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Research Article

Chaotic Attractor Generation via a Simple Linear Time-Varying System

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A novel generation method of chaotic attractor is introduced in this paper. The underlying mechanism involves a simple three-dimensional time-varying system with simple time functions as control inputs. Moreover, it is demonstrated by simulation that various attractor patterns are generated conveniently by adjusting suitable system parameters. The largest Lyapunov exponent of the system has been obtained.

1. Introduction

Chaos and Chaotic systems, which have been extensively investigated during the last four decades, have been found to be very useful in a variety of applications such as science, mathematics, engineering communities [1], and various techniques such as identification and synchronization [2]. This provides a strong motivation for the current research on exploiting new chaotic systems.

Many chaotic attractors in dynamical systems have been found numerically and experimentally, such as Lorenz attractor [3] and Rössler attractor [4]. The Chua's circuit system [5] that has double scroll attractor is probably the best known and the simplest chaotic system. Suykens and Vandewalle [6] proposed some effective methods for generating n-scroll attractors. Moreover, multiscroll attractors are also found in some simple systems; it stimulates the current research on generating various complex multiscroll chaotic attractors by using some simple electronic circuits and devices.

In general case, it is relatively easy to generate chaotic systems numerically, but it is usually very hard to analyze or verify the dynamical characteristics of nonsmooth systems, even for the switched systems with low dimensions [7, 8]. To deal with the stability of the equilibrium of switched (linear) systems, many efforts have been made and strict analysis has been carried out [9–11]. In [12], a chaotic attractor in a new funnel shape is introduced, simply by designing a switched system with hysteresis-switching signal.

Motivated by previous works of chaotic attractor generation, we have made further effort to generate more chaotic behaviors, by introducing time functions, that is, a time-varying system with various time functions is investigated. To our happiness, chaotic attractors are observed. And moreover, the shape of the created attractors can be changed easily by changing parameters, and various complex patterns can be obtained. The statistic behavior is also discussed, which reveals the regularities in the complex dynamics.

The rest of this paper is organized as follows. Section 2 presents the analysis of chaotic attractor generation. Section 3 introduces a simple three-dimensional time-varying system to generate a new chaotic attractor; then Section 4 concentrates on the pattern changes of the generated attractors with parameters variation. A brief conclusion is given in Section 5.

2. The Analysis of Chaotic Attractor Generation

There are two major researching methods of the chaotic attractor, analytical method and numerical analysis. The analytical methods had two general methods, the Melnikov method and the Kalashnikov method. However, in general case, we used the numerical method due to the difficulties of the analytical method.

Numerical method bases on theoretical achievements and computer simulation. Recently, there were a lot of those achievements, such as the Poincare cross-section, power spectrum, subsampling frequency, and continuous feedback control. In addition, the spectrum analysis, Lyapunov index, fractal dimension and topological entropy, and so forth, were common methods to describe the statistical properties of chaotic.

Some analysis results of chaotic attractor generation were given [13–16], such as horseshoe map [13], Shil'nikov theorem [14], and Li-Yorke theorem [15].

In this section, we studied the conditions of chaotic attractor generation. We first concern general linear system as follows:

$$\dot{x} = Ax, \quad x \in R^3, \tag{2.1}$$

where A is the Jacobian J.

We assume that A has three eigenvalues γ and $\sigma \pm j\omega$, where γ, σ, and ω are real numbers, $\omega \neq 0$. Then, the equilibrium is stable if and only if $\gamma < 0$, $\sigma < 0$; otherwise the equilibrium is unstable.

The range of motion of chaotic attractor can be defined, and there should be both of convergent motions and divergent motions of chaotic attractor in the certain region. In consideration of the above, it can be simulated in the case of the equilibrium being stable (where convergent) or the equilibrium is unstable (where divergent).

Therefore, a necessary condition of chaotic attractor generation for system (2.1) is obtained.

