infinite. In this situation, we consider that the constituent game,* Γ^{T_H} *(denoted with* Γ^{∞}*), is repeated an infinite number of times.*

According to what was said, a repeated game with heterogeneous structure, Γ_e, is a sequence of constituent games (finite or infinite). In each sequence of stages, T_h, the same unchanged constituent game, Γ^{T_h}, is repeated T_h times.

In a repeated game with heterogeneous structure, Γ_e, in extensive form, the game tree shows, in a visible way, the place of the players in time, as well as the possible strategies they can choose in each decision point. In a game in extensive form, a strategy of a player,$i \in I$, must specify the action choice in each point in which the decision must be made. Each point in which a player makes a decision is identified through a history that goes up to that point. A history until the stage $t \in T$, denoted by h^t(or a t- history) is a sequence of strategy combinations, played until that stage $\left\{ s^1, s^2, ..., s^h \right\}$. We will consider, in the following part, only pure strategy combinations. The results remain the same if the players use mixed strategy combinations, provided that these can be observed. The structure of a t- history is the following:

$$\begin{array}{ll}
s^1 = \left(s_1^1, s_2^1, ..., s_k^1, ..., s_{T_1}^1 \right); & s_k^1 \in S_{T_1}^1 \times S_{T_1}^2 \times ... \times S_{T_1}^I, \\
s^2 = \left(s_1^2, s_2^2, ..., s_k^2, ..., s_{T_2}^2 \right); & s_k^2 \in S_{T_2}^1 \times S_{T_2}^2 \times ... \times S_{T_2}^I, \\
\dots\dots\dots\dots\dots\dots & \dots\dots\dots\dots\dots\dots \\
s^h = \left(s_1^h, s_2^h, ..., s_k^h, ..., s_{T_m}^h \right); & s_k^h \in S_{T_h}^1 \times S_{T_h}^2 \times ... \times S_{T_h}^I,
\end{array}$$

where s_k^h is a strategy combination played by I players in the game Γ^{T_h}. This strategy combination belongs to the I- fold Cartesian product of the sets of all strategy combinations in the constituent game, Γ^{T_h}. Moreover, we consider

$m = \left(t - \sum_{k=1}^{h-1} cardT_k\right) - 1$, where $card\ T_k$ is the number of elements of the set T_k. This means that the t stage belongs to the set T_h. The strategies $s_i \in S^i_{T_h}$ of a player i within the constituent game Γ^{T_h} will be called actions within this game in order to distinguish them from the strategies of the repeated game with heterogeneous structure, denoted by σ_i.

Remark 3. *Even if the structure of a t-history, defined as above, is more complex, the definition of the other elements of a repeated game with heterogeneous structure is almost identical with the classical definition of a repeated game.*

A strategy, σ_i, of each player, $i \in I$, in a repeated game with heterogeneous structure consists of an action that depends on the chosen history. We consider $a^t_i\left(h^t\right) \in S^i_{T_h}$, $h \in H$, the action chosen by player i in stage t, after observing the history h^t. A strategy of a player i, in the repeated game with heterogeneous structure, takes the following form:

$$\sigma_i = \left(a^1_i\left(h^1\right), a^2_i\left(h^2\right), ..., a^T_i\left(h^T\right)\right),$$

for a finite horizon game and

$$\sigma_i = \left(a^1_i\left(h^1\right), a^2_i\left(h^2\right), ...\right)$$

for a infinite horizon game.

A strategy in a repeated game with heterogeneous structure is a sequence of chosen actions that depend on the history up to stage t. An action at stage t, $a^t_i\left(\circ\right)$, is a function that links each history, h^t, with an action from $S^i_{T_k}$, $h \in H$, $i \in I$. The set of possible histories up to stage t is denoted by PH_t. The set of possible actions in stage t is defined as a set of functions, denoted by A^t_i:

$$A^t_i = \left\{a^t_i \left| a^t_i : PH_t \rightarrow S^i_{T_k}\right.\right\}.$$

