

Research Article

On the Solutions of a General System of Difference Equations

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We deal with the solutions of the systems of the difference equations $x_{n+1} = 1/x_{n-p}y_{n-p}$, $y_{n+1} = x_{n-p}y_{n-p}/x_{n-q}y_{n-q}$, and $x_{n+1} = 1/x_{n-p}y_{n-p}z_{n-p}$, $y_{n+1} = x_{n-p}y_{n-p}z_{n-p}/x_{n-q}y_{n-q}z_{n-q}$, $z_{n+1} = x_{n-q}y_{n-q}z_{n-q}/x_{n-r}y_{n-r}z_{n-r}$, with a nonzero real numbers initial conditions. Also, the periodicity of the general system of k variables will be considered.

1. Introduction

In this paper, we deal with the solutions of the systems of the difference equations

$$\begin{aligned} x_{n+1} &= \frac{1}{x_{n-p}y_{n-p}}, & y_{n+1} &= \frac{x_{n-p}y_{n-p}}{x_{n-q}y_{n-q}}, \\ x_{n+1} &= \frac{1}{x_{n-p}y_{n-p}z_{n-p}}, & y_{n+1} &= \frac{x_{n-p}y_{n-p}z_{n-p}}{x_{n-q}y_{n-q}z_{n-q}}, & z_{n+1} &= \frac{x_{n-q}y_{n-q}z_{n-q}}{x_{n-r}y_{n-r}z_{n-r}}, \end{aligned} \quad (1.1)$$

with a nonzero real numbers initial conditions. Also, the periodicity of the general system of k variables will be considered.

Recently, there has been a great interest in studying nonlinear difference equations and systems (cf. [1–14] and the references therein). One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real-life situations in population biology, economy, probability theory,

genetics, psychology, sociology, and so forth. Such equations also appear naturally as discrete analogues of differential equations which model various biological and economical systems.

Cinar [3] has obtained the positive solution of the difference equation system:

$$x_{n+1} = \frac{1}{y_n}, \quad y_{n+1} = \frac{y_n}{x_{n-1}y_{n-1}}. \quad (1.2)$$

Also, Çinar and Yalçinkaya [4] have obtained the positive solution of the difference equation system:

$$x_{n+1} = \frac{1}{z_n}, \quad y_{n+1} = \frac{x_n}{x_{n-1}}, \quad z_{n+1} = \frac{1}{x_{n-1}}. \quad (1.3)$$

Clark and Kulenović [5] have investigated the global stability properties and asymptotic behavior of solutions of the system

$$x_{n+1} = \frac{x_n}{a + cy_n}, \quad y_{n+1} = \frac{y_n}{b + dx_n}. \quad (1.4)$$

Elabbasy et al. [6] have obtained the solution of particular cases of the following general system of difference equations:

$$\begin{aligned} x_{n+1} &= \frac{a_1 + a_2 y_n}{a_3 z_n + a_4 x_{n-1} z_n}, & y_{n+1} &= \frac{b_1 z_{n-1} + b_2 z_n}{b_3 x_n y_n + b_4 x_n y_{n-1}}, \\ z_{n+1} &= \frac{c_1 z_{n-1} + c_2 z_n}{c_3 x_{n-1} y_{n-1} + c_4 x_{n-1} y_n + c_5 x_n y_n}. \end{aligned} \quad (1.5)$$

Elsayed [10] has obtained the solution of systems of difference equations of rational form.

Also, the behavior of the solutions of the following systems:

$$x_{n+1} = \frac{x_{n-1}}{\pm 1 + x_{n-1} y_n}, \quad y_{n+1} = \frac{y_{n-1}}{\mp 1 + y_{n-1} x_n}, \quad (1.6)$$

has been studied by Elsayed [15].

Özban [16] has investigated the positive solutions of the system of rational difference equations:

$$x_{n+1} = \frac{1}{y_{n-k}}, \quad y_{n+1} = \frac{y_n}{x_{n-m} y_{n-m-k}}. \quad (1.7)$$

Özban [17] has investigated the solutions of the following system:

$$x_{n+1} = \frac{a}{y_{n-3}}, \quad y_{n+1} = \frac{b y_{n-3}}{x_{n-q} y_{n-q}}. \quad (1.8)$$

Yang et al. [18] have investigated the positive solutions of the systems:

$$x_n = \frac{a}{y_{n-p}}, \quad y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}. \quad (1.9)$$

Similar nonlinear systems of rational difference equations were investigated see [15–32].