At the equilibrium x^* of the linear system (2.1), the three eigenvalues γ and $\sigma \pm j\omega$ which change with time switch between $\gamma < 0$, $\sigma < 0$, and other conditions.

3. Generating Chaotic Attractor

From the analysis in Section 2, the three eigenvalues of the linearized system must be variables for generating chaotic attractor, and the time function is introduced for the convenience of understanding and calculation. Consider the following time-varying system:

$$\dot{\underline{x}} = \underline{\underline{A}}(t)\underline{x}, \quad \underline{x} \in R^3, \tag{3.1}$$

where

$$\underline{\underline{A}}(t) = \begin{pmatrix} 1/t+\delta & b & 0 \\ c & a\cos(t+e) & 0 \\ 0 & 0 & d\cos(t) \end{pmatrix}, \tag{3.2}$$

$a,\ b,\ c,\ d$ are constants, e is controllable parameter, $\delta > 0$ is infinitesimal. We can prove the system (3.1) satisfies the conditions from Section 2. Solving $\dot{x} = \dot{y} = \dot{z} = 0$ yields the unique system equilibrium $(0,0,0)^T$.

We assume that:

$$e = 0, \quad a = 3, \quad b = -2.5, \quad c = d = 1, \quad \delta = 0.0001, \quad t_1 = \frac{\pi}{2}, \quad t_2 = \pi. \tag{3.3}$$

Then, $1/(t+\delta) \longrightarrow 0$, when $t \longrightarrow +\infty$.

Considering the system trajectory nearby $t_1 = \pi/2$ and $t_2 = \pi$. Assume $\Delta t > 0$ is infinitesimal and $\Delta t = 0.0001$. Then,

$$\dot{x}(t) \approx \begin{pmatrix} \dfrac{1}{\pi/2+0.0001} & -2.5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} x(t), \quad t \in (t_1-\Delta t,\ t_1+\Delta t), \tag{3.4}$$

$$\dot{x}(t) \approx \begin{pmatrix} \dfrac{1}{\pi+0.0001} & -2.5 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & -1 \end{pmatrix} x(t), \quad t \in (t_2-\Delta t,\ t_2+\Delta t). \tag{3.5}$$

