THE BEST FINANCIAL ADMINISTRATION OF PRODUCT'S STOCKS OR OF THE CASH MONEY STOCKS IN PROBABILISTIC CONDITIONS

by
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Abstract. The problem for the best financial administration of stocks or of the available assets from a certain product, any kind of its nature is, may be put in any domain of activity also for the fact that any kind of economic activity means finally money, straightly or indirectly, in a natural way it is in fact all about the financial administration of some money stocks. Except this general case of financial administration of money stocks appears also the concrete problem of the best financial administration in case of the cash money (or a ready money stocks).
Keywords: financial administration, product's stocks, cash money stocks

1. Expressing the problem

Let us suppose as known some important elements of some problem of financial administration of stocks such as: necessary for the product or the asset that we consider, expenses of re-supply or diverse transactions, expenses of stocking or stocks immobilization, the stock standard, the demanded standard, rhythm of re-supply, etc.

Next, we consider that on a certain period of time, the demanded of the product studded it is such a random variable X:

$$X: \begin{pmatrix} x \\ p(x) \end{pmatrix}; \ x \geq 0 \qquad (1)$$

such that in a discret case x $\in$ N and $\sum_x p(x) = 1$, and in the continuu case x $\in$ $[0,\infty)$ and $\int_0^\infty p(x)dx = 1$. Let be s the stock level of considered product. We can have one of this situations:

1) X $\leq$ s, and it means that the stock cover the demand and it will remain (s-x) unites in the stock which attract some expenses of stocking (of immobilization or over supply);

2) X > s, which means that the stock does cover anymore the demand and because of that (x-s) unites required can't be honored and that goes to certain expenses of shortcoming (or under supply).

Each one of the two situations brings to certain expenses which can not be analyzed separately (over stocking separately of under stocking) because of the random character of the demand. We have to define a total medium cost of financial administration, which we can note it with C(s), and the problem that appears now is how we can determinate the best standard level s_0 of the stock for which total medium cost of the financial administration should be minimum, meaning

$$C(s_0) = \min_s C(s) \quad (2)$$

concluding in this way a case of the general problem (1). Defining or constructing the function total medium cost of the financial administration depends on the way in which the financial administration of the considered product was conceived or with other words depends on the demand and the supply of the product that was analyzed.

2. Model with random demand, unit cost of penalization for over stocking, unit cost of penalization for shortcoming and neglect stock cost

In the conditions of this model defining the objective function total medium cost of financial administration consider only the unit cost of penalization for over stocking (or for surplus of stock), which we will note with c_1 and the unit cost of penalization for shortcoming (or absence) in stock (or under stocking) which we will note with c_2, untaken in calculus neither time of the stocking operation and neither proper stocking expenses.

Let us note with Q(s) the random variable cost of penalization which we define it thus:

$$Q(s) = \begin{cases} \begin{pmatrix} (s-x)c_1 \\ p(x) \end{pmatrix}, & if \quad 0 \le x \le s \\ \begin{pmatrix} (x-s)c_2 \\ p(x) \end{pmatrix}, & if \quad s \langle x \langle \infty \end{cases} \quad (3)$$

We can talk about the total medium cost of financial administration of the stock as being the medium value of the random variable Q(s) and which we will note with C(s). We easily deduct that in the conditions of this model of financial administration considered by us, total medium cost of stock's financial administration is:

$$C(s) = M[Q(s)] = c_1 \sum_{x=0}^{s} (s-x)\,p(x) + c_2 \sum_{x=s+1}^{\infty} (x-s)\,p(x) \qquad (4)$$

if the X demand is a random discrete variable, or

$$C(s) = M[Q(s)] = c_1 \int_0^s (s-x)\,p(x)dx + c_2 \int_s^{\infty} (x-s)\,p(x)dx \qquad (5)$$

if the X demand is a random continuous variable.

We specify that M [Q(s)] represents the medium value of the random variable Q(s).

Definition 1

If the X demand is a random discrete variable then we will say that s_0 minimize the total medium cost of stock's financial administration if

$$C(s_0-1) \geq C(s_0) \leq C(s_0+1) \qquad (6)$$

Proposition 1

If the X demand is a random discrete variable then the best level of stock s_0 which minimize the total medium cost of stock's financial administration C(s) from (4) is the solution of the inequation

$$P(X \leq s_0-1) \leq \frac{c_2}{c_1+c_2} \leq P(X \leq s_0) \qquad (7)$$

where $P(X \leq a)$ is the probability as X could take smaller or at most equal values with a.