Definition 1.1 (Periodicity). A sequence $\{x_n\}_{n=-k}^{\infty}$ is said to be periodic with period p if $x_{n+p} = x_n$ for all $n \geq -k$.

2. Main Results

2.1. The First System

In this section, we deal with the solutions of the system of the difference equations

$$x_{n+1} = \frac{1}{x_{n-p}y_{n-p}}, \quad y_{n+1} = \frac{x_{n-p}y_{n-p}}{x_{n-q}y_{n-q}}, \quad (2.1)$$

with a nonzero real numbers initial conditions and $p \neq q$.

Theorem 2.1. *Suppose that $\{x_n, y_n\}$ are solutions of system (2.1). Also, assume that the initial conditions are arbitrary nonzero real numbers. Then all solutions of equation system (2.1) are eventually periodic with period $(2q + 2)$.*

Proof. From (2.1), we see that

$$\begin{aligned} x_{n+1} &= \frac{1}{x_{n-p}y_{n-p}}, & y_{n+1} &= \frac{x_{n-p}y_{n-p}}{x_{n-q}y_{n-q}}, \\ x_{n+2} &= \frac{1}{x_{n-p+1}y_{n-p+1}}, & y_{n+2} &= \frac{x_{n-p+1}y_{n-p+1}}{x_{n-q+1}y_{n-q+1}}, \\ x_{n+3} &= \frac{1}{x_{n-p+2}y_{n-p+2}}, & y_{n+3} &= \frac{x_{n-p+2}y_{n-p+2}}{x_{n-q+2}y_{n-q+2}}, \\ & & & \vdots \\ x_{n-p+q-2} &= \frac{1}{x_{n-2p+q-3}y_{n-2p+q-3}}, & y_{n-p+q-2} &= \frac{x_{n-2p+q-3}y_{n-2p+q-3}}{x_{n-p-3}y_{n-p-3}}, \\ x_{n-p+q-1} &= \frac{1}{x_{n-2p+q-2}y_{n-2p+q-2}}, & y_{n-p+q-1} &= \frac{x_{n-2p+q-2}y_{n-2p+q-2}}{x_{n-p-2}y_{n-p-2}}, \\ x_{n-p+q} &= \frac{1}{x_{n-2p+q-1}y_{n-2p+q-1}}, & y_{n-p+q} &= \frac{x_{n-2p+q-1}y_{n-2p+q-1}}{x_{n-p-1}y_{n-p-1}}, \end{aligned}$$

