Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2010, Article ID 189017, 14 pages
doi:10.1155/2010/189017

Research Article

A Production Model for Deteriorating Inventory Items with Production Disruptions

Yong He[1] and Ju He[2]

[1] *School of Economics and Management, Southeast University, Nanjing 210096, China*
[2] *School of Management and Engineering, Nanjing University, Nanjing 210093, China*

Correspondence should be addressed to Yong He, heyong@126.com

Received 27 January 2010; Revised 4 June 2010; Accepted 14 July 2010

Academic Editor: Aura Reggiani

Copyright © 2010 Y. He and J. He. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Disruption management has recently become an active area of research. In this study, an extension is made to consider the fact that some products may deteriorate during storage. A production-inventory model for deteriorating items with production disruptions is developed. Then the optimal production and inventory plans are provided, so that the manufacturer can reduce the loss caused by disruptions. Finally, a numerical example is used to illustrate the model.

1. Introduction

In real life, the effect of decay and deterioration is very important in many inventory systems. In general, deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility, or loss of marginal value of a commodity that results in decreasing usefulness [1]. Most of the physical goods undergo decay or deterioration over time, the examples being medicine, volatile liquids, blood banks, and others. Consequently, the production and inventory problem of deteriorating items has been extensively studied by researchers. Ghare and Schrader [2] were the first to consider ongoing deterioration of inventory with constant demand. As time progressed, several researchers developed inventory models by assuming either instantaneous or finite production with different assumptions on the patterns of deterioration. In this connection, researchers may refer to [3–7]. Interested readers may refer to review [8, 9]. Recently, several related papers were presented, dealing with such inventory problems [10–17].

At the beginning of each cycle, the manufacturer should decide the optimal production time, so that the production quantity should satisfy the following two requirements: one, it should meet demand and deterioration; second, all products should be sold out in each cycle, that is, at the end of each cycle, the inventory level should decrease to zero. Some researchers have studied such production model for deteriorating items under different condition. For example, Yang and Wee [18] derived the optimal production time for a single-vendor, multiple-buyers system. Liao [19] derived a production model for the lot-size inventory system with finite production rate, taking into consideration the effect of decay and the condition of permissible delay in payments. Lee and Hsu [20] developed a two-warehouse inventory model with time-dependent demand. He et al. [21] provided a solution procedure to find the optimal production time under the premise that the manufacturer sells his products in multiple markets. The above papers all assume that production rate is known and keeps constant during each cycle. They do not consider how to adjust the production plan once the production rate is changed during production time.

However, after the plan was implemented, the production run is often disrupted by some emergent events, such as supply disruptions, machine breakdowns, earthquake, H1N1 epidemic, financial crisis, political event and policy change. For example, the Swedish mining company Boliden AB suffered the production disruptions at its Tara zinc mine in Ireland due to an electric motor breakdown at one of the grinding mills. As a result of the breakdown, the production of zinc and lead concentrates is expected to fall by some 40% over the next six weeks [22]. These production disruptions will lead to a hard decision in production and inventory plans. Recently, there is a growing literature on production disruptions. For example, some researchers studied the production rescheduling problems with the machine disruptions [23–26]. Some researchers analyzed the optimal inventory policy with supply disruptions [27–30].

In most of the existing literature, products are assumed to be no deterioration when the production disruptions are considered. But, in real situation the deterioration is popular in many kinds of products. Hence, if the deterioration rate is not small enough, the deterioration factor cannot be ignored when the production system is disrupted.

Therefore, in this paper, we develop a production-inventory model for deteriorating items with production disruptions. Once the production rate is disrupted, the following questions are considered in this paper.

(i) Whether to replenish from spot markets or not?

(ii) How to adjust the production plan if the new production system can still satisfy the demand?

(iii) How to replenish from spot markets if the new production system no longer satisfies the demand?

The paper is organized as follows. Section 2 is concerned with the mathematical development and the method for finding the optimal solutions. In Section 3, we present a numerical example to illustrate the model. In Section 4, conclusions and topics for further research are presented.

2. Mathematical Modeling and Analysis

Suppose a manufacturer produces a certain product and sells it in a market. All items are produced and sold in each cycle. The following assumptions are used to formulate the problem.

(a) A single product and a single manufacturer are assumed.

(b) Demand rate is deterministic and constant.

(c) Normal Production rate is greater than demand rate.

(d) Lead time is assumed to be negligible.

(e) Deterioration rate is deterministic and constant.

(f) Shortages are not allowed.

(g) Time horizon is finite.

(h) There is only one chance to order the products from spot markets during the planning horizon.