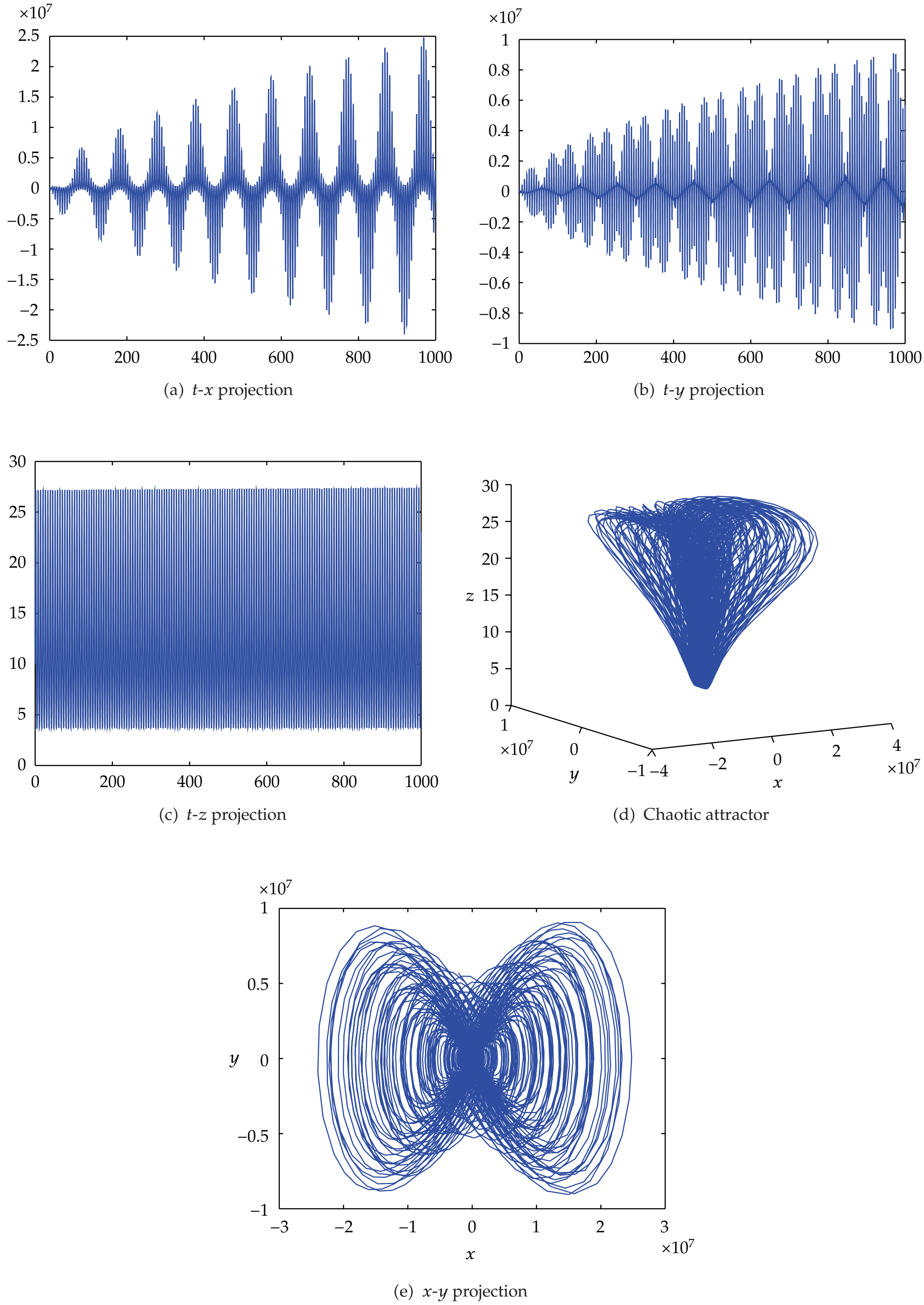
And the characteristic polynomials of (3.4) and (3.5) are

$$\lambda^3 - 0.64\lambda^2 + 2.5\lambda, \tag{3.6}$$

$$\lambda^3 + 3.68\lambda^2 + 4.22\lambda + 2.5. \tag{3.7}$$

The characteristic polynomial in (3.6) has at least one root with positive real part, meaning that the system in (3.4) is divergent, whereas the characteristic polynomial in (3.7) is convergent, meaning that the system in (3.5) is convergent. Meanwhile, the equilibrium position of system (3.1) is unstable as $t = \pi/2 + k\pi\ (k \in Z)$, and the equilibrium position of system (3.1) is stable as $t = (2k+1)\pi\ (k \in Z)$.

The system (3.1) switches in an alternating manner between two situations: convergence and divergence. A chaotic attractor is generated as shown in Figure 1. However, this is not a sufficient condition for a chaotic regime to exist.



(a) t-x projection

(b) t-y projection

(c) t-z projection

(d) Chaotic attractor

(e) x-y projection

Figure 1: Chaotic attractor generated by system (3.3).