$$\begin{aligned}
x_{n-p+q+1} &= \frac{1}{x_{n-2p+q}y_{n-2p+q}}, & y_{n-p+q+1} &= \frac{x_{n-2p+q}y_{n-2p+q}}{x_{n-p}y_{n-p}}, \\
x_{n-p+q+2} &= \frac{1}{x_{n-2p+q+1}y_{n-2p+q+1}}, & y_{n-p+q+2} &= \frac{x_{n-2p+q+1}y_{n-2p+q+1}}{x_{n-p+1}y_{n-p+1}}, \\
& & & \vdots \\
x_{n-p+2q-1} &= \frac{1}{x_{n-2p+2q-2}y_{n-2p+2q-2}}, & y_{n-p+2q-1} &= x_{n-2p+2q-2}y_{n-2p+2q-2}x_{n-p-3}y_{n-p-3}, \\
x_{n-p+2q} &= \frac{1}{x_{n-2p+2q-1}y_{n-2p+2q-1}}, & y_{n-p+2q} &= x_{n-2p+2q-1}y_{n-2p+2q-1}x_{n-p-2}y_{n-p-2}, \\
x_{n-p+2q+1} &= \frac{1}{x_{n-2p+q}y_{n-2p+q}}, & y_{n-p+2q+1} &= x_{n-2p+2q}y_{n-2p+2q}x_{n-p-1}y_{n-p-1}, \\
x_{n-p+2q+2} &= \frac{1}{x_{n-2p+q+1}y_{n-2p+q+1}}, & y_{n-p+2q+2} &= x_{n-2p+2q+1}y_{n-2p+2q+1}x_{n-p}y_{n-p}, \\
& & & \vdots \\
x_{n+q-1} &= \frac{1}{x_{n-p+q-2}y_{n-p+q-2}}, & y_{n+q-1} &= \frac{x_{n-p+q-2}y_{n-p+q-2}}{x_{n-2}y_{n-2}}, \\
x_{n+q} &= \frac{1}{x_{n-p+q-1}y_{n-p+q-1}}, & y_{n+q} &= \frac{x_{n-p+q-1}y_{n-p+q-1}}{x_{n-1}y_{n-1}}, \\
x_{n+q+1} &= \frac{1}{x_{n-p+q}y_{n-p+q}}, & y_{n+q+1} &= \frac{x_{n-p+q}y_{n-p+q}}{x_n y_n}, \\
x_{n+q+2} &= \frac{1}{x_{n-p+q+1}y_{n-p+q+1}}, & y_{n+q+2} &= \frac{x_{n-p+q+1}y_{n-p+q+1}}{x_{n+1}y_{n+1}} = x_{n-p+q+1}y_{n-p+q+1}x_{n-q}y_{n-q}, \\
x_{n+q+3} &= \frac{1}{x_{n-p+q+2}y_{n-p+q+2}}, & y_{n+q+3} &= \frac{x_{n-p+q+2}y_{n-p+q+2}}{x_{n+2}y_{n+2}} = x_{n-p+q+2}y_{n-p+q+2}x_{n-q+1}y_{n-q+1}, \\
x_{n+q+4} &= \frac{1}{x_{n-p+q+3}y_{n-p+q+3}}, & y_{n+q+4} &= \frac{x_{n-p+q+3}y_{n-p+q+3}}{x_{n+3}y_{n+3}} = x_{n-p+q+3}y_{n-p+q+3}x_{n-q+2}y_{n-q+2}, \\
& & & \vdots \\
x_{n+2q} &= \frac{1}{x_{n-p-3}y_{n-p-3}}, & y_{n+2q} &= x_{n-p-3}y_{n-p-3}x_{n-2}y_{n-2}, \\
x_{n+2q+1} &= \frac{1}{x_{n-p-2}y_{n-p-2}}, & y_{n+2q+1} &= x_{n-p-2}y_{n-p-2}x_{n-1}y_{n-1}, \\
x_{n+2q+2} &= \frac{1}{x_{n-p-1}y_{n-p-1}}, & y_{n+2q+2} &= x_{n-p-1}y_{n-p-1}x_n y_n, \\
x_{n+2q+3} &= \frac{1}{x_{n-p+2q+2}y_{n-p+2q+2}} = x_{n+1}, & y_{n+2q+3} &= \frac{x_{n-p+2q+2}y_{n-p+2q+2}}{x_{n+q+2}y_{n+q+2}} = y_{n+1},
\end{aligned}$$

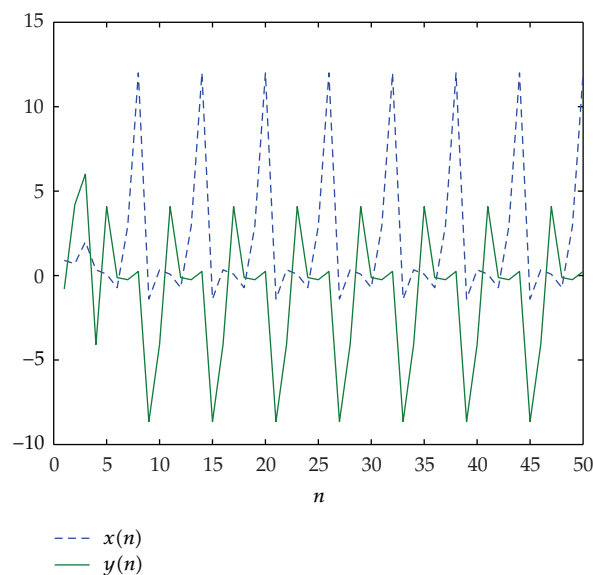


Figure 1

$$\begin{aligned}
 x_{n+2q+4} &= \frac{1}{x_{n-p+2q+3}y_{n-p+2q+3}} = x_{n+2}, \\
 y_{n+2q+4} &= \frac{x_{n-p+2q+3}y_{n-p+2q+3}}{x_{n+q+3}y_{n+q+3}} = y_{n+2}.
 \end{aligned} \tag{2.2}$$

Hence, the proof is completed. \square

Numerical Example

In order to illustrate the results of this section and to support our theoretical discussions, we consider the following numerical example.

