

Research Article

An Impulse Model for Computer Viruses

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Computer virus spread model concerning impulsive control strategy is proposed and analyzed. We prove that there exists a globally attractive infection-free periodic solution when the vaccination rate is larger than θ_0 . Moreover, we show that the system is uniformly persistent if the vaccination rate is less than θ_1 . Some numerical simulations are finally given to illustrate the main results.

1. Introduction

Computer virus is a kind of computer program that can replicate itself and spread from one computer to others. Viruses mainly attack the file system and worms use system vulnerability to search and attack computers. As hardware and software technology develop and computer networks become an essential tool for daily life, the computer virus starts to be a major threat. Consequently, the trial on better understanding of the computer virus propagation dynamics is an important matter for improving the safety and reliability in computer systems and networks. Similar to the biological virus, there are two ways to study this problem: microscopic and macroscopic models. Following a macroscopic approach, since [1, 2] took the first step towards modeling the spread behavior of computer virus, much effort has been done in the area of developing a mathematical model for the computer virus propagation [3–13]. These models provide a reasonable qualitative understanding of the conditions under which viruses spread much faster than others.

In [4], the authors investigated a differential *SIRS* model by making the following assumptions.

- (H1) The total population of computers is divided into three groups: susceptible, infected, and recovered computers. Let $S, I,$ and R denote the numbers of susceptible, infected and recovered computers, respectively.

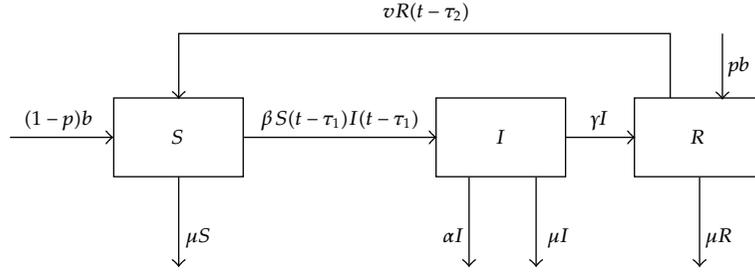


Figure 1: Original model.

- (H2) New computers are attached to the computer network with constant rate b . For the sake of antivirus software, some new nodes have temporary immunity with probability p , some have not with probability $1 - p$. Hence, new nodes are added into the susceptible class with rate $(1 - p)b$ and into the recovered class with rate pb .
- (H3) Computers are disconnected to the computer network with the constant rate μ and are disconnected from the attack of malicious object with probability α .
- (H4) S computers become I with constant rate β and with constant time delay τ_1 ; R computers become S with constant rate γ and with constant time delay τ_1 ; I computers become R with constant rate γ .

According to the above assumptions, the following model (see Figure 1) is derived :

$$\begin{aligned} \frac{dS}{dt} &= (1 - p)b - \mu S - \beta S(t - \tau_1)I(t - \tau_1) + vR(t - \tau_2), \\ \frac{dI}{dt} &= \beta S(t - \tau_1)I(t - \tau_1) - (\mu + \gamma + \alpha)I, \\ \frac{dR}{dt} &= pb - \gamma I - \mu R - vR(t - \tau_2). \end{aligned} \quad (1.1)$$

As we know, antivirus software is a kind of computer program which can detect and eliminate known viruses. There are two common methods that an antivirus software application uses to detect viruses: using a list of virus signature definitions and using a heuristic algorithm to find viruses based on common behaviors. It has been observed that it does not always work in detecting a novel computer virus by using the heuristic algorithm. On the other hand, obviously, it is impossible for antivirus software to find new computer virus signature definitions on the dated list. So, to keep the antivirus soft in high efficiency, it is important to ensure that it is updated. Based on the above facts, we propose an impulsive system to model the process of periodic installing or updating antivirus software on susceptible computers at fixed time for controlling the spread of computer virus.

Based on above facts, we propose the following assumptions.

- (H5) The antivirus software is installed or updated at time $t = kT$ ($k \in \mathbb{N}$), where T is the period of the impulsive effect.
- (H6) S computers are successfully vaccinated from S class to R class with rate θ ($0 < \theta < 1$).

According to the above assumptions (H1)–(H6), and for the reason of simplicity we propose the following model with one time delay (see Figure 2):

$$\begin{aligned}
\frac{dS}{dt} &= (1-p)b - \mu S - \beta SI(t-\tau) + \nu R, \\
\frac{dI}{dt} &= \beta SI(t-\tau) - (\mu + \gamma + \alpha)I, \quad t \neq kT, \quad k \in \mathbb{Z}^+, \\
\frac{dR}{dt} &= pb + \gamma I - \mu R - \nu R, \\
S(t^+) &= (1-\theta)S(t), \\
I(t^+) &= I(t), \quad t = kT, \\
R(t^+) &= R(t) + \theta S(t).
\end{aligned} \tag{1.2}$$

The total population size $N(t)$ can be determined by $N(t) = S(t) + I(t) + R(t)$ to form the differential equation

$$\dot{N}(t) = b - \mu N(t) - \alpha I(t), \tag{1.3}$$

which is derived by adding the equations in system (1.1). Thus the total population size N may vary in time. From (1.2), we have

$$b - (\mu + \alpha)N(t) \leq \dot{N}(t) \leq b - \mu N(t). \tag{1.4}$$

It follows that $b/(\mu + \alpha) \leq \inf \lim_{x \rightarrow \infty} N(t) \leq \lim_{x \rightarrow \infty} \sup N(t) \leq b/\mu$.