Demonstration:

From relation (4) we deduct that:

$$C(s+1) = c_1 \sum_{x=0}^{s+1} (s+1-x)\,p(x) + c_2 \sum_{x=s+2}^{\infty} (x-s-1)\,p(x)$$

and making the necessary calculations, we have

$$C(s+1) = C(s) + (c_1+c_2)\,P(X \leq s) - c_2$$

from which, replacing on s through (s-1), we obtain this relation:

$$C(s-1) = C(s) - (c_1+c_2)\,P(X \leq s-1) + c_2$$

Seeing the inequalities (6) and the expressions C(s-1), C(s) and C(s+1) from above, we easily deduct the relation (7) and the sentence is demonstrated.

Observation

Taking in consideration the repartition function of the random variable X demand defined through equality $F(x) = P(X < x)$, the relation (7) could be presented also in this form:

$$F(s_0) \leq \frac{c_2}{c_1 + c_2} \leq F(s_0+1) \tag{8}$$

Observation

If the inequalities (7) are strictly, then the problem has the best unique solution. If still:

1) $P(X \leq s_0-1) = \frac{c_2}{c_1 + c_2} < P(X \leq s_0) \Rightarrow$ even s_0 and still (s_0-1) are the best solutions;

2) $P(X \leq s_0-1) < \frac{c_2}{c_1 + c_2} = P(X \leq s_0) \Rightarrow$ even s_0 and still (s_0+1) are the best solutions.

Also we can observe, taking in consideration the monotony of the repartition function, that we can not have double equality simultaneous in relation (7).

As a result of the proposition 1 we can easily deduct:

Proposition 2

In the conditions of proposition 1,

1) if c_1 and s_0 are fixed, then c_2 can take the values

$$\frac{p}{1-p} c_1 \leq c_2 \leq \frac{q}{1-q} c_1 \tag{9}$$

2) if c_2 and s_0 are fixed, then c_1 can take the values

$$\frac{1-q}{q} c_2 \leq c_1 \leq \frac{1-p}{p} c_2 \tag{10}$$

where $p = P(X \leq s_0-1) = F(s_0)$ and $q = P(X \leq s_0) = F(s_0+1)$.

Without a demonstration, we try to present an important result concerning derivation of some integrals with parameters:

Proposition 3

Let be $f(x,t)$ the continuous function on the rectangle $[a,b]\times[c,d]$, u and v two derivable functions on $[c,d]$ with values in $[a,b]$. In these conditions, if f function admits on $[a,b]\times[c,d]$ the continuous partial derivate compared with t, then the function

$$F(t) = \int_{u(t)}^{v(t)} f(x,t)dx \qquad (11)$$

is derivable on [c,d] and we have the relation:

$$F'(t) = \int_{u(t)}^{v(t)} \frac{\partial f(x,t)}{\partial t}dx + f(v(t),t)v'(t) - f(u(t),t)u'(t) \qquad (12)$$

for $\forall$ $t\in[c,d]$.

Proposition 4

If the demand X is a random continuous variable then the best level of stock s_0 which minimize the total medium cost of financial administration C(s) given by (5) is the equation's solution:

$$\int_0^{s_0} p(x)dx = \frac{c_2}{c_1 + c_2} \qquad (13)$$

Demonstration

From relation (5), taking in consideration (12), we deduct that:

$$C'(s) = (c_1+c_2)\int_0^{s} p(x)dx - c_2$$

and as a conclusion, the determination condition for the stationary points of the total medium cost function C(s) given by the equation $C'(s) = 0$ involve the relation (13). Also, because for $\forall$ $s \geq 0$ we have the relation

$$C''(s) = (c_1+c_2)p(s) > 0,$$

results that the stationary point given by (13) equation minimize the total medium cost of financial administration and the proposition 4 is demonstrated also.

Observation

From (13) relation can be determined the level of one of the penalization cost when the other cost is fixed (with one value or with an interval of variations) and the best stock has a given value (or fixed in certain conditions).