Example 2.2. Consider the difference system (2.1) with $p = 1$, $q = 2$, and the initial conditions $x_{-2} = 0.9$, $x_{-1} = 0.7$, $x_0 = 2$, $y_{-2} = -0.8$, $y_{-1} = 4.2$, and $y_0 = 6$. (See Figure 1).

2.2. The Second System

In this section, we deal with the solutions of the system of the difference equations

$$x_{n+1} = \frac{1}{x_{n-p}y_{n-p}z_{n-p}}, \quad y_{n+1} = \frac{x_{n-p}y_{n-p}z_{n-p}}{x_{n-q}y_{n-q}z_{n-q}}, \quad z_{n+1} = \frac{x_{n-q}y_{n-q}z_{n-q}}{x_{n-r}y_{n-r}z_{n-r}}, \tag{2.3}$$

with a nonzero real numbers initial conditions and $p \neq q$, $q \neq r$.

Theorem 2.3. Suppose that $\{x_n, y_n, z_n\}$ are solutions of system (2.3). Also, assume that the initial conditions are arbitrary nonzero real numbers. Then all solutions of equation system (2.3) are eventually periodic with period $(2r + 2)$.

Proof. From (2.3), we see that

$$\begin{aligned}
x_{n+1} &= \frac{1}{x_{n-p}y_{n-p}z_{n-p}}, & y_{n+1} &= \frac{x_{n-p}y_{n-p}z_{n-p}}{x_{n-q}y_{n-q}z_{n-q}}, & z_{n+1} &= \frac{x_{n-q}y_{n-q}z_{n-q}}{x_{n-r}y_{n-r}z_{n-r}}, \\
x_{n+2} &= \frac{1}{x_{n-p+1}y_{n-p+1}z_{n-p+1}}, & y_{n+2} &= \frac{x_{n-p+1}y_{n-p+1}z_{n-p+1}}{x_{n-q+1}y_{n-q+1}z_{n-q+1}}, & z_{n+2} &= \frac{x_{n-q+1}y_{n-q+1}z_{n-q+1}}{x_{n-r+1}y_{n-r+1}z_{n-r+1}}, \\
x_{n+3} &= \frac{1}{x_{n-p+2}y_{n-p+2}z_{n-p+2}}, & y_{n+3} &= \frac{x_{n-p+2}y_{n-p+2}z_{n-p+2}}{x_{n-q+2}y_{n-q+2}z_{n-q+2}}, & z_{n+3} &= \frac{x_{n-q+2}y_{n-q+2}z_{n-q+2}}{x_{n-r+2}y_{n-r+2}z_{n-r+2}}, \\
& & & \vdots & & \\
x_{n-p+r-1} &= \frac{1}{x_{n-2p+r-2}y_{n-2p+r-2}z_{n-2p+r-2}}, \\
y_{n-p+r-1} &= \frac{x_{n-2p+r-2}y_{n-2p+r-2}z_{n-2p+r-2}}{x_{n-q-p+r-2}y_{n-q-p+r-2}z_{n-q-p+r-2}}, \\
z_{n-p+r-1} &= \frac{x_{n-q-p+r-2}y_{n-q-p+r-2}z_{n-q-p+r-2}}{x_{n-p-2}y_{n-p-2}z_{n-p-2}}, \\
x_{n-p+r} &= \frac{1}{x_{n-2p+r-1}y_{n-2p+r-1}z_{n-2p+r-1}}, \\
y_{n-p+r} &= \frac{x_{n-2p+r-1}y_{n-2p+r-1}z_{n-2p+r-1}}{x_{n-q-p+r-1}y_{n-q-p+r-1}z_{n-q-p+r-1}}, \\
z_{n-p+r} &= \frac{x_{n-q-p+r-1}y_{n-q-p+r-1}z_{n-q-p+r-1}}{x_{n-p-1}y_{n-p-1}z_{n-p-1}}, \\
x_{n-p+r+1} &= \frac{1}{x_{n-2p+r}y_{n-2p+r}z_{n-2p+r}}, \\
y_{n-p+r+1} &= \frac{x_{n-2p+r}y_{n-2p+r}z_{n-2p+r}}{x_{n-q-p+r}y_{n-q-p+r}z_{n-q-p+r}}, \\
z_{n-p+r+1} &= \frac{x_{n-q-p+r}y_{n-q-p+r}z_{n-q-p+r}}{x_{n-p}y_{n-p}z_{n-p}}, \\
x_{n-p+r+2} &= \frac{1}{x_{n-2p+r+1}y_{n-2p+r+1}z_{n-2p+r+1}}, \\
y_{n-p+r+2} &= \frac{x_{n-p+r+1}y_{n-p+r+1}z_{n-p+r+1}}{x_{n-q-p+r+1}y_{n-q-p+r+1}z_{n-q-p+r+1}}, \\
z_{n-p+r+2} &= \frac{x_{n-q-p+r+1}y_{n-q-p+r+1}z_{n-q-p+r+1}}{x_{n-p+1}y_{n-p+1}z_{n-p+1}},
\end{aligned}$$