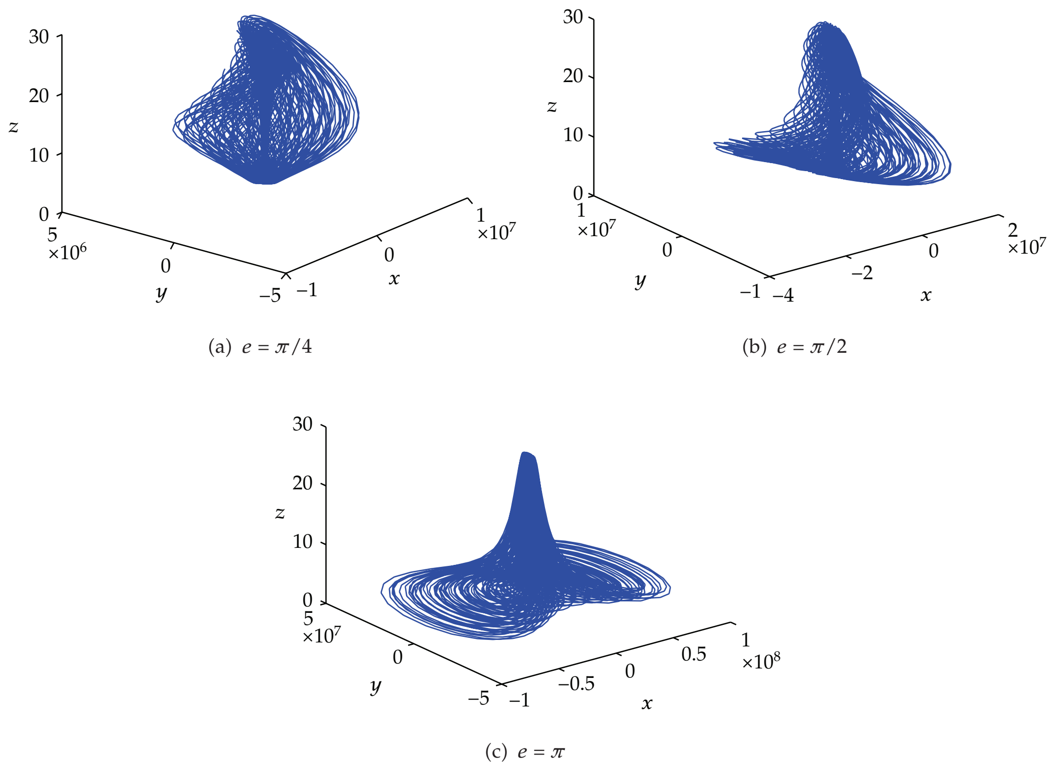


Figure 2: Chaotic attractor changed by e.

4. Various Patterns with Parameter Changing

In this section, we pay attention to the dynamical behaviors of the time-varying system (3.3) with parameter selected in the "chaotic" regions in order to show the effective generation of various patterns of attractors based on the parameter selection.

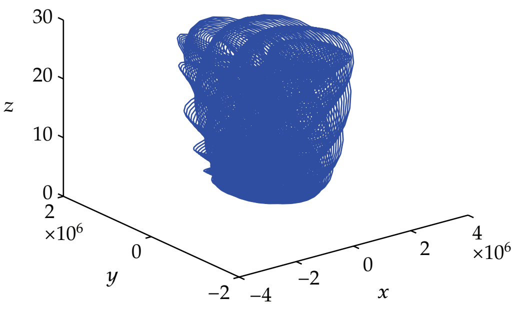
At first, we do not change constant parameters from (3.4) but let e change in the stability intervals. Then, the system display different patterns for different values of e, as shown in Figure 2.

In the three cases, the largest Lyapunov exponents are $LE = 0.0087(e = \pi/4)$, $LE = 0.0085(e = \pi/2)$, $LE = 0.0073(e = \pi)$.

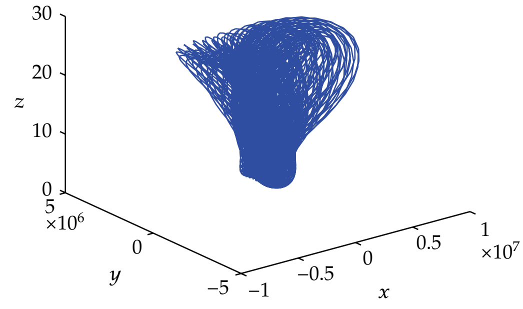
Then, we chose parameters from (3.4) but, respectively, change constant parameters a, b, c, d. Various attractors are produced with different values of a, b, c, d, as shown in Figures 3 and 4.