Let the basic parameters be as follows:

p: normal production rate,

d: demand rate,

θ: constant deterioration rate of finished products,

H: planning horizon,

T_p: the normal production period without disruptions,

T_d: the production disruptions time,

T_p^d: the new production period with disruptions,

T_r: the replenishment time from spot markets once shortage appears,

Q_r: the order quantity from spot markets once shortage appears,

$I_i(t)$: inventory level in the ith interval ($i = 1, 2, \ldots n$), n can be different in different scenario.

2.1. The Basic Model without Disruptions

At first, the manufacturer makes decisions about the optimal production time T_p under the normal production rate. The inventory model for deteriorating items with normal production rate can be depicted as in Figure 1.

The instantaneous inventory level at any time $t \in [0, H]$ is governed by the following differential equations:

$$\begin{aligned} \frac{dI_1(t)}{dt} + \theta I_1(t) &= p - d, \quad 0 \le t \le T_p, \\ \frac{dI_2(t)}{dt} + \theta I_2(t) &= -d, \quad T_p \le t \le H. \end{aligned} \tag{2.1}$$

Using the boundary condition $I_1(0) = 0$ and $I_2(H) = 0$, the solutions of above differential equations are

$$\begin{aligned} I_1(t) &= \frac{p-d}{\theta}\left(1 - e^{-\theta t}\right), \quad 0 \le t \le T_p, \\ I_2(t) &= \frac{d}{\theta}\left[e^{\theta(H-t)} - 1\right], \quad T_p \le t \le H. \end{aligned} \tag{2.2}$$

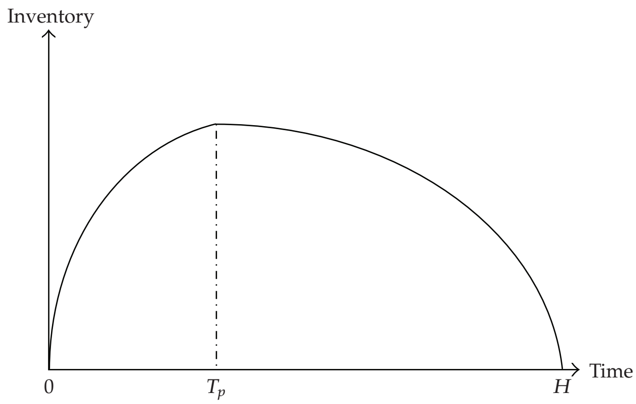

Figure 1: Inventory system without disruptions.

The condition $I_1(T_p) = I_2(T_p)$ yields

$$\frac{p-d}{\theta}\left(1-e^{-\theta T_p}\right) = \frac{d}{\theta}\left[e^{\theta(H-T_p)}-1\right]. \tag{2.3}$$

From (2.3), the production time T_p satisfies the following equation:

$$T_p = \frac{1}{\theta}\ln\frac{p-d+de^{\theta H}}{p}. \tag{2.4}$$

In order to facilitate analysis, we do an asymptotic analysis for $I_i(t)$. Expanding the exponential functions and neglecting second and higher power of θ for small value of θ, (2.2) becomes

$$\begin{aligned} I_1(t) &\approx (p-d)\left(t-\frac{1}{2}\theta t^2\right), \quad 0 \le t \le T_p, \\ I_2(t) &\approx d\left[(H-t)+\frac{1}{2}\theta(H-t)^2\right], \quad T_p \le t \le H, \end{aligned} \tag{2.5}$$

and T_p approximately satisfies the equation

$$(p-d)\left(T_p-\frac{1}{2}\theta T_p^2\right) = d\left[(H-T_p)+\frac{1}{2}\theta(H-T_p)^2\right]. \tag{2.6}$$

From Misra [31], we have

$$T_p \approx \frac{d}{p-d}(H-T_p)\left[1+\frac{1}{2}\theta(H-T_p)\right]. \tag{2.7}$$

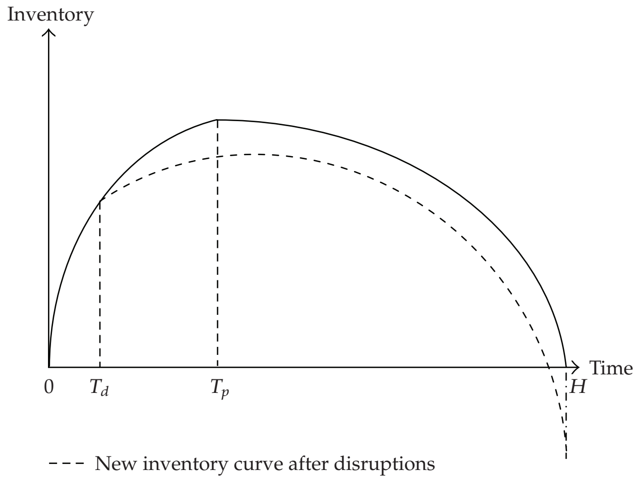

Figure 2: Inventory system with production disruptions.