$$\begin{aligned}
x_{n-p+r+3} &= \frac{1}{x_{n-2p+r+2}y_{n-2p+r+2}z_{n-2p+r+2}}, \\
y_{n-p+r+3} &= \frac{x_{n-p+r+2}y_{n-p+r+2}z_{n-p+r+2}}{x_{n-q-p+r+2}y_{n-q-p+r+2}z_{n-q-p+r+2}}, \\
z_{n-p+r+3} &= \frac{x_{n-q-p+r+2}y_{n-q-p+r+2}z_{n-q-p+r+2}}{x_{n-p+2}y_{n-p+2}z_{n-p+2}}, \\
&\vdots \\
x_{n-p+2r} &= \frac{1}{x_{n-2p+2r-1}y_{n-2p+2r-1}z_{n-2p+2r-1}}, \\
y_{n-p+2r} &= \frac{x_{n-2p+2r-1}y_{n-2p+2r-1}z_{n-2p+2r-1}}{x_{n-q-p+2r-1}y_{n-q-p+2r-1}z_{n-q-p+2r-1}}, \\
z_{n-p+2r} &= \frac{x_{n-q-p+2r-1}y_{n-q-p+2r-1}z_{n-q-p+2r-1}}{x_{n-p+r-1}y_{n-p+r-1}z_{n-p+r-1}} \\
&= x_{n-q-p+2r}y_{n-q-p+2r}z_{n-q-p+2r}x_{n-p-2}y_{n-p-2}z_{n-p-2}, \\
x_{n-p+2r+1} &= \frac{1}{x_{n-2p+2r}y_{n-2p+2r}z_{n-2p+2r}}, \\
y_{n-p+2r+1} &= \frac{x_{n-2p+2r}y_{n-2p+2r}z_{n-2p+2r}}{x_{n-q-p+2r}y_{n-q-p+2r}z_{n-q-p+2r}}, \\
z_{n-p+2r+1} &= \frac{x_{n-q-p+2r}y_{n-q-p+2r}z_{n-q-p+2r}}{x_{n-p+r}y_{n-p+r}z_{n-p+r}} \\
&= x_{n-q-p+2r}y_{n-q-p+2r}z_{n-q-p+2r}x_{n-p-1}y_{n-p-1}z_{n-p-1}, \\
x_{n-p+2r+2} &= \frac{1}{x_{n-2p+2r+1}y_{n-2p+2r+1}z_{n-2p+2r+1}}, \\
y_{n-p+2r+2} &= \frac{x_{n-p+2r+1}y_{n-p+2r+1}z_{n-p+2r+1}}{x_{n-q-p+2r+1}y_{n-q-p+2r+1}z_{n-q-p+2r+1}}, \\
z_{n-p+2r+2} &= \frac{x_{n-q-p+2r+1}y_{n-q-p+2r+1}z_{n-q-p+2r+1}}{x_{n-p+r+1}y_{n-p+r+1}z_{n-p+r+1}} \\
&= x_{n-q-p+2r+1}y_{n-q-p+2r+1}z_{n-q-p+2r+1}x_{n-p}y_{n-p}z_{n-p}, \\
x_{n-p+2r+3} &= \frac{1}{x_{n-2p+2r+2}y_{n-2p+2r+2}z_{n-2p+2r+2}}, \\
y_{n-p+2r+3} &= \frac{x_{n-p+2r+2}y_{n-p+2r+2}z_{n-p+2r+2}}{x_{n-q-p+2r+2}y_{n-q-p+2r+2}z_{n-q-p+2r+2}}, \\
z_{n-p+2r+3} &= \frac{x_{n-q-p+2r+2}y_{n-q-p+2r+2}z_{n-q-p+2r+2}}{x_{n-p+r+2}y_{n-p+r+2}z_{n-p+r+2}} \\
&= x_{n-q-p+2r+2}y_{n-q-p+2r+2}z_{n-q-p+2r+2}x_{n-p+1}y_{n-p+1}z_{n-p+1}, \\
&\vdots
\end{aligned}$$

