NUMERICAL BLOW-UP SOLUTIONS FOR SOME SEMILINEAR HEAT EQUATIONS∗

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Abstract. This paper concerns the study of the numerical approximation for the following initial-boundary value problem,

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\begin{align*}
    u_t &= u_{xx} + \frac{b}{x}u_x + u^p, & x \in (0, 1), & t \in (0, T), \\
    u_x(0, t) &= 0, & u(1, t) &= 0, & t \in (0, T), \\
    u(x, 0) &= u_0(x), & x \in [0, 1],
\end{align*}
\]

where \( b > 0 \) and \( p > 1 \). We give some conditions under which the solution of a semidiscrete form of the above problem blows up in a finite time and estimate its semidiscrete blow-up time. Under some assumptions, we also show that the semidiscrete blow-up time converges to the continuous blow-up time when the mesh size goes to zero. Finally, we give some numerical results to illustrate our analysis.

Key words. semidiscretizations, discretizations, semilinear heat equations, semidiscrete blow-up time

AMS subject classifications. 35B40, 35K65, 65M06

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