Since

$$\frac{\mathrm{d}T_p}{\mathrm{d}\theta} = \frac{1}{2}\frac{d\left(H - T_p\right)^2}{p + \theta d\left(H - T_p\right)} > 0, \tag{2.8}$$

we can get the following corollary.

Corollary 2.1. *Assuming that $\theta \ll 1$, then T_p is increasing in θ.*

Corollary 2.1 implies that the manufacturer has to produce more products when deterioration rate increases. Hence, decreasing deterioration rate is an effective way to reduce the product cost of manufacture.

2.2. The Production-Inventory Model under Production Disruptions

In the above model, the production rate is assumed to be deterministic and known. In practice, the production system is often disrupted by various unplanned and unanticipated events. Here, we assume the production disruptions time is T_d. Without loss of generality, we assume that the new disrupted production rate is $p + \Delta p$, where $\Delta p < 0$ if production rate decreases suddenly, or $\Delta p > 0$ if production rate increases.

Proposition 2.2. *If $\Delta p \geq -(p - d)(1 - e^{-\theta H})/(1 - e^{-\theta(H - T_d)})$, then the manufacturer can still satisfy the demand after production disruptions. Otherwise, that is, $-p \leq \Delta p < -(p - d)(1 - e^{-\theta H})/(1 - e^{-\theta(H - T_d)})$, there will exist shortages due to the production disruptions.*

Proof. Without considering the stop time of production or replenishment, the inventory system with production disruptions can be depicted as Figure 2.

From Section 2.1, we know

$$I_1(t) = \frac{p-d}{\theta}\left(1 - e^{-\theta t}\right), \quad 0 \le t \le T_d. \tag{2.9}$$

The inventory system after disruptions can be represented by the following differential equation:

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = p + \Delta p - d, \quad T_d \le t \le H. \tag{2.10}$$

Using $I_1(T_d) = I_2(T_d) = ((p-d)/\theta)(1 - e^{-\theta T_d})$, we have

$$I_2(t) = \frac{1}{\theta}\left[p + \Delta p - d - \Delta p e^{-\theta(t-T_d)} - (p-d)e^{-\theta t}\right], \quad T_d \le t \le H. \tag{2.11}$$

Hence, we know that

$$I_2(H) = \frac{1}{\theta}\left[p + \Delta p - d - \Delta p e^{-\theta(H-T_d)} - (p-d)e^{-\theta H}\right]. \tag{2.12}$$

Hence, if $I_2(H) \ge 0$, that is, $\Delta p \ge -(p-d)(1 - e^{-\theta H})/(1 - e^{-\theta(H-T_d)})$, this means that the manufacturer can still satisfy the demand after production disruptions.

But if $I_2(H) < 0$, that is, $-p \le \Delta p < -(p-d)(1 - e^{-\theta H})/(1 - e^{-\theta(H-T_d)})$, we know that the manufacturer will face shortage since the production rate decreases deeply. The proposition is proved. □

From Proposition 2.2, we know that if $\Delta p \ge -(p-d)(1 - e^{-\theta H})/(1 - e^{-\theta(H-T_d)})$, the production-inventory problem is to find the new optimal production period T_p^d. If $-p \le \Delta p < -(p-d)(1 - e^{-\theta H})/(1 - e^{-\theta(H-T_d)})$, the production-inventory problem is to find the optimal replenishment time T_r and replenishment quantity Q_r.

Proposition 2.3. *If $\Delta p \ge -(p-d)(1 - e^{-\theta H})/(1 - e^{-\theta(H-T_d)})$, then the manufacturer's production time with production disruptions is*

$$T_p^d = \frac{1}{\theta}\ln\frac{p + d\left(e^{\theta H} - 1\right) + \Delta p e^{\theta T_d}}{p + \Delta p}. \tag{2.13}$$

Proof. From Proposition 2.2, we know that the new production time $T_p^d \in [T_d, H]$ if $\Delta p \ge -(p-d)(1 - e^{-\theta H})/(1 - e^{-\theta(H-T_d)})$. The inventory model can be depicted as in Figure 3.