The largest Lyapunov exponents are given as follows:

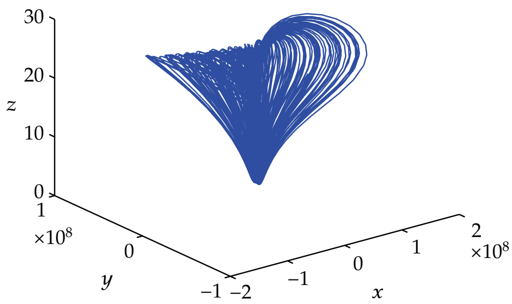
$$\begin{gathered}
LE = 0.0075(a = 0.5), \qquad LE = 0.0081(a = 1.5), \qquad LE = 0.0115(a = 3.5), \\
LE = 0.0084(b = -6), \qquad LE = 0.0086(b = -4), \qquad LE = 0.0103(b = -2.05), \\
LE = 0.0676(c = 0.8), \qquad LE = 0.0087(c = 1.3), \qquad LE = 0.0604(c = 1.5), \\
LE = 0.0083(c = 1.6), \qquad LE = 0.0086(d).
\end{gathered} \tag{4.1}$$



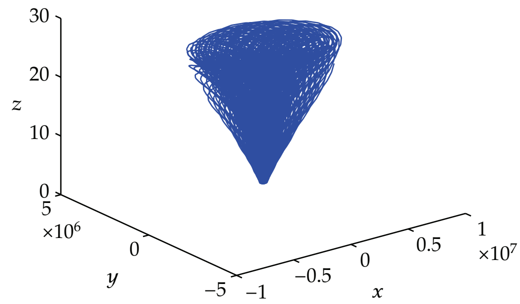
(a) $a = 0.5$



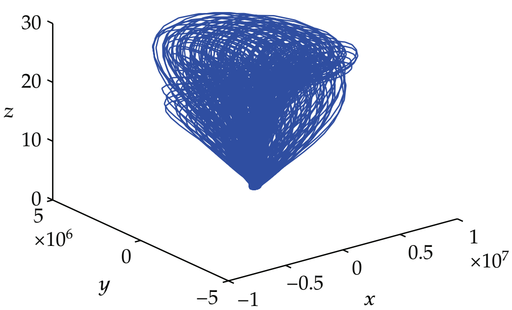
(b) $a = 1.5$



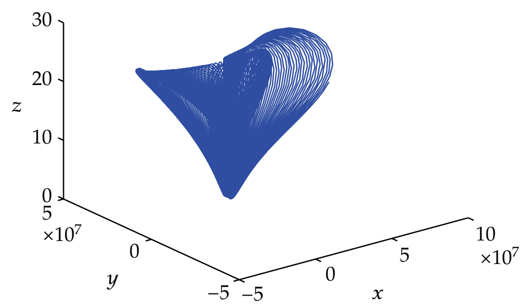
(c) $a = 3.5$



(d) $b = -6$

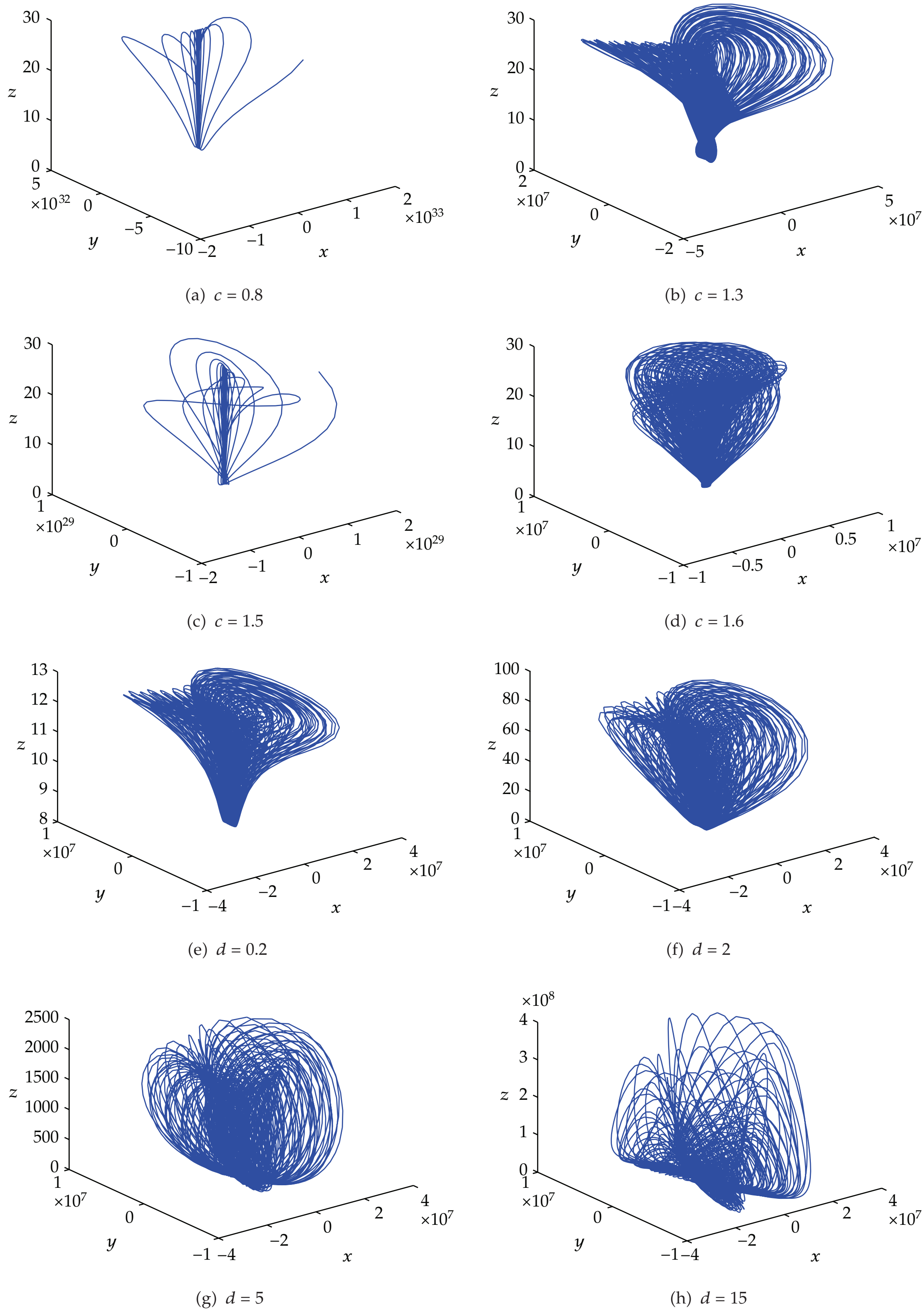


(e) $b = -4$



(f) $b = -2.05$

Figure 3: Different values of parameters a, b.



(a) $c = 0.8$
(b) $c = 1.3$
(c) $c = 1.5$
(d) $c = 1.6$
(e) $d = 0.2$
(f) $d = 2$
(g) $d = 5$
(h) $d = 15$

Figure 4: Different values of parameters c, d.

From these numerical simulations, it is shown that the system (3.1) demonstrates rich complex patterns when adjusting system parameters. From this, we can see that the proposed time-varying system is quite effective in the generation of attractor with obviously quasi-periodic or chaotic behaviors based on the change of parameters.

5. Conclusion

This paper has given a novel chaotic attractor generation method. The generation of novel chaotic attractors via a simple three-dimensional time-varying system with various time functions has been introduced. The results once again support the long-accepted belief that properly designed simple systems can perform complex dynamical behaviors. Moreover, this system can produce various attractor patterns within a wide range of parameter values, and the statistic behavior which reveals the regularities in the complex dynamics is also discussed. In addition, the method has been developed in this article can also be applied to nonlinear dynamical systems and other fields. It is desirable that one could design more chaos generators by means of the method proposed in this paper.

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