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Sequential closures of σ-subalgebras for a vector measure

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Abstract: Let X be a locally convex space, $m : \Sigma \to X$ be a vector measure defined on a σ-algebra Σ, and $L^1(m)$ be the associated (locally convex) space of m-integrable functions. Let $\Sigma(m)$ denote $\{\chi_E; E \in \Sigma\}$, equipped with the relative topology from $L^1(m)$. For a subalgebra $\mathcal{A} \subseteq \Sigma$, let $\mathcal{A}_\sigma$ denote the generated σ-algebra and $\overline{\mathcal{A}}_s$ denote the sequential closure of $\chi(\mathcal{A}) = \{\chi_E; E \in \mathcal{A}\}$ in $L^1(m)$. Sets of the form $\overline{\mathcal{A}}_s$ arise in criteria determining separability of $L^1(m)$; see [6]. We consider some natural questions concerning $\overline{\mathcal{A}}_s$ and, in particular, its relation to $\chi(\mathcal{A}_\sigma)$. It is shown that $\overline{\mathcal{A}}_s \subseteq \Sigma(m)$ and moreover, that $\{E \in \Sigma; \chi_E \in \overline{\mathcal{A}}_s\}$ is always a σ-algebra and contains $\mathcal{A}_\sigma$. Some properties of X are determined which ensure that $\chi(\mathcal{A}_\sigma) = \overline{\mathcal{A}}_s$, for any X-valued measure m and subalgebra $\mathcal{A} \subseteq \Sigma$; the class of such spaces X turns out to be quite extensive.

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