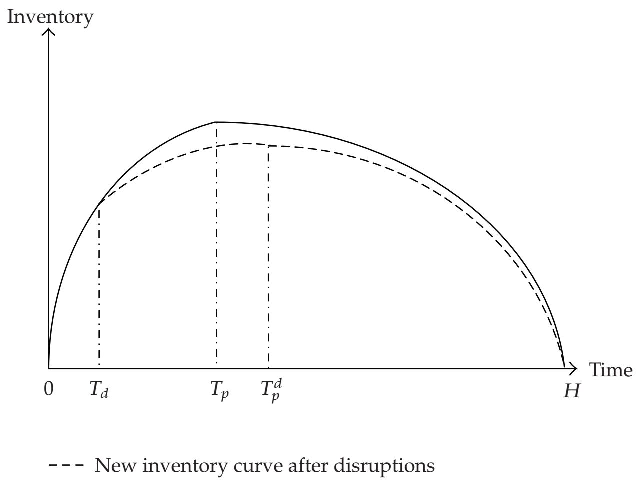

Figure 3: Inventory system ($T_p^d \in [T_d, H]$).

So if $\Delta p \geq -(p-d)(1-e^{-\theta H})/(1-e^{-\theta(H-T_d)})$, the inventory system after disruptions can be represented by the following differential equations:

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = p + \Delta p - d, \quad T_d \leq t \leq T_p^d,$$
$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -d, \quad T_p^d \leq t \leq H. \tag{2.14}$$

Using the boundary conditions $I_1(T_d) = I_2(T_d) = ((p-d)/\theta)(1-e^{-\theta T_d})$ and $I_3(H) = 0$, we know

$$I_2(t) = \frac{1}{\theta}\left[p + \Delta p - d - \Delta p e^{-\theta(t-T_d)} - (p-d)e^{-\theta t}\right], \quad T_d \leq t \leq T_p^d,$$
$$I_3(t) = \frac{d}{\theta}\left[e^{\theta(H-t)} - 1\right], \quad T_p^d \leq t \leq H. \tag{2.15}$$

Using the boundary condition $I_2(T_p^d) = I_3(T_p^d)$, we have

$$T_p^d = \frac{1}{\theta} \ln \frac{p + d\left(e^{\theta H} - 1\right) + \Delta p e^{\theta T_d}}{p + \Delta p}. \tag{2.16}$$

The proposition is proved. □

Since $\mathrm{d}T_p^d/\mathrm{d}T_d = (p+\Delta p)\Delta p e^{\theta T_d}/(p + d(e^{\theta H} - 1) + \Delta p e^{\theta T_d})$, we can easily get Corollary 2.4.

Corollary 2.4. *If* $-(p-d)(1-e^{-\theta H})/(1-e^{-\theta(H-T_d)}) \le \Delta p < 0$, *then* T_p^d *is decreasing in* T_d. *If* $\Delta p > 0$, *then* T_p^d *is increasing in* T_d.

Expanding the exponential functions and neglecting second and higher power of θ for small value of θ, (2.15) becomes

$$\begin{aligned} I_2(t) &\approx \Delta p(t-T_d)\left[1-\frac{1}{2}\theta(t-T_d)\right]+(p-d)t\left(1-\frac{1}{2}\theta t\right), \quad T_d \le t \le T_p^d, \\ I_3(t) &\approx d(H-t)\left[1+\frac{1}{2}\theta(H-t)\right], \quad T_p^d \le t \le H, \end{aligned} \tag{2.17}$$

and T_p^d approximately satisfies the equation

$$\Delta p\left(T_p^d-T_d\right)\left[1-\frac{1}{2}\theta\left(T_p^d-T_d\right)\right]+(p-d)T_p^d\left[1-\frac{1}{2}\theta T_p^d\right]=d\left(H-T_p^d\right)\left[1+\frac{1}{2}\theta\left(H-T_p^d\right)\right]. \tag{2.18}$$

From Misra [31], we have

$$T_p^d \approx \frac{\Delta p T_d + d\left(H-T_p^d\right)\left[1+(1/2)\theta\left(H-T_p^d\right)\right]}{p+\Delta p-d}. \tag{2.19}$$

According to (2.19), we know that

$$\frac{dT_p^d}{d\theta}=\frac{1}{2}\frac{d\left(H-T_p^d\right)^2}{p+\Delta p+d\theta\left(H-T_p^d\right)}>0. \tag{2.20}$$

Hence, we can get the following corollary.

Corollary 2.5. *Assuming that* $\theta \ll 1$, *then* T_p^d *is increasing in* θ.

If $-p \le \Delta p < -(p-d)(1-e^{-\theta H})/(1-e^{-\theta(H-T_d)})$, there will exist shortage. The manufacturer will have to produce products during the whole planning horizon, that is, $T_p^d = H$. In order to avoid shortage, the manufacturer needs to order products from spot markets to satisfy the demand. The inventory model can be depicted as in Figure 4.