Choosing a strategy in a repeated game with heterogeneous structure signifies choosing a sequence of functions that specify what a player must do, for each possible history. In a repeated game with heterogeneous structure, the set of strategies of the player i, denoted by Σ_i, is a T- fold Cartesian product of the set of functions A_i^t, that is $\Sigma_i = A_i^1 \times A_i^2 \times ... \times A_i^T$. We denote by the $\Sigma = \Sigma_1 \times \Sigma_2 \times ... \times \Sigma_I$ set of strategy combinations in a repeated game with heterogeneous structure and by $\sigma = (\sigma_1, \sigma_2, ..., \sigma_I)$ an element from this set. A strategy combination, σ, presumes a plan of action for each player in which we specify the action chosen by the player, for any possible history. The set of plans contained in a strategy combination, σ, uniquely determines the sequence of action combinations that is played. Such a sequence of played action combinations is denoted by $\pi(\sigma) = (\pi^1(\sigma), \pi^2(\sigma), ...)$.

In this paragraph, we intend to define the notion of payoff in a repeated game with heterogeneous structure. A natural notion of payoff in a repeated game is the sum of the discounted or undiscounted payoffs of the constituent games. For each constituent game, Γ^{T_h}, we consider a discount factor, $\delta_h \in [0, 1]$.

In a finite horizon game, the average payoff is given by:

$$P_i^T(\sigma) = \left(\sum_{h=1}^{H} \sum_{t=1}^{T_h} \delta_h^{J_t} \right)^{-1} \left(\sum_{h=1}^{H} \sum_{t=1}^{T_h} \delta_h^{J_t} \cdot p_i \left(\pi^{J_t - 1}(\sigma) \right) \right),$$

where $J_t = \left(t + \sum_{k=0}^{h-1} cardT_k \right) - 1$, $T_0 = \Phi$, $card\ T_0 = 0$, and $p_i \left(\pi^{J_t - 1}(\sigma) \right)$ is the payoff for the i player in stage $J_t - 1$, knowing that the chosen strategy is σ. J_t represents a time indexation in order to establish the set T_h, $h \in H$, to which it belongs.

For infinitely repeated games, the following average payoff is used as a winning function:

$$P_i^{T\infty}(\sigma) = P_i^T(\sigma) + P_i^\infty(\sigma),$$

where $P_i^\infty(\sigma) = \left(\sum_{t=l}^{\infty} \delta_H^{t-1} \right)^{-1} \left(\sum_{t=l}^{\infty} \delta_H^{t-1} \cdot p_i(\pi^t(\sigma)) \right)$, for $\delta < 1$, $l = \sum_{h=1}^{H-1} (cardT_h)$ and the constituent game, Γ^{T_H}, is infinitely repeated.

For $\delta = 1$, bounded payoffs of the constituent game, Γ^{T_h}, guarantee, in general, that only some sequences of average payoffs will not diverge. However, it is possible that average payoffs cycle in a bounded range. In this case, the average payoff is defined as the limit point of the smallest converging subsequence that one can select, as it follows:

$$P_i^{T\infty}(\sigma) = \lim_{T \to \infty} \inf \frac{1}{T} \left(\sum_{i=1}^{T} p_i \left(\pi^t(\sigma) \right) \right)$$

for $\delta = 1$.

In the case of a repeated game with heterogeneous structure, Γ_e, the discount factors, $\delta_h \in [0,1]$, $h = \overline{1,H}$, are specific for every subset, T_h. One can analyze the T- times repeated game as a normal form game:

$$\Gamma_e^T(\delta) = (I, \Sigma, P),$$

where Σ is the set of strategies and $P = \left(P_1^T, P_2^T, ..., P_I^T\right)$ is the set of winning functions. Within the following paragraphs, we will present some theoretical results concerning repeated games, which are adapted for the case of repeated games with heterogeneous structure.