$$\begin{aligned}
x_{n-q+r-1} &= \frac{1}{x_{n-p-q+r-2}y_{n-p-q+r-2}z_{n-p-q+r-2}}, \\
y_{n-q+r-1} &= \frac{x_{n-p-q+r-2}z_{n-p-q+r-2}}{x_{n-2q+r-2}y_{n-2q+r-2}z_{n-q-p+r-2}}, \\
z_{n-q+r-1} &= \frac{x_{n-2q+r-2}y_{n-2q+r-2}z_{n-q-p+r-2}}{x_{n-q-2}y_{n-q-2}z_{n-q-2}}, \\
x_{n-q+r} &= \frac{1}{x_{n-p-q+r-1}y_{n-p-q+r-1}z_{n-p-q+r-1}}, \\
y_{n-q+r} &= \frac{x_{n-p-q+r-1}z_{n-p-q+r-1}}{x_{n-2q+r-1}y_{n-2q+r-1}z_{n-2q+r-1}}, \\
z_{n-q+r} &= \frac{x_{n-2q+r-1}y_{n-2q+r-1}z_{n-2q+r-1}}{x_{n-q-1}y_{n-q-1}z_{n-q-1}}, \\
x_{n-q+r+1} &= \frac{1}{x_{n-p-q+r}y_{n-p-q+r}z_{n-p-q+r}}, \\
y_{n-q+r+1} &= \frac{x_{n-p-q+r}z_{n-p-q+r}}{x_{n-2q+r}y_{n-2q+r}z_{n-2q+r}}, \\
z_{n-q+r+1} &= \frac{x_{n-2q+r}y_{n-2q+r}z_{n-2q+r}}{x_{n-q}y_{n-q}z_{n-q}}, \\
x_{n-q+r+2} &= \frac{1}{x_{n-p-q+r+1}y_{n-p-q+r+1}z_{n-p-q+r+1}}, \\
y_{n-q+r+2} &= \frac{x_{n-p-q+r+1}z_{n-p-q+r+1}}{x_{n-2q+r+1}y_{n-2q+r+1}z_{n-2q+r+1}}, \\
z_{n-q+r+2} &= \frac{x_{n-2q+r+1}y_{n-2q+r+1}z_{n-2q+r+1}}{x_{n-q+1}y_{n-q+1}z_{n-q+1}}, \\
x_{n-q+r+3} &= \frac{1}{x_{n-p-q+r+2}y_{n-p-q+r+2}z_{n-p-q+r+2}}, \\
y_{n-q+r+3} &= \frac{x_{n-p-q+r+2}z_{n-p-q+r+2}}{x_{n-2q+r+2}y_{n-2q+r+2}z_{n-2q+r+2}}, \\
z_{n-q+r+3} &= \frac{x_{n-2q+r+2}y_{n-2q+r+2}z_{n-2q+r+2}}{x_{n-q+2}y_{n-q+2}z_{n-q+2}}, \\
&\vdots \\
x_{n-q+2r} &= \frac{1}{x_{n-p-q+2r-1}y_{n-p-q+2r-1}z_{n-p-q+2r-1}}, \\
y_{n-q+2r} &= \frac{x_{n-p-q+2r-1}z_{n-p-q+2r-1}}{x_{n-2q+2r-1}y_{n-2q+2r-1}z_{n-2q+2r-1}}, \\
z_{n-q+2r} &= \frac{x_{n-2q+2r-1}y_{n-2q+2r-1}z_{n-2q+2r-1}}{x_{n-q+r-1}y_{n-q+r-1}z_{n-q+r-1}} \\
&= x_{n-2q+2r-1}y_{n-2q+2r-1}z_{n-2q+2r-1}x_{n-q-2}y_{n-q-2}z_{n-q-2},
\end{aligned}$$