Before going into any details, we simplify model (1.1) and restrict our attention to the following model:

$$\begin{aligned}
\frac{dS}{dt} &= (1-p)b + \beta SI(t-\tau) + \nu(N - S - I), \\
\frac{dI}{dt} &= \beta SI(t-\tau) - (\mu + \gamma + \alpha)I, \quad t \neq kT, \quad k \in \mathbb{Z}^+, \\
\frac{dN}{dt} &= b - \mu N - \alpha I, \\
S(t^+) &= (1-\theta)S(t), \\
I(t^+) &= I(t), \quad t = kT, \\
N(t^+) &= N(t).
\end{aligned} \tag{1.5}$$

The initial conditions for (1.5) are

$$(\phi_1(\xi), \phi_2(\xi), \phi_3(\xi)) \in C_+ = C([- \omega, 0], \mathbb{R}_+^3), (\phi_2(\xi)) > 0, \quad i = 1, 2, 3. \tag{1.6}$$

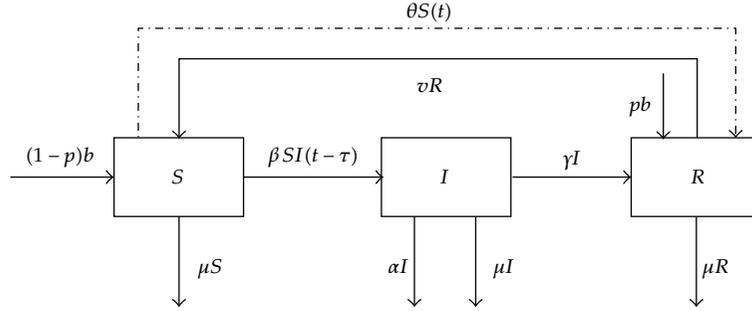


Figure 2: Impulse Model.

From physical considerations, we discuss system (1.5) in the closed set

$$\Omega = \left\{ (S, I, N) \in R_+^3 \mid 0 \leq S + I \leq \frac{b}{\mu}, 0 \leq N \leq \frac{b}{\mu} \right\}, \quad (1.7)$$

where R_+^3 denotes the nonnegative cone of R^3 including its lower-dimensional faces. Note that it is positively invariant with respect to (1.7). The organization of this paper is as follows. In Section 2, we first state three lemmas which are essential to our proofs and establish sufficient condition for the global attractivity of infection-free periodic solution. The sufficient condition for the permanence of the model is obtained in Section 3. Some numerical simulations are performed in Section 4. In the final section, a brief conclusion is given and some future research directions are also pointed out.

2. Global Attractivity of Infection-Free Periodic Solution

In this section, we prove that the infection-free periodic solution is globally attractive under some conditions. To prove the main results, two lemmas (given in [14]) which are essential to the proofs are stated here.

Lemma 2.1 (see [14], Lemma 1). *Consider the following impulsive system:*

$$\begin{aligned} \dot{u}(t) &= a - bu(t), \quad t \neq k\tau, \\ u(t^+) &= (1 - \theta)u(t), \quad t = k\tau, \end{aligned} \quad (2.1)$$

where $a > 0$, $b > 0$, $0 < \theta < 1$. Then there exists a unique positive periodic solution of system (2.1)

$$\bar{u}_e(t) = \frac{a}{b} + \left(u^* - \frac{a}{b} \right) e^{-b(t-k\tau)}, \quad k\tau < t \leq (k+1)\tau, \quad (2.2)$$

which is globally asymptotically stable, where $u^* = a(1 - \theta)(1 - e^{-b\tau}) / (b(1 - (1 - \theta)e^{-b\tau}))$.

Lemma 2.2 (see [14], Lemma 2). Consider the following linear neutral delay equation:

$$\dot{x}(t) + \lambda \dot{x}(t - \omega) + a_2 x(t) - a_1 x(t - \omega) = 0. \quad (2.3)$$

If $|\lambda| < 1$, $a_1^2 < a_2^2$ or $-a_1 = a_2 \neq 0$, then increasing τ does not change the stability of (2.3).