Proposition 2.6. *If* $-p \le \Delta p < -(p-d)(1-e^{-\theta H})/(1-e^{-\theta(H-T_d)})$, *then the replenishment time and quantity are*

$$\begin{aligned} T_r &= \frac{1}{\theta}\ln\frac{p-d+\Delta p e^{\theta T_d}}{p+\Delta p-d}, \\ Q_r &= \frac{p+\Delta p-d}{\theta}\left[1-e^{\theta(H-T_r)}\right]. \end{aligned} \tag{2.21}$$

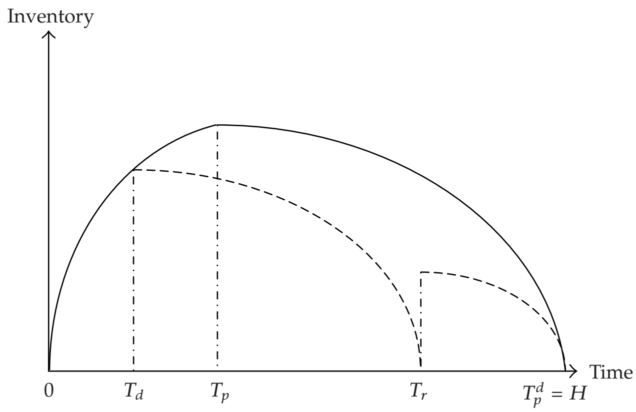

Figure 4: Inventory system ($T_p^d = H$).

Proof. First, we need to determine the order time point T_r. Let $I_2(t) = (1/\theta)[p + \Delta p - d - \Delta p e^{-\theta(t-T_d)} - (p-d)e^{-\theta t}] = 0$, we have

$$T_r = \frac{1}{\theta}\ln\frac{p-d+\Delta p e^{\theta T_d}}{p+\Delta p - d}. \tag{2.22}$$

So,

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = p + \Delta p - d, \quad T_r \le t \le H. \tag{2.23}$$

Using the boundary condition $I_3(H) = 0$, we have

$$I_3(t) = \frac{p+\Delta p - d}{\theta}\left[1 - e^{\theta(H-t)}\right], \quad T_r \le t \le H. \tag{2.24}$$

Hence, the order quantity is

$$Q_r = I_3(T_r) = \frac{p+\Delta p - d}{\theta}\left[1 - e^{\theta(H-T_r)}\right]. \tag{2.25}$$

The proposition is proved. □

If $-p \le \Delta p < -(p-d)(1-e^{-\theta H})/(1-e^{-\theta(H-T_d)})$, according to (2.22), we have

$$\begin{aligned} \frac{\mathrm{d}T_r}{\mathrm{d}T_d} &= \frac{p+\Delta p - d}{p-d+\Delta p e^{\theta T_d}}\Delta p e^{\theta T_d} < 0, \\ \frac{\mathrm{d}Q_r}{\mathrm{d}T_d} &= \left(p+\Delta p - d\right)e^{\theta(H-T_r)}\frac{\mathrm{d}T_r}{\mathrm{d}T_d} < 0. \end{aligned} \tag{2.26}$$

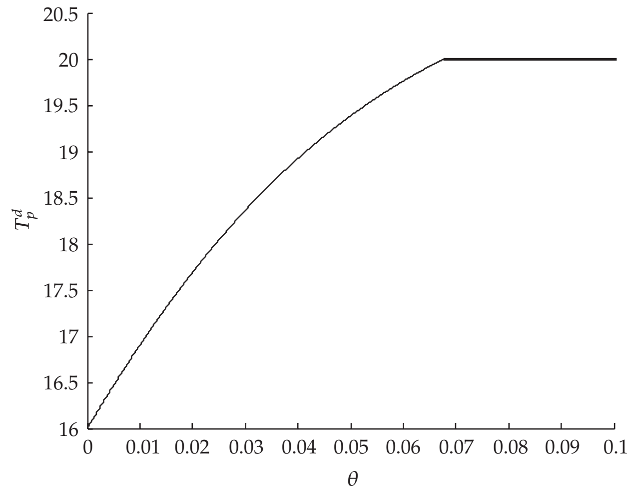

Figure 5: T_p^d with respect to θ.

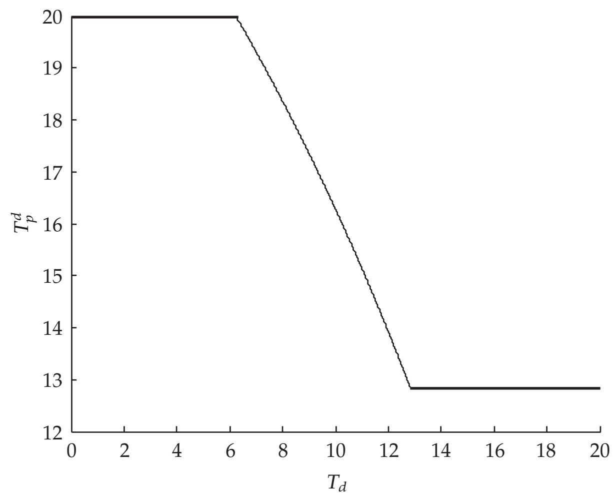

Figure 6: T_p^d with respect to T_d.