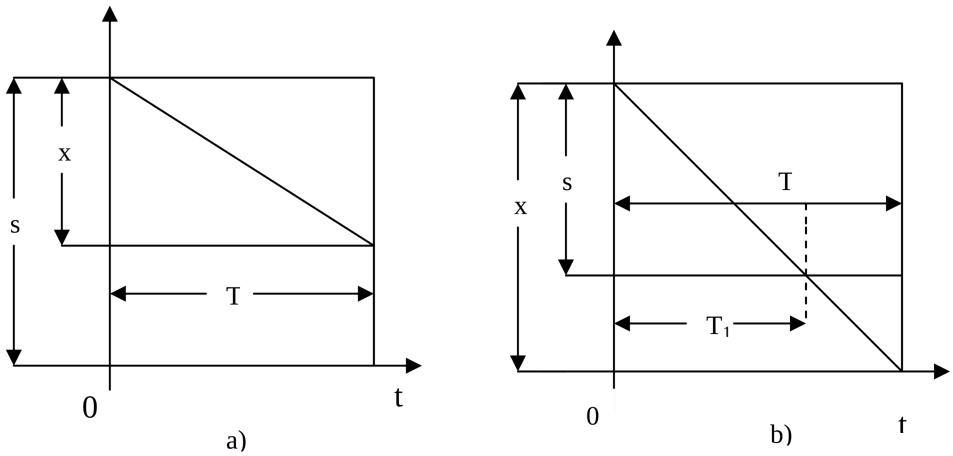
3. Model with random demand, unit cost of stocking and unit cost of penalization for shortcoming

Let suppose that it is all about stocking one product for a T period of time, for which the demand is also a random variable, discrete or continuous one. Making a comparison with the first model, on this time we take in consideration the unit cost of stocking c_s and the unit cost of penalization for the shortcoming c_p (both expressed in monetary unities on product unity and time unity). We can have two situations:

1) the demand do not surpass the stock, that means that there are only stocking expenses;

2) the demand surpass the stock, that means that involve stocking expenses on $T_1 < T$ period of un-surpassing stock and even penalization expenses for shortcoming $T_2 = T - T_1$ when the demand surpass the stock.

The two situations can be illustrated this way:



Observation

The two figures above illustrate the assumption of the linear decrease of product's stock considered and we deduct from this assumption that:

1) if the demand do not surpass the stock, then the medium demand on T period is equal to $\frac{x}{2}$ which goes to a medium stock equal to (s $-\frac{x}{2}$) which can give stocking expenses equal to $c_s \cdot T \cdot (s - \frac{x}{2})$;

2) if the demand surpass the stock, then:

a) on T_1 period we can talk about a medium stock equal to $\frac{s}{2}$ which goes to stocking expenses equal to $c_s \cdot T_1 \cdot \frac{s}{2}$;

b) on $T_2 = T - T_1$ period, we can talk about a medium shortcoming equal to $\left(\frac{x-s}{2}\right)$ which goes to the shortcoming expenses equal to $c_p \cdot T_2 \cdot \left(\frac{x-s}{2}\right)$.

Simulation of some obvious triangles from b) figure goes to these relations:

$$\frac{s}{x} = \frac{T_1}{T} \quad \text{and} \quad \frac{x-s}{x} = \frac{T_2}{T} \quad \text{or} \quad T_1 = \frac{s}{x}T \quad \text{and} \quad T_2 = \frac{x-s}{x}T$$

(14)

and, in conclusion, if the demand surpass the stock then on T_1 period appear the stocking expenses $c_s \cdot \frac{s^2}{2x} \cdot T$, and on T_2 period appear the shortcoming expenses $c_p \cdot \frac{(x-s)^2}{2x} \cdot T$.

With all these we can define the random variable, cost of stock's financial administration on unit time, Q(s), in this way:

$$Q(s) = \begin{cases} \begin{pmatrix} \left(s - \frac{x}{2}\right)c_s \\ p(x) \end{pmatrix}, & if \quad 0 \le x \le s \\ \begin{pmatrix} \frac{s^2}{2x}c_s + \frac{(x-s)^2}{2x}c_p \\ p(x) \end{pmatrix}, & if \quad s \le x \le \infty \end{cases} \tag{15}$$

and in conclusion we will obtain the total medium cost of stock's financial administration on unit time as medium value of Q(s) meaning C(s) = M[Q(s)] having:

$$C(s) = c_s \sum_{x=0}^{s}\left(s - \frac{x}{2}\right)p(x) + \frac{s^2}{2}c_s \sum_{x=s+1}^{\infty}\frac{p(x)}{x} + \frac{c_p}{2}\sum_{x=s+1}^{\infty}\frac{(x-s)^2}{x}p(x) \quad (16)$$

if the demand is a random discrete variable, and respective

$$C(s) = c_s \int_0^s \left(s - \frac{x}{2}\right)p(x)dx + \frac{s^2}{2}c_s \int_s^{\infty}\frac{p(x)}{x}dx + \frac{c_p}{2}\int_s^{\infty}\frac{(x-s)^2}{x}p(x)dx \quad (17)$$

if the demand is a random continue variable.