When $\lambda = 0$, (2.3) becomes

$$\dot{x}(t) = a_1 x(t - \omega) - a_2 x(t). \quad (2.4)$$

Corollary 2.3. Consider system (2.4) and assume that $a_1, a_2, \omega > 0$; $x(t) > 0$ for $-\omega \leq t \leq 0$. Then we have the following statements:

- (i) assume that $a_1 < a_2$, then $\lim_{t \rightarrow \infty} x(t) = 0$;
- (ii) assume that $a_1 > a_2$, then $\lim_{t \rightarrow \infty} x(t) = +\infty$.

From the third and sixth equations of system (1.5), we have $\lim_{t \rightarrow \infty} N(t) = b/\mu$. Further, if $I(t) \equiv 0$, we have the following limit system

$$\begin{aligned} \frac{dS}{dt} &= (1-p)b + \frac{\nu b}{\mu} - (\mu + \nu)S, \quad t \neq kT, \quad k \in Z^+, \\ \frac{dN}{dt} &= b - \mu N, \\ S(t^+) &= (1-\theta)S(t), \quad t = kT, \\ N(t^+) &= N(t). \end{aligned} \quad (2.5)$$

From the second and fourth equations of system (3.5), we have $\lim_{t \rightarrow \infty} N(t) = b/\mu$ and have the following limit systems of (3.5):

$$\begin{aligned} \frac{dS}{dt} &= (1-p)b + \frac{\nu b}{\mu} - (\mu + \nu)S, \quad t \neq kT, \\ S(t^+) &= (1-\theta)S(t), \quad t = kT. \end{aligned} \quad (2.6)$$

According to Lemma 2.1, we know that the periodic solution of system (3.10) is of the form

$$\bar{S}_e(t) = \frac{(\mu(1-p) + \nu)b}{\mu(\mu + \nu)} + \left(S^* - \frac{(\mu(1-p) + \nu)b}{\mu(\mu + \nu)} \right) e^{-(\mu + \nu)(t - k\tau)}, \quad k\tau < t \leq (k+1)\tau, \quad (2.7)$$

and it is globally asymptotically stable, where $S^* = ((1-p)b + \nu b/\mu)(1-\theta)(e^{(\mu + \nu)\tau} - 1)/((\mu + \nu)(e^{(\mu + \nu)\tau} - 1 + \theta))$.

Theorem 2.4. *The infection-free periodic solution $(\bar{S}_e(t), 0, b/\mu)$ of system (1.5) is globally attractive provided that $R_0 < 1$, where*

$$R_0 = \frac{((1-p)b + \nu b/\mu)\beta(e^{(\mu+\nu)\tau} - 1)}{(\mu + \gamma + \alpha)(\mu + \nu)(e^{(\mu+\nu)\tau} - 1 + \theta)}. \quad (2.8)$$

Proof. Since $R_0 < 1$, we can choose $\varepsilon_1 > 0$ sufficiently small such that

$$\beta \left(\frac{((1-p)b + \nu b/\mu)(e^{(\mu+\nu)\tau} - 1)}{(\mu + \nu)(e^{(\mu+\nu)\tau} - 1 + \theta)} + \varepsilon_1 \right) < \mu + \gamma + \alpha. \quad (2.9)$$

It follows from the third equation of system (1.5) that

$$\dot{N}(t) \leq b - \mu N(t). \quad (2.10)$$

There exists an integer $k_1 > 0$ such that $N(t) \leq b/\mu$, $t > k_1\tau$.

From the first equation of system (1.5), we have

$$\dot{S}(t) \leq (1-p)b + \frac{\nu b}{\mu} - (\mu + \nu)S. \quad (2.11)$$

For $t > k_1\tau$, $k > k_1$, we consider the following comparison differential system:

$$\dot{x}(t) = (1-p)b + \frac{\nu b}{\mu} - (\mu + \nu)x(t), \quad t \neq k\tau, \quad (2.12)$$

$$x(t^+) = (1-\theta)x(t), \quad t = k\tau.$$

In view of Lemma 2.1, we know that the unique periodic solution of system (2.12) is of the form

$$S(t) < \bar{S}_e(t) + \varepsilon_1 \leq \frac{((1-p)b + \nu b/\mu)(e^{(\mu+\nu)\tau} - 1)}{(\mu + \nu)(e^{(\mu+\nu)\tau} - 1 + \theta)} + \varepsilon_1 \doteq \delta, \quad k\tau < t \leq (k+1)\tau, \quad k > k_2, \quad (2.13)$$

and it is globally asymptotically stable. From (1.5), we have

$$\dot{I}(t) \leq \beta\delta I(t-\tau) - (\mu + \gamma + \alpha)I(t), \quad t > k\tau, \quad k > k_2. \quad (2.14)$$