Hence, we can obtain the following corollary.

Corollary 2.7. *If* $-p \le \Delta p < -(p-d)(1-e^{-\theta H})/(1-e^{-\theta(H-T_d)})$, *then* T_r *and* Q_r *are decreasing in* T_d.

3. A Numerical Example

Our objective in this section is to gain further insights based on a numerical example. We use the following numbers as the base values of the parameters: $p = 350$, $d = 200$, $\theta = 0.03$, $H = 20$, $T_d = 8$, and $\Delta p = -200$. Using (2.4), we obtain $T_p = 12.8$. Next, we observe how T_p^d, T_r, and Q_r would change as θ and T_d. Figures 5 and 6 depict T_p^d with respect to θ and T_d,

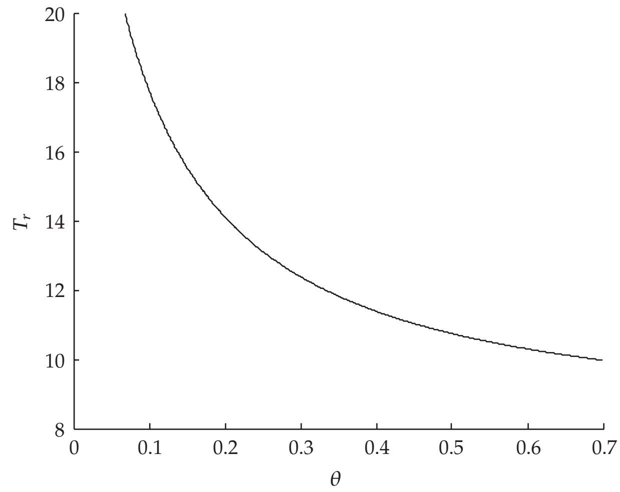

Figure 7: T_r with respect to θ.

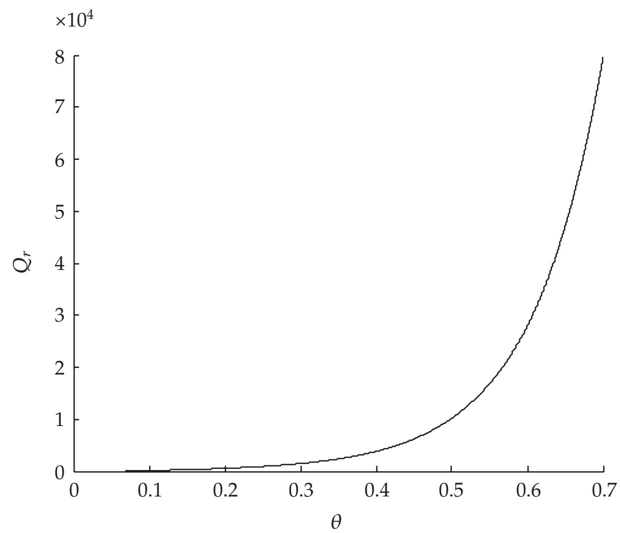

Figure 8: Q_r with respect to θ.

respectively. Figures 7 and 9 depict T_r with respect to θ and T_d, respectively. Figures 8 and 10 depict Q_r with respect to θ and T_d, respectively.

From Figure 5, we can find that T_p^d is increasing in θ when $\theta \leq 0.068$. When $\theta > 0.068$, since the deterioration rate is so high that the manufacturer cannot satisfy the demand by self-producing, he has to buy products from spot markets in order to avoid shortage. From Figures 7 and 8, we can see that T_r is decreasing in θ, and Q_r is increasing in θ when $\theta > 0.068$.

From Figure 6, we can find that T_p^d is decreasing in T_d when $6.2 \leq T_d \leq 12.8$. If $0 \leq T_d < 6.2$, the manufacturer will have to replenish inventory from spot markets. From Figures 9 and 10, we can see that T_r is increasing in T_d, and Q_r is decreasing in θ when $0 \leq T_d < 6.2$.