Theorem 1. *The necessary condition for a repeated game with heterogeneous structure to display a Nash equilibrium is that every constituent game should display a Nash equilibrium.*

Proof. If s^h is a Nash equilibrium of the constituent game, Γ^{T_h}, and σ^h is a strategy combination of the game, defined by $a_i^t(h^t) = s_i^h$, $t = \overline{1,T}$, $i \in I$, $\delta_h \in [0,1]$, $h = \overline{1,H}$, then σ^h is a Nash equilibrium for $\Gamma_e^T(\delta)$. The rest of the demonstration is the same as in the case of repeated games with homogeneous structure.

According to what was said, a repeated game, in the new suggested sense, is a sequence of constituent games, Γ^{T_h}, (finite or infinite). In each sequence of stages, T_h, the same unchanged constituent game, Γ^{T_h}, is repeated. Now let us return to the existence of other equilibriums in a repeated game with heterogeneous structure. Suppose that all players, except player i, cooperate in punishing the latter. Player i can choose the best response to the strategy combination that the opponents play. Thus, the highest punishment that the opponents can inflict on player i in a constituent game, Γ^{T_h}, $h = \overline{1,H}$, is:

$$w_i^h = \min_{s_{-i}} \max_{s_i} p_i(s_i, s_{-i}).$$

Let us denote by r_{-i}^i the strategy combination that player i' s opponents have to choose for his punishment,

$$r_{-i}^i \in \arg\min_{s_{-i}} \max_{s_i} p_i(s_i, s_{-i}).$$

The punishment w_i^h is the worst payoff that other players can inflict on player i, by choosing the strategy combination r_{-i}^i. The worst punishment that opponents can impose a player, w_i^h, usually exceeds the security level of this player, v_i^h, defined as:

$$v_i^h = \max_{s_i} \min_{s_{-i}} p_i \left(s_i, s_{-i}\right).$$

for a constituent game, Γ^{T_h},$h = \overline{1, H}$.

By choosing a best response strategy to the punishment strategy combination, r_{-i}^i, in each stage of the constituent game Γ^{T_h}, player i will obtain an average payoff of w_i^h in the repeated game Γ^{T_h}. It is impossible to achieve a payoff combination in the repeated game Γ^{T_h} that does not give each player an average payoff of at least w_i^h. The set of all feasible payoff combinations, strictly greater than $w_h = \left(w_1^h, w_2^h, ..., w_I^h\right)$, is a potential outcome in a repeated game, Γ^{T_h}. This reasoning is valid for each constituent game, Γ^{T_h}, $h = \overline{1, H}$. Each constituent game is a game repeated a finite/ infinite number of times. For a constituent game,Γ^{T_h}, we denote by $P\left(\Gamma^{T_h}\right)$ the set of all feasible and individually rational payoff vectors, for each constituent game:

$$P\left(\Gamma^{T_h}\right) = \left\{\left(p_1\left(s\right), ..., p_I\left(s\right)\right) \middle| s \in S_{T_h}^1 \times ... \times S_{T_h}^I, p_i\left(s\right) > w_i^h, i \in I\right\},$$

for h=$\overline{1, H}$.

The Folk Theorem demonstrates that virtually any payoff vector that gives each player more than his worst punishment payoff, can be considered as a Nash equilibrium of an infinitely repeated game, provided that players are sufficiently patient, that is, that their discount factor exceeds a critical value, δ_0. The suggested generalization is perfectly consistent with the Folk Theorem. This means that there must be an h, $1 \leq h \leq H$, that makes Γ^{T_h} to repeat infinitely. Only when these conditions are respected, does the Folk theorem remain valid.

Theorem 2. *(Folk Theorem). For the constituent game,*Γ^{T_h} *, and any individual rational payoff vector,* $p \in P\left(\Gamma^{T_h}\right)$*, there exists* $\delta^0 \in (0, 1)$*, so that, for any* $\delta \geq \delta^0$*,* $\Gamma_e^{\infty}\left(\delta\right)$ *has a Nash equilibrium,* σ^**, so that* $P_i^{\infty}\left(\sigma^*\right) = p_i$*, for all* $i \in I$*, holds.*

Proof. We denoted by $\Gamma_e^{\infty}\left(\delta\right)$ the normal form game that is associated to the infinitely repeated constituent game, Γ^{T_h}. Therefore, the demonstration of this theorem is the same with (Eichberger, 1993).