From (2.9), we have that $\beta\delta < \mu + \gamma + \alpha$. According to Corollary 2.3 we have

$$\lim_{t \rightarrow \infty} I(t) = 0. \quad (2.15)$$

Therefore, for any $\varepsilon_1 > 0$ (sufficiently small), there exists an integer $k_3 > k_2$ such that $I(t) < \varepsilon$ for all $t > k_3\tau$. From the third equation of system (1.5), we have

$$N(t) > b - \mu N(t) - \alpha\varepsilon_1, \quad t > k_3\tau. \quad (2.16)$$

Consider the comparison equation

$$\dot{z}(t) = b - \alpha\varepsilon_1 - \mu z(t), \quad \text{for } t > k_3\tau. \quad (2.17)$$

It is easy to see that $\lim_{t \rightarrow \infty} z(t) = (b - \alpha\varepsilon_1)/\mu$. It follows by the comparison theorem that there exists an integer $k_4 > k_3$ such that

$$N(t) \geq \frac{b - \alpha\varepsilon_1}{\mu} - \varepsilon_1, \quad \forall t > k_4\tau. \quad (2.18)$$

Since ε_1 is arbitrarily small, from $\lim_{t \rightarrow \infty} \sup N(t) \leq b/\mu$ and (2.18) we have

$$\lim_{t \rightarrow \infty} N(t) = \frac{b}{\mu}. \quad (2.19)$$

It follows from (2.15) and (2.19) that there exists $k_5 > k_4$ such that

$$I(t) < \varepsilon_1, \quad N(t) > \frac{b}{\mu} - \varepsilon_1, \quad \text{for } t > k_5\tau. \quad (2.20)$$

Hence, from the first equation of system (1.5) we have that

$$\begin{aligned} \dot{S}(t) &\geq (1-p)b - \mu S(t) - \beta S(t)\varepsilon_1 + v \left(\frac{b}{\mu} - s\varepsilon_1 - S(t) \right) \\ &= (1-p)b + v \frac{b}{\mu} - 2\varepsilon_1 v - (\mu + \beta\varepsilon_1 + v)S(t), \quad \text{for } t > k_5\tau. \end{aligned} \quad (2.21)$$

Consider the following comparison impulsive differential equations for $t > k_5\tau$ and $k > k_5$,

$$\begin{aligned} \dot{u}(t) &= (1-p)b + v \frac{b}{\mu} - 2\varepsilon_1 v - (\mu + \beta\varepsilon_1 + v)u(t), \quad t \neq k\tau, \\ u(t^+) &= (1-\theta)u(t), \quad t = k\tau. \end{aligned} \quad (2.22)$$

In view of Lemma 2.1, we periodic solution of system

$$\bar{u}_c(t) = \Gamma + (u^* - \Gamma)e^{-(\mu + \beta\varepsilon_1 + v)(t - k\tau)}, \quad k\tau < t \leq (k+1)\tau, \quad (2.23)$$

which is globally asymptotically stable, where

$$\Gamma = \frac{(1-p)b + v(b/\mu) - 2\varepsilon_1 v}{\mu + \beta\varepsilon_1 + v}, \quad u^* = \Gamma \frac{(1-\theta)(1 - e^{-(\mu+\beta\varepsilon_1+v)\tau})}{(1 - (1-\theta)e^{-(\mu+\beta\varepsilon_1+v)\tau})}. \quad (2.24)$$

According to the comparison theorem for impulsive differential equation, there exists an integer $k_6 > k_5$ such that

$$S(t) > \bar{u}_e(t) - \varepsilon_1, \quad k\tau < t \leq (k+1)\tau, \quad k > k_6. \quad (2.25)$$

Because ε_1 is arbitrarily small, it follows from (2.25) that

$$\bar{S}_e(t) = \frac{((1-p)b + v(b/\mu))(e^{(\mu+v)\tau} - 1)}{(\mu + v)(e^{(\mu+v)\tau} - 1 + \theta)}, \quad k\tau < t \leq (k+1)\tau, \quad (2.26)$$

is globally attractive, that is,

$$\lim_{x \rightarrow \infty} S(t) = \bar{S}_e(t). \quad (2.27)$$

It follows from (2.15), (2.19), (2.27), and the restriction $N(t) = S(t) + I(t) + R(t)$ that $\lim_{x \rightarrow \infty} R(t) = b/\mu - \bar{S}_e(t)$. Hence, the infection-free periodic solution $(\bar{S}_e(t), 0, b/\mu)$ of system (1.5) is globally attractive. The proof is completed. \square

Corollary 2.5. *The infection-free periodic solution $(\bar{S}_e(t), 0, b/\mu)$ of system (1.5) is globally attractive, if $\theta > \theta_0$ where $\theta_0 = (e^{(\mu+v)\tau} - 1)((1-p + v/\mu)/((\mu + \gamma)(\mu + \gamma + \alpha))) - 1$.*

Theorem 2.4 determines the global attractivity of (1.5) in Ω for the case $R_0 < 1$. Its realistic implication is that the infected computers vanish so the computer virus removed from the network. Corollary 2.5 implies that the computer virus will disappear if the vaccination rate is larger than θ_0 .