Observation

Like in the precedent model, this one also follows to minimizing the total medium cost of stock's financial administration.

Proposition 5

If the demand X is a random discrete variable, then the best level of stock s_0 which minimize the total medium cost of stock's financial administration C(s) given by relation (16) is the inequation solution:

$$L(s_0-1) \leq \frac{c_p}{c_s + c_p} \leq L(s_0) \quad (18)$$

where

$$L(s) = \sum_{x=0}^{s} p(x) + \left(s + \frac{1}{2}\right)\sum_{x=s+1}^{\infty}\frac{p(x)}{x} \quad (19)$$

Demonstration

Taking in consideration the relation (16) and making the necessary calculations we can easily deduct that:

$C(s+1) = C(s) + (c_p+c_s)L(s) – c_p$

relation from which, replacing on s with (s-1), result the equality:

$C(s) = C(s-1) + (c_s+c_p)L(s-1) – c_p$.

Imposing (6) condition result the double inequation (18) and the sentence is demonstrated.

Observation

The total medium cost on unit time is given by relation (16) and through the multiplication of the relation with T we can deduct the minimum total medium cost on all T time period.

Analogy to proposition 2 we easily deduct:

Proposition 6

In proposition 5 conditions:

1) if c_s and s_0 are fixed, then

$$c_s \frac{L(s_0 - 1)}{1 - L(s_0 - 1)} \le c_p \le c_s \frac{L(s_0)}{1 - L(s_0)} \tag{20}$$

2) if c_p and s_0 are fixed, then

$$c_p \frac{1 - L(s_0)}{L(s_0)} \le c_s \le c_p \frac{1 - L(s_0 - 1)}{L(s_0 - 1)} \tag{21}$$

Proposition 7

If the demand X is a random continuous variable then the best level of stock s_0 which minimize the total medium cost of stock's financial administration C(s) given by (17) is the equation solution:

$$L(s_0) = \frac{c_p}{c_s + c_p} \tag{22}$$

where

$$L(s) = \int_o^s p(x)dx + s\int_s^\infty \frac{p(x)}{x} dx \tag{23}$$

Demonstration

Taking in consideration the proposition (3) we immediately can deduct the relation:

$$C'(s) = c_s \int_o^s p(x)dx + (c_s + c_p)s\int_s^\infty \frac{p(x)}{x} dx + c_p \int_0^s p(x)dx - c_p$$

and, in conclusion, the equation $C'(s) = 0$, through which we determine the stationary points of C(s) function, goes to the relation (22). On the other side, we deduct also that

$$C''(s) = (c_s+c_p)\int_s^\infty \frac{p(x)}{x} dx > 0, \text{ for } \forall\ s \ge 0$$

and, in conclusion, the stationary point obtained as a solution of (22) equation minimizes the total medium cost of stock's financial administration and demonstration is closed.

Observation

The (22) relation allow the determination to the one of the unit costs (of stocking or of penalization for shortcoming) when the other cost as well as the best level of the stock are given (fixed) in a certain way.

4. Applications in the financial administration of cash money

The models of stock's financial administration with random demand previously presented had a general character in comparison to the product, the asset or the service which are referring to. In a particular case, the asset or the product administrated can be money and then the stock notion is unfit, even that this notion could be correctly understand. From this reason it is more adequate to talk about the financial administration of cash money or ready money as well as this operation has a special importance in the bank's current activity, in the insurance society's activity as well as other diverse commercial societies.

When we talk about the administration of cash money, the two models of financial administration of stocks presented above have the next significations:

1) the demand of liquid capital for diverse payments can be a random discrete or continuous variable;
2) the penalization cost for over stocking c_1 from (4) can be assimilate with a penalization or a lost caused by immobilizing a cash money stock bigger that will be necessary at a certain moment or for a certain period of time;
3) the penalization cost for under stocking c_2 from (4) can be assimilate with a penalization or a lost caused by the absence of the cash money from a bank, insurance society or commercial society considered at that moment when this should effect some payments to its customers;
4) the unit cost of stocking c_s from (16) can be assimilate with the annual rate of the interest, the interest for the liquid stock immobilized being a lost (equivalent with a pay) for the one that own the cash money;
5) the unit cost of penalization for shortcoming c_p from (16), can be assimilate with the typical penalizations of the cases of

impossibility to effect some payments, from absence of money (increasing interests and other penalizations).

Specifying all these and with the practical and adequate interpretations of the solutions, the financial administration's models previously presented are applied without any modification for the best financial administration of cash money with sporadic demand.

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