Remark 4. *A repeated game with heterogeneous structure displays at least one equilibrium, if each constituent game displays at least one equilibrium. In every constituent game, one can virtually choose a more convenient equilibrium than the Nash equilibrium, provided that it exists.*

In contrast to infinitely repeated games with homogeneous structure, the Nash equilibrium is often the only equilibrium of a finitely repeated game with heterogeneous structure. In fact, the following result can be proven:

Theorem 3.
(1) If, for each constituent game, Γ^{T_h}, $h = \overline{1, H}$, *there is a unique Nash equilibrium,* s^h, *such that* $p_i\left(s^h\right) = w_i^h$, *for all* $i \in I$, *holds, then the only payoff vector that can be obtained as a Nash equilibrium in* $\Gamma_e^{\infty}(\delta)$ *is:*

$$\left\{\left(p_1\left(s^1\right), p_2\left(s^1\right), ..., p_I\left(s^1\right)\right); ...; \left(p_1\left(s^H\right), p_2\left(s^H\right), ..., p_I\left(s^H\right)\right)\right\}.$$

(2) If, for a constituent game, Γ^{T_h}, *there is a Nash equilibrium,* s^h, *so that* $p_i\left(s^h\right) > w_i^h$, *for all* $i \in I$, *holds, then any payoff vector in* $P\left(\Gamma^{T_h}\right)$ *can be obtained as a Nash equilibrium of the game* $\Gamma^{T_h}(\delta)$, *for* δ *close to one card* T_h *and sufficiently high.*

Proof. We denoted by $\Gamma^{T_h}(\delta)$ the constituent game, Γ^{T_h}, characterized by a discount factor, δ, and *card* T_h sufficiently high. $\Gamma^{T_h}(\delta)$ from (2) is a particular case of repeated game with homogeneous structure. In this way, the proofs of these results are analogous to the ones in Van Damme (1991).

Two examples, well-known in the literature and presented here, will allow us to make a few comments.

Example 1. Let us consider a matrix of the well known game, called the "prisoner's dilemma":

		Player 2	
Player 1		N	C
	N	2, 2	-1, 4
	C	4, -1	0, 0

By using the classical notations, we can immediately notice that there is only one Nash equilibrium (C,C) based on a dominant strategy. This strategy leads to much smaller winnings for both players than the (N,N) strategy. In the case of a repeated prisoner's game, the (C,C) strategy is no longer the best solution for the players. The (N,N) strategy is more profitable for the two of them. However, their tendency to deviate from it is quite large because there is a chance of doubling the winnings. Let us assume that this game is repeated three times. In this repeated game, the Nash equilibrium payoff coincides with the worst punishment, $w_i^1 = 0$. For the first constituent game, Γ^{T_1}, $T_1 = \{1, 2, 3\}$, $card\ T_1 = 3$, the set of feasible and individually rational payoffs consists of just one element, $P\left(\Gamma^{T_1}\right) = \{(2, 2)\}$.

In the next two stages, we assume the matrix of the game can take the following form:

	Player 2		
Player 1		N	C
	N	3, 3	-1, 4
	C	4, -1	0, 0

Let us define Γ^{T_2}, which differs from Γ^{T_1} only by the payoff vector that corresponds to the strategy (N,N). In this case, $T_2 = \{4, 5\}$, $card\ T_2 = 2$ and $P\left(\Gamma^{T_2}\right) = \{(3, 3)\}$. If the game is finished, the following repeated game with heterogeneous structure is obtained: $\Gamma_e = \left\{\Gamma^{T_1}, \Gamma^{T_2}\right\}$. We can easily notice that the set of strategies, $\{(C, C), (C, C)\}$ represent a Nash equilibrium for the game . The strategy (C, C) is a Nash equilibrium for the constituent games and .