3. Permanence

In this section, we say the computer virus is local if the infectious population persists above a certain positive level for sufficiently large time. The locality viruses can be well captured and studied through the notion of permanence.

Definition 3.1. System (1.5) is said to be uniformly persistent if there is an $\varphi > 0$ (independent of the initial data) such that every solution $(S(t), I(t), R(t), N(t))$ with initial conditions (1.7) of system (1.5) satisfies

$$\liminf_{t \rightarrow \infty} S(t) \geq \varphi, \quad \liminf_{t \rightarrow \infty} I(t) \geq \varphi, \quad \liminf_{t \rightarrow \infty} R(t) \geq \varphi. \quad (3.1)$$

Definition 3.2. System (1.5) is said to be permanent if there exists a compact region $\Omega_0 \in \text{int } \Omega$ such that every solution of system (1.5) with initial data (1.7) will eventually enter and remain in region Ω_0 . Denote

$$R_1 = \frac{\beta(1-p)b(e^{\mu\tau} - 1)}{(\mu + \gamma + \alpha)(e^{\mu\tau} - 1 + \theta)}. \quad (3.2)$$

Theorem 3.3. *Suppose that $R_1 > 1$. Then there is a positive constant m_I such that each positive solution $(S(t), I(t), R(t))$ of system (1.5) satisfies $I(t) \geq m_I$, for t large enough.*

Proof. Now, we will prove there exist $m_I > 0$ and a sufficiently large t_p such that $I(t) \geq m_I$ holds for all $t > t_p$. Suppose that $I(t) < m_I^*$ for all $t > t_0$. From the first equation of (1.5), we have

$$\dot{S}(t) > (1-p)b - (\mu + \beta m_I^*)S. \quad (3.3)$$

Consider the following comparison system:

$$\begin{aligned} \dot{z}_2(t) &= (1-p)b - (\mu + \beta m_I^*)z_2(t), \quad t \neq k\tau, \\ z_2(t^+) &= (1-\theta)z_2(t), \quad t = k\tau \end{aligned} \quad (3.4)$$

By Lemma 2.1, we know that, there exists t_1 such that

$$S(t) \geq z_2(t) > z_2^*(t) - \varepsilon \geq \frac{(1-p)b(e^{(\mu+\beta m_I^*)\tau} - 1)}{(\mu + \beta m_I^*)(e^{(\mu+\beta m_I^*)\tau} - 1 + \theta)} - \varepsilon \doteq \bar{S} > 0, \quad \text{for } t > t_1. \quad (3.5)$$

It follows from the second equation of (1.5) that $\dot{I}(t) > \beta \bar{S} I(t - \tau) - (\mu + \gamma + \alpha)I(t)$. Consider the comparison system $\dot{z}_3(t) = \beta \bar{S} I(t - \tau) - (\mu + \gamma + \alpha)z_3(t)$. Noting that $R_1 > 1$ and ε is sufficiently small, we have $\beta \bar{S} > (\mu + \gamma + \alpha)$.

Corollary 2.3 implies that $t \rightarrow \infty$, $I(t) > z_3(t) \rightarrow \infty$. This contradicts $I(t) \leq b$. Hence, we can claim that, for any $t_0 > 0$, it is impossible that

$$I(t) < m_I^* \quad \forall t \geq t_0. \quad (3.6)$$

By the claim, we are left to consider two cases. First, $I(t) \leq m_I^*$ for t large enough. Second, $I(t)$ oscillates about m_I^* for t large enough. Obviously, there is nothing to prove for the first case. For the second case, we can choose $t_2 > t_1$ and $\xi > 0$ satisfy

$$I(t_2) = I(t_2 + \xi) = m_I^*, \quad I(t) < m_I^*, \quad \text{for } t_2 < t < t_2 + \xi \quad (3.7)$$

$I(t)$ is uniformly continuous since the positive solutions to (1.5) are ultimately bounded and $I(t)$ is not effected by impulses.

Therefore, it is certain that there exists η ($0 < \eta < \tau$, and η is independent of the choice of t_2) such that

$$I(t) \geq \frac{m_I^*}{2}, \quad \text{for } t_2 \leq t \leq t_2 + \eta. \quad (3.8)$$

In this case, we shall consider the following three possible cases in term of the sizes of η , ξ and τ .

Case 1. If $\xi \leq \eta < \tau$, then it is obvious that $I(t) \geq m_I^*/2$, for $t \in [t_2, t_2 + \xi]$.

Case 2. If $\eta < \xi \leq \tau$, then from the second equation of system (1.5), we obtain $\dot{I}(t) > -(\mu + \gamma + \alpha)I(t)$. Since $I(t_2) = m_I^*$, it is obvious that $I(t) > m_I^*e^{-(\gamma + \alpha + \mu)t} \doteq m_I^{**}$, for $t \in [t_2, t_2 + \xi]$.