$$\begin{aligned}
x_{n-q+2r+1} &= \frac{1}{x_{n-p-q+2r}y_{n-p-q+2r}z_{n-p-q+2r}}, \\
y_{n-q+2r+1} &= \frac{x_{n-p-q+2r}y_{n-p-q+2r}z_{n-p-q+2r}}{x_{n-2q+2r}y_{n-2q+2r}z_{n-2q+2r}}, \\
z_{n-q+2r+1} &= \frac{x_{n-2q+2r}y_{n-2q+2r}z_{n-2q+2r}}{x_{n-q+r}y_{n-q+r}z_{n-q+r}}, \\
&= x_{n-2q+2r}y_{n-2q+2r}z_{n-2q+2r}x_{n-q-1}y_{n-q-1}z_{n-q-1}, \\
x_{n-q+2r+2} &= \frac{1}{x_{n-p-q+2r+1}y_{n-p-q+2r+1}z_{n-p-q+2r+1}}, \\
y_{n-q+2r+2} &= \frac{x_{n-p-q+2r+1}y_{n-p-q+2r+1}z_{n-p-q+2r+1}}{x_{n-2q+2r+1}y_{n-2q+2r+1}z_{n-2q+2r+1}}, \\
z_{n-q+2r+2} &= x_{n-2q+2r+1}y_{n-2q+2r+1}z_{n-2q+2r+1}x_{n-q}y_{n-q}z_{n-q}, \\
x_{n-q+2r+3} &= \frac{1}{x_{n-p-q+2r+2}y_{n-p-q+2r+2}z_{n-p-q+2r+2}}, \\
y_{n-q+2r+3} &= \frac{x_{n-p-q+2r+2}y_{n-p-q+2r+2}z_{n-p-q+2r+2}}{x_{n-2q+2r+2}y_{n-2q+2r+2}z_{n-2q+2r+2}}, \\
z_{n-q+2r+3} &= x_{n-2q+2r+2}y_{n-2q+2r+2}z_{n-2q+2r+2}x_{n-p+1}y_{n-p+1}z_{n-p+1}, \\
&\vdots \\
x_{n+r} &= x_{n-p-2}y_{n-p-2}z_{n-p-2}, \quad y_{n+r} = \frac{x_{n-q-2}y_{n-q-2}z_{n-q-2}}{x_{n-p-2}y_{n-p-2}z_{n-p-2}}, \\
z_{n+r} &= \frac{1}{x_{n-q-2}y_{n-q-2}z_{n-q-2}x_{n-1}y_{n-1}z_{n-1}}, \\
x_{n+r+1} &= x_{n-p-1}y_{n-p-1}z_{n-p-1}, \quad y_{n+r+1} = \frac{x_{n-q-1}y_{n-q-1}z_{n-q-1}}{x_{n-p-1}y_{n-p-1}z_{n-p-1}}, \\
z_{n+r+1} &= \frac{1}{x_{n-q-1}y_{n-q-1}z_{n-q-1}x_n y_n z_n}, \\
x_{n+r+2} &= x_{n-p}y_{n-p}z_{n-p}, \quad y_{n+r+2} = \frac{x_{n-q+1}y_{n-q+1}z_{n-q+1}}{x_{n-p}y_{n-p}z_{n-p}}, \\
z_{n+r+2} &= \frac{x_{n-r}y_{n-r}z_{n-r}}{x_{n-q+1}y_{n-q+1}z_{n-q+1}}, \\
x_{n+r+3} &= x_{n-p+1}y_{n-p+1}z_{n-p+1}, \quad y_{n+r+3} = \frac{x_{n-q+2}y_{n-q+2}z_{n-q+2}}{x_{n-p+1}y_{n-p+1}z_{n-p+1}}, \\
z_{n+r+3} &= \frac{x_{n-r+1}y_{n-r+1}z_{n-r+1}}{x_{n-q+2}y_{n-q+2}z_{n-q+2}}, \\
x_{n+r+4} &= x_{n-p+2}y_{n-p+2}z_{n-p+2}, \quad y_{n+r+4} = \frac{x_{n-q+3}y_{n-q+3}z_{n-q+3}}{x_{n-p+2}y_{n-p+2}z_{n-p+2}},
\end{aligned}$$