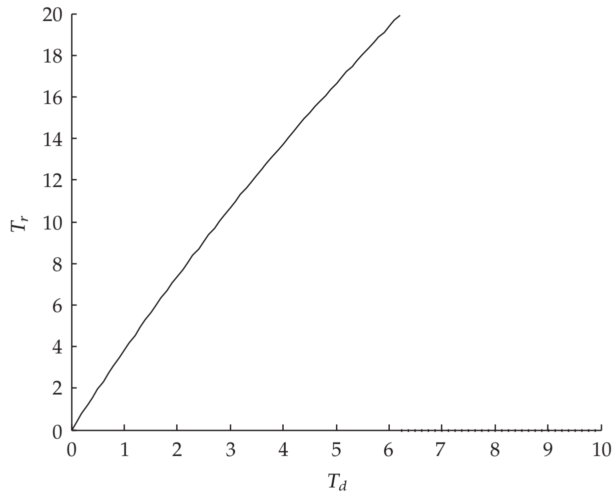

Figure 9: T_r with respect to T_d.

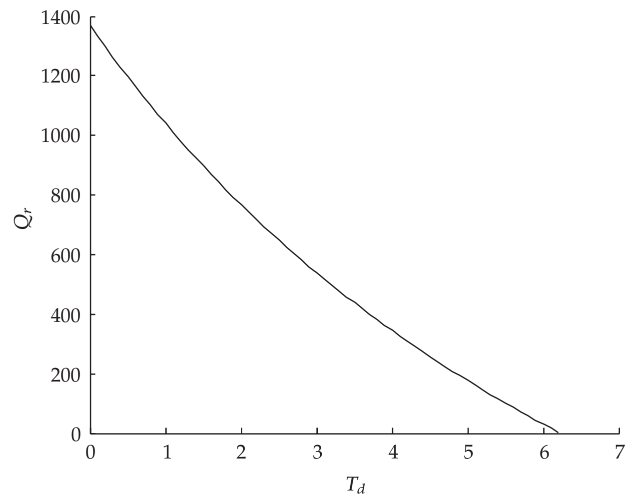

Figure 10: Q_r with respect to T_d.

4. Conclusions

In this paper, we propose a production-inventory model for a deteriorating item with production disruptions. Here, we analyze this inventory system under different situations. We have showed that our method helps the manufacturer reduce the loss caused by production disruptions.

In this study, the proposed model considers the deterioration rate as constant. In real life, we may consider the deterioration rate as a function of time, stock, and so on. This will be done in our future research.

Acknowledgments

The authors thank the valuable comments of the referees for an earlier version of this paper. Their comments have significantly improved the paper. This research is supported by a grant from the Ph.D. Programs Foundation of Ministry of Education of China (no. 200802861030). Also, this research is partly supported by the Project Sponsored by the Scientific Research Foundation for the Returned Overseas Chinese Scholars, the National Natural Science Foundation of China, the China Postdoctoral Science Foundation (no. 20070411043), and the Postdoctoral Foundation of Jiangsu Province of China (no. 0701045C).

References

[1] H.-M. Wee, "Economic production lot size model for deteriorating items with partial back-ordering," *Computers and Industrial Engineering*, vol. 24, no. 3, pp. 449–458, 1993.

[2] P. M. Ghare and S. F. Schrader, "A model for exponentially decaying inventory," *Journal of Industrial Engineering*, vol. 14, no. 5, pp. 238–243, 1963.

[3] R. P. Covert and G. C. Philip, "An EOQ model for items with Weibull distribution deterioration," *AIIE Transactions*, vol. 5, no. 4, pp. 323–326, 1973.

[4] T. Chakrabarty, B. C. Giri, and K. S. Chaudhuri, "An EOQ model for items with weibull distribution deterioration, shortages and trended demand: an extension of philip's model," *Computers and Operations Research*, vol. 25, no. 7-8, pp. 649–657, 1998.

[5] U. Dave, "On a discrete-in-time order-level inventory model for deteriorating items," *Journal of the Operational Research Society*, vol. 30, no. 4, pp. 349–354, 1979.

[6] E. A. Elasayed and C. Teresi, "Analysis of inventory systems with deteriorating items," *International Journal of Production Research*, vol. 21, no. 4, pp. 449–460, 1983.

[7] S. Kang and I. Kim, "A study on the price and production level of the deteriorating inventory system," *International Journal of Production Research*, vol. 21, no. 6, pp. 899–908, 1983.

[8] F. Raafat, "Survey of literature on continuously deteriorating inventory models," *Journal of the Operational Research Society*, vol. 42, no. 1, pp. 27–37, 1991.

[9] S. K. Goyal and B. C. Giri, "Recent trends in modeling of deteriorating inventory," *European Journal of Operational Research*, vol. 134, no. 1, pp. 1–16, 2001.

[10] K.-J. Chung and J.-J. Liao, "The optimal ordering policy in a DCF analysis for deteriorating items when trade credit depends on the order quantity," *International Journal of Production Economics*, vol. 100, no. 1, pp. 116–130, 2006.