Case 3. If $\eta < \tau \leq \xi$, it is easy to obtain that $I(t) > m_I^{**}$ for $t \in [t_2, t_2 + \tau]$. Then, proceeding exactly the proof for above claim, we have that $I(t) > m_I^{**}$ for $t \in [t_2 + \tau, t_2 + \xi]$.

Owing to the randomness of t_2 , we obtain that there exists $m_I \doteq \min\{m_I^*/2, m_I^{**}\}$, such that $I(t) > m_I$ holds for all $t > t_p$. The proof of Theorem 3.3 is completed. \square

Theorem 3.4. *Suppose $R_1 > 1$. Then system (1.5) is permanent.*

Proof. Let $(S(t), I(t), N(t))$ be any solution to system (1.5). First, from the first equation of system (1.5), we have $\dot{S}(t) > (1-p)b - (\mu + \beta b)S$. Consider the following comparison system:

$$\begin{aligned} \dot{z}_1(t) &= (1-p)b - (\mu + \beta b)z_1(t), \quad t \neq k\tau, \\ z_1(t^+) &= (1-\theta)z_1(t), \quad t = k\tau. \end{aligned} \quad (3.9)$$

By Lemmas 2.1 and 2.2, we know that for any sufficiently small $\varepsilon > 0$, there exists t_1 (t_1 is sufficiently large) such that

$$S(t) \geq z_2(t) > z_2^*(t) - \varepsilon \geq \frac{(1-p)b(e^{(\mu + \beta m_I^*)\tau} - 1)}{(\mu + \beta m_I^*)(e^{(\mu + \beta m_I^*)\tau} - 1 + \theta)} - \varepsilon \doteq m_S > 0, \quad k\tau < t \leq (k+1)\tau. \quad (3.10)$$

From the third equation of (1.5), we have $\dot{N}(t) > n - \mu N(t) - \alpha N(t)$. It is easy to see that $N(t) > b/(\mu + \alpha) - \varepsilon \doteq m_N$. Let $\Omega_0 = \{(S, I, N) \in \mathbb{R}_+^3 \mid m_S \leq S, m_I \leq I, m_N \leq N \leq b/\mu, S+1 \leq b/\mu\}$. By Theorem 3.3 and above discussions, we know that the set Ω_0 is a global attractor in Ω , and of course, every solution of system (1.5) with initial conditions (1.7) will eventually enter and remain in region Ω_0 . Therefore, system (1.5) is permanent. The proof is completed. \square

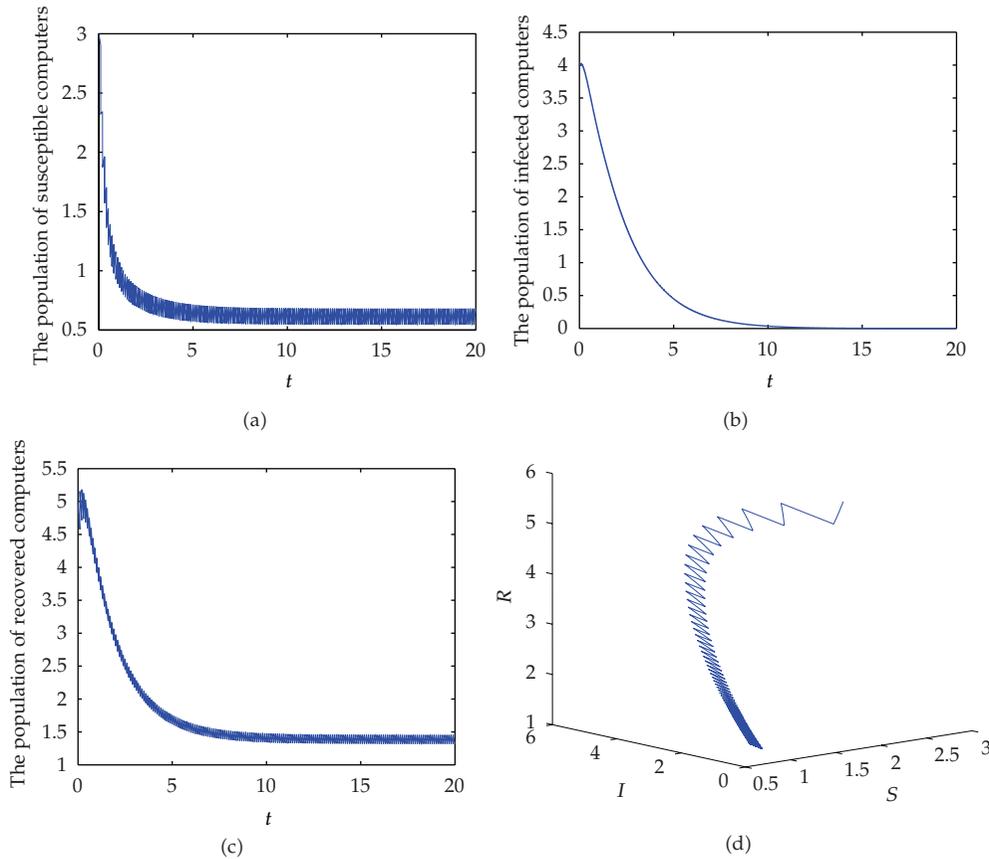


Figure 3: Global attractivity of infection-free periodic solution.