Example 2. The practical issue of equilibrium stability is more obvious in the "Cournot duopoly". A few remarks must be made here. Let us consider two firms producing the same product with marginal costs, c. The maximum price that can be obtained on the market depends on the total quantity that can be sold, $q = q_1 + q_2$. Consequently, each producer's profit depends on the quantity produced by him, but also on the production of the competitor. Let us assume the inverse demand function to be $p_1\left(q_1, q_2\right) = a_1 + b_1\left(q_1 + q_2\right)$,

$a_1 > c_1 > 0$ and $b_1 > 0$. The profit function for both the firms has the following expression:

$$\pi_i (q_1, q_2) = (a_1 - b_1 (q_1 + q_2)) q_i - c_1 q_i, i = \overline{1, 2}.$$

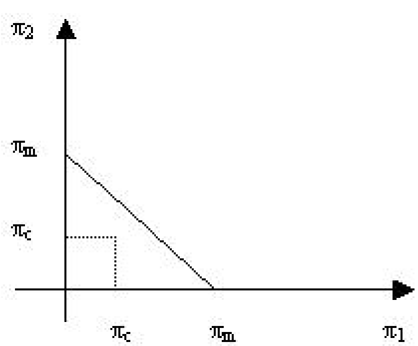
The best choice is based on the best response function:

$$r_i (q_j) = (a_1 - c_1) / b_1 - q_j / 2, i \neq j.$$

The Nash equilibrium strategy, (q_1^1, q_2^1), is at the intersection of the two functions, $r_1 (q_2)$ and $r_2 (q_1)$. Let us assume that this competition has three consecutive stages, during which the requirements concerning the demand do not change. After that, there are four stages that are characterized by a different demand function. The new Nash equilibrium is (q_1^2, q_2^2), as a result of the changes in the marginal cost, c_2, as well as in the demand function:

$$p (q_1, q_2) = a_2 + b_2 (q_1 - q_2), a_2 > c_2 > 0, b_2 > 0.$$

Consequently, the first constituent game, Γ^{T_1}, corresponds to the first three stages of the competition, $T_1 = \{1, 2, 3\}$, $card\ T_1 = 3$. The maximal joint profit equals the monopoly profit, denoted by π_m. Producing the monopoly output in different proportions allows the duopolies to share this profit in any desired proportion, without affecting the total profit. Other feasible output combinations yield profit combinations below the line from $(0, \pi_m)$ to $(\pi_m, 0)$:



The Nash equilibrium gives both firms the same positive profit, π_c. However, the total profit of the duopolies is lower than the monopoly profit,

$\pi_c + \pi_c < \pi_m$. By increasing its output to the quantity for which the price equals the marginal cost, each producer can force the other producer's profits to drop to zero. Thus, $\omega_i^1 = 0$ is the worst punishment for each producer. The diagram shows the set $P\left(\Gamma^{T_1}\right)$ in the case of the duopoly game (the triangular aria).

In the next four stages, the elements of the triangle within the above diagram change. The new Nash equilibrium, (q_1^2, q_2^2), is determined by the new demand function of the new considered period. In the same way, the components of the constituent game Γ^{T_2} are established. In this case, the change in the structure of the game consists of the modification of both the strategy set and the payoff vector. The repeated game does not display a homogeneous structure. The new repeated game with heterogeneous structure Γ_e^T is more adequate in modeling a repeated conflict situation.

3.Conclusions

The analysis of the stability of the equilibrium has a great theoretical and practical importance in the evolution of conflict situations in general and in the evolution of the economic situations, in particular. Finding the logical and favorable solutions for the partners involved in these situations represents an objective for the scientists in the field. The theoretical and methodological substantiation, which determines the solutions for the conflict situations, is well presented in Varien (1992), Kreps (1990), Shubik (1991). Finally, there are some conclusions to be drawn out of everything presented until now:

- the equilibrium state of a conflict situation cannot resist, in general, for a long period of time. That is why it is necessary to divide this period in intervals of time, taking in consideration the changes in the structural elements of the related component game.

- the generalization suggested in this thesis, concerning the repeated games, assures a more realistic and pragmatic vision over the evolution of conflict situation equilibrium.

- a more suitable theoretical development of repeated games with heterogeneous structure assures the fundamental elements that are necessary to elaborate the theory of interactive decisions.

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