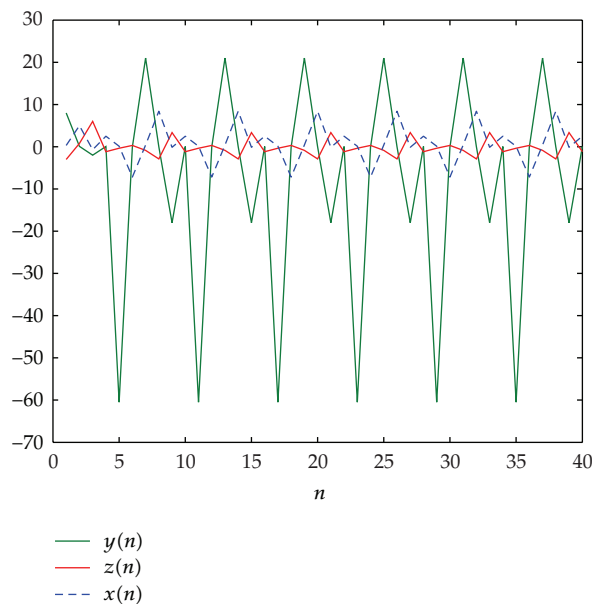


Figure 2

$$\begin{aligned}
 z_{n+r+4} &= \frac{x_{n-r+2}y_{n-r+2}z_{n-r+2}}{x_{n-q+3}y_{n-q+3}z_{n-q+3}}, \\
 &\vdots \\
 x_{n+2r+1} &= \frac{1}{x_{n-p-2}y_{n-p-2}z_{n-p-2}}, & y_{n+2r+1} &= \frac{x_{n-p-2}y_{n-p-2}z_{n-p-2}}{x_{n-q-2}y_{n-q-2}z_{n-q-2}}, \\
 z_{n+2r+1} &= \frac{x_{n-q-2}y_{n-q-2}z_{n-q-2}}{x_{n-1}y_{n-1}z_{n-1}}, \\
 x_{n+2r+2} &= \frac{1}{x_{n-p-1}y_{n-p-1}z_{n-p-1}}, & y_{n+2r+2} &= \frac{x_{n-p-1}y_{n-p-1}z_{n-p-1}}{x_{n-q-1}y_{n-q-1}z_{n-q-1}}, \\
 z_{n+2r+2} &= \frac{x_{n-q-1}y_{n-q-1}z_{n-q-1}}{x_n y_n z_n}, \\
 x_{n+2r+3} &= x_{n+1}, & y_{n+2r+3} &= y_{n+1}, & z_{n+2r+3} &= z_{n+1}.
 \end{aligned} \tag{2.4}$$

Hence, the proof is completed. \square

Example 2.4. Consider the difference system (2.3) with $p = 1$, $q = 0$, $r = 2$, and the initial conditions $x_{-2} = 0.3$, $x_{-1} = 5$, $x_0 = -0.7$, $y_{-2} = 8$, $y_{-1} = 0.1$, $y_0 = 2$, $z_{-2} = 3$, $z_{-1} = 0.8$, and $z_0 = 6$ (See Figure 2).

Proposition 2.5. *It is easy to see by induction that the following general system is periodic with period $(2p_k + 2)$:*

$$\begin{aligned} x_{n+1}^1 &= \frac{1}{\prod_{i=1}^k x_{n-p_1}^i}, & x_{n+1}^2 &= \frac{\prod_{i=1}^k x_{n-p_1}^i}{\prod_{i=1}^k x_{n-p_2}^i}, & x_{n+1}^3 &= \frac{\prod_{i=1}^k x_{n-p_2}^i}{\prod_{i=1}^k x_{n-p_3}^i}, \dots, \\ x_{n+1}^j &= \frac{\prod_{i=1}^k x_{n-p_{j-1}}^i}{\prod_{i=1}^k x_{n-p_j}^i}, \dots, & x_{n+1}^k &= \frac{\prod_{i=1}^k x_{n-p_{k-1}}^i}{\prod_{i=1}^k x_{n-p_k}^i}. \end{aligned} \quad (2.5)$$

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