Corollary 3.5. *It follows from Theorem 3.4 that the system (1.5) is uniformly persistent provided that $\theta < \theta_1$, where $\theta_1 = (e^{\mu\tau} - 1)((\beta(1 - p)b / ((\mu + \alpha)(\mu + \alpha + \gamma))) - 1)$.*

4. Numerical Simulations

In this section, we perform some numerical simulations to show the geometric impression of our results. To demonstrate the global attractivity of infection-free periodic solution to system (1.5), we take the following parameter values: $b = 1$, $\mu = 0.5$, $\gamma = 0.3$, $\alpha = 0.02$, $\beta = 0.3$, $\nu = 0.7$, $\theta = 0.4$, and $\tau = 1$. In this case, we have $R_0 = 0.5894 < 1$. In Figures 3(a), 3(b), and 3(c) display respectively the susceptible, infected and recovered population of system (1.5) with initial conditions: $S(0) = 3$, $I(0) = 4$ and $R(0) = 5$. Figure 3(d) shows their corresponding phase-portrait.

To demonstrate the permanence of system (1.5) we take following set parameter values: $b = 10$, $\mu = 0.3$, $\gamma = 0.3$, $\alpha = 0.02$, $\beta = 0.3$, $\theta = 0.2$, $\nu = 0.7$, and $\tau = 1$. In this case, we have $R_1 = 2.2195 > 1$. In Figures 4(a), 4(b), and 4(c) display, respectively, the susceptible, infected and recovered population of system (1.5) with initial conditions: $S(0) = 3$, $I(0) = 4$, and $R(0) = 5$. Figure 4(d) shows their corresponding phase portrait.

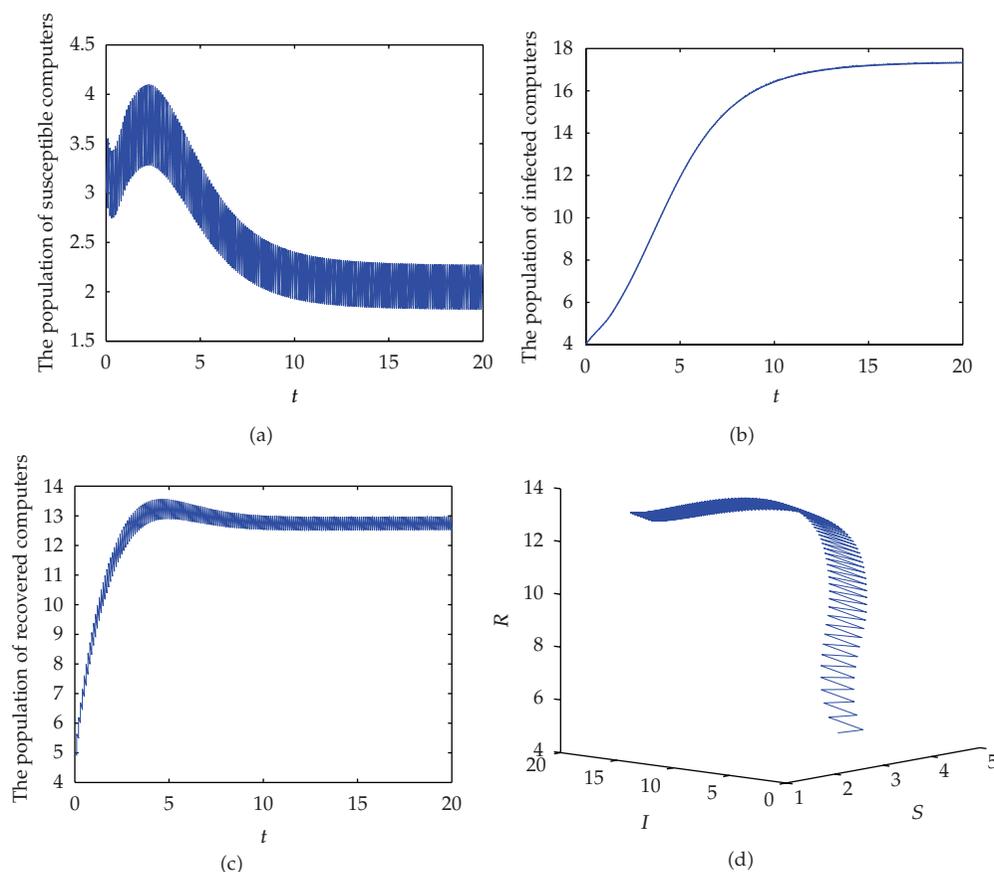


Figure 4: permanence of system.