[11] A. K. Maity, K. Maity, S. Mondal, and M. Maiti, "A Chebyshev approximation for solving the optimal production inventory problem of deteriorating multi-item," *Mathematical and Computer Modelling*, vol. 45, no. 1-2, pp. 149–161, 2007.

[12] M. Rong, N. K. Mahapatra, and M. Maiti, "A two warehouse inventory model for a deteriorating item with partially/fully backlogged shortage and fuzzy lead time," *European Journal of Operational Research*, vol. 189, no. 1, pp. 59–75, 2008.

[13] M.-S. Chern, H.-L. Yang, J.-T. Teng, and S. Papachristos, "Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation," *European Journal of Operational Research*, vol. 191, no. 1, pp. 127–141, 2008.

[14] S. S. Mishra and P. P. Mishra, "Price determination for an EOQ model for deteriorating items under perfect competition," *Computers & Mathematics with Applications*, vol. 56, no. 4, pp. 1082–1101, 2008.

[15] L. Y. Ouyang, J. T. Teng, S. K. Goyal, and C. T. Yang, "An economic order quantity model for deteriorating items with partially permissible delay in payments linked to order quantity," *European Journal of Operational Research*, vol. 194, no. 2, pp. 418–431, 2009.

[16] C. T. Yang, L. Y. Ouyang, and H. H. Wu, "Retailer's optimal pricing and ordering policies for non-instantaneous deteriorating items with price-dependent demand and partial backlogging," *Mathematical Problems in Engineering*, Article ID 198305, 18 pages, 2009.

[17] K. Skouri and I. Konstantaras, "Order level inventory models for deteriorating seasonable/fashionable products with time dependent demand and shortages," *Mathematical Problems in Engineering*, Article ID 679736, 24 pages, 2009.

[18] P. C. Yang and H. M. Wee, "A single-vendor and multiple-buyers production-inventory policy for a deteriorating item," *European Journal of Operational Research*, vol. 143, no. 3, pp. 570–581, 2002.

[19] J.-J. Liao, "On an EPQ model for deteriorating items under permissible delay in payments," *Applied Mathematical Modelling*, vol. 31, no. 3, pp. 393–403, 2007.

[20] C. C. Lee and S. L. Hsu, "A two-warehouse production model for deteriorating inventory items with time-dependent demands," *European Journal of Operational Research*, vol. 194, no. 3, pp. 700–710, 2009.

[21] Y. He, S.-Y. Wang, and K. K. Lai, "An optimal production-inventory model for deteriorating items with multiple-market demand," *European Journal of Operational Research*, vol. 203, no. 3, pp. 593–600, 2010.

[22] "Boliden AB reports production disruptions at Tara mine in Ireland," Nordic Business Report, 2005.

[23] S. W. Chiu, "Robust planning in optimization for production system subject to random machine breakdown and failure in rework," *Computers & Operations Research*, vol. 37, no. 5, pp. 899–908, 2010.

[24] G. C. Lin and D. E. Kroll, "Economic lot sizing for an imperfect production system subject to random breakdowns," *Engineering Optimization*, vol. 38, no. 1, pp. 73–92, 2006.

[25] B. C. Giri and T. Dohi, "Exact formulation of stochastic EMQ model for an unreliable production system," *Journal of the Operational Research Society*, vol. 56, no. 5, pp. 563–575, 2005.

[26] B. Liu and J. Cao, "Analysis of a production-inventory system with machine breakdowns and shutdowns," *Computers & Operations Research*, vol. 26, no. 1, pp. 73–91, 1999.

[27] A. M. Ross, Y. Rong, and L. V. Snyder, "Supply disruptions with time-dependent parameters," *Computers and Operations Research*, vol. 35, no. 11, pp. 3504–3529, 2008.

[28] D. Heimann and F. Waage, "A closed-form approximation solution for an inventory model with supply disruptions and non-ZIO reorder policy," *Journal of Systemics, Cybernetics and Informatics*, vol. 5, pp. 1–12, 2007.

[29] Z. Li, S. H. Xu, and J. Hayya, "A periodic-review inventory system with supply interruptions," *Probability in the Engineering and Informational Sciences*, vol. 18, no. 1, pp. 33–53, 2004.

[30] H. J. Weiss and E. C. Rosenthal, "Optimal ordering policies when anticipating a disruption in supply or demand," *European Journal of Operational Research*, vol. 59, no. 3, pp. 370–382, 1992.

[31] R. B. Misra, "Optimal production lot size model for a system with deteriorating inventory," *International Journal of Production Research*, vol. 15, pp. 495–505, 1975.