5. Conclusion

We have analyzed the delayed *SIRS* model with pulse vaccination and varying total population size. We have shown that $R_1 > 1$ or $\theta < \theta_1$ implies that the disease will be endemic, whereas $R_0 < 1$ or $\theta > \theta_0$ implies that the disease will fade out. We have also established sufficient condition for the permanence of the model. Our results indicate that a short interpulse time or a large pulse vaccination rate will lead to eradication of the computer virus.

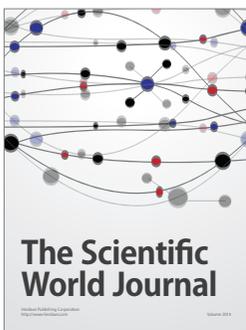
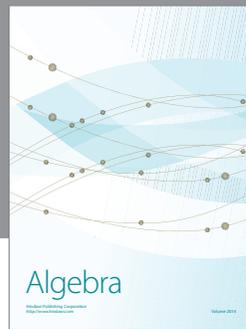
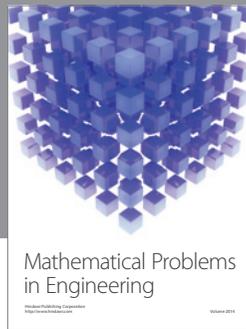
In this paper, we have only discussed two cases: (i) $R_0 < 1$ (or $\theta > \theta_0$) and (ii) $R_1 > 1$ (or $\theta < \theta_1$). But for closed interval $[R_0, R_1]$ (or $[\theta_1, \theta_0]$), the dynamical behavior of model (3) have not been studied, and the threshold parameter for the reproducing number (or the pulse vaccination rate) between the extinction of the computer viruses and the uniform persistence of the viruses have not been obtained. These issues would be left to our future consideration.

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References

- [1] J. O. Kephart and S. R. White, "Directed-graph epidemiological models of computer viruses," in *Proceedings of the IEEE Symposium on Security and Privacy*, pp. 343–359, 1991.
- [2] J. O. Kephart, S. R. White, and D. M. Chess, "Computers and epidemiology," *IEEE Spectrum*, vol. 30, no. 5, pp. 20–26, 1993.
- [3] L. Billings, W. M. Spears, and I. B. Schwartz, "A unified prediction of computer virus spread in connected networks," *Physics Letters A*, vol. 297, no. 3-4, pp. 261–266, 2002.
- [4] X. Han and Q. Tan, "Dynamical behavior of computer virus on Internet," *Applied Mathematics and Computation*, vol. 217, no. 6, pp. 2520–2526, 2010.
- [5] B. K. Mishra and N. Jha, "Fixed period of temporary immunity after run of anti-malicious software on computer nodes," *Applied Mathematics and Computation*, vol. 190, no. 2, pp. 1207–1212, 2007.
- [6] B. K. Mishra and D. Saini, "Mathematical models on computer viruses," *Applied Mathematics and Computation*, vol. 187, no. 2, pp. 929–936, 2007.
- [7] B. K. Mishra and S. K. Pandey, "Dynamic model of worms with vertical transmission in computer network," *Applied Mathematics and Computation*, vol. 217, no. 21, pp. 8438–8446, 2011.
- [8] J. R. C. Piqueira and V. O. Araujo, "A modified epidemiological model for computer viruses," *Applied Mathematics and Computation*, vol. 213, no. 2, pp. 355–360, 2009.
- [9] J. R. C. Piqueira, "Dynamic models for computer viruses," *Computers and Security*, vol. 27, no. 7-8, pp. 355–359, 2008.
- [10] J. Ren, X. Yang, L.-X. Yang, Y. Xu, and F. Yang, "A delayed computer virus propagation model and its dynamics," *Chaos, Solitons & Fractals*, vol. 45, no. 1, pp. 74–79, 2012.
- [11] J. Ren, X. Yang, Q. Zhu, L.-X. Yang, and C. Zhang, "A novel computer virus model and its dynamics," *Nonlinear Analysis*, vol. 13, no. 1, pp. 376–384, 2012.
- [12] J. C. Wierman and D. J. Marchette, "Modeling computer virus prevalence with a susceptible-infected-susceptible model with reintroduction," *Computational Statistics & Data Analysis*, vol. 45, no. 1, pp. 3–23, 2004.
- [13] H. Yuan and G. Chen, "Network virus-epidemic model with the point-to-group information propagation," *Applied Mathematics and Computation*, vol. 206, no. 1, pp. 357–367, 2008.
- [14] J. Ren, X. Yang, Q. Zhu, L.-X. Yang, and C. Zhang, "A novel computer virus model and its dynamics," *Nonlinear Analysis*, vol. 13, no. 1, pp. 376–384, 2012.



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