Research Article

Stackelberg Game for Product Renewal in Supply Chain

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The paper studied the process of product renewal in a supply chain, which is composed of one manufacturer and one retailer. There are original product and renewal product in the supply chain. A market share shift model for renewal product was firstly built on a increment function and a shift function. Based on the model, the decision-making plane consisting of two variables was divided into four areas. Since the process of product renewal was divided into two stages, Stackelberg-Nash game model and Stackelberg-merger game model could be built to describe this process. The optimal solutions of product pricing strategy of two games were obtained. The relationships between renewal rate, cost, pricing strategy, and profits were got by numerical simulation. Some insights were obtained from this paper. Higher renewal rate will make participants' profits and total profit increase at the same margin cost. What is more important, the way of the optimal decision making of the SC was that RP comes onto the market with a great price differential between OP and RP.

1. Introduction

With the development of IT, the speed of product renewal process becomes faster and faster. This situation brings some new problems to traditional supply chain (SC), such as the dynamic nature of an SC and the relationship coordination between participants of an SC. The dynamic nature of an SC includes the dynamic variety of products; that is to say, the market requests SC to satisfy diverse needs of customer with the fastest speed, as well as the best quality, which calls for SC improving performance to adapt to the product variety. A valid way for SC enterprises to follow the variety of market is to renew their existing products, which can make full use of enterprise's existing resources and supply/distribution outlet of SC. Thus, SC will adapt to a variational market economically and quickly. Product renewal is an effective method that strengthens a SC enterprise's core competitiveness, and it will even impact on the survival of a enterprise. There are quite a few cases that enterprises collapse because of the mistakes of product renewal decision making, such as the failure of WANGAN Computer Corp.

Now, it is necessary to differentiate "renewal product (RP)" from "innovated product (IP)." IP's structure and principle are different from original product (OP), while RP's main structure and principle are the same as OP. Furthermore, RP has some appended components or upgraded functions. Thereby, RP is the renewal of existing product. At present, most researches are mainly focused on complete new product innovation, including management of product innovation process, optimization of product innovation investment, promotion of new product, and design of product innovation drive mechanism. Dereli et al. proposed a framework of the rapid response for innovative product development using reverse engineering approach [1]. Bourreau et al. studied the effects of modular design on firms' product innovation strategies and postinnovation competition in digital market [2]. Ju and Xiao built a model of product lifecycle evaluation based on context knowledge [3]. Wan et al. built a new product investment model [4]. Researches about product innovation based on SC frame have also emerged. Huo has studied the R&D strategy in SC using game theory [5].

At present, the researches concerning product renewal have been carried out. Bass P. I. and Bass F. M. had proposed a new product expansion model and studied the process of multigeneration renewal products coming onto market [6, 7]. Wu examined the reuse/redesign, quality, speed-to-market,

and marketing decisions for two consecutive generations of a multicomponent modular product, utilizing stylized models [8]. Druehl et al. developed a model to gain insight into which factors drive the pace of product updates [9]. Huang and Ling studied the tracks of product update of ICT enterprises [10]. Koca et al. studied the product rollover strategy decision, where a firm decides whether to phase out an old generation product to be replaced by a new with either a dual or single roll [11]. Quan analyzed the relationship of the optimal launch time with parameters in the process of introducing a renewal product [12].

The other focus of product renewal is on the marketing process of renewal product, especially pricing decision. Luo and Tu studied a price decision problem for product renewal supply chain based on Nash game [13]. Wei studied the optimal decision of inventory and pricing in a supply chain, in which there are original products and renewal products [14]. But the work of product renewal is far less than enough.

The RP and OP are virtually differentiated products. When they are on sale at the same market, they can substitute for each other. Compared with common differentiated products, the substitution of RP for OP is unidirectional and partial. Therefore, the coming of RP onto market will cause some influences on OP market. At the same time, each entity in SC has independent profit. Therefore, the coordination of SC will become much more complex if there are RP and OP in the SC. A potent tool to coordinate SC is price. The fluctuation of price will make supply and demand tend to an equilibrium. On the other hand, price's fluctuation can make the profit of each entity in SC distributed reasonably, so as to achieve the aim of coordination of SC. Therefore pricing strategy is the main content of the paper.

The SC including OP and RP is called product renewal SC. The change of market share caused by product renewal in the SC is studied in this paper. When there is a RP in SC, the enterprises in SC will make strategies for both of OP and RP to achieve an equilibrium of enterprises' profits. Compared with other researches, this paper firstly studied Stackelberg game for product renewal, aiming at solving complex decision-making question in actual renewal SC, and obtained some novel insights.

The rest of this paper is organized as follows. Product renewal model is formulated in Section 2. In Section 3, we provide a Stackelberg-Nash Game based on product renewal model. Stackelberg-merger game is played in Section 4, and numerical examples are provided in Section 5. Finally, we conclude our paper in Section 6.

2. Model Description

2.1. Assumption. Consider a two-stage SC which is composed of one manufacturer and one retailer. First, suppose the manufacturer produces product *A* and sells it in market via the retailer. After a period of renewal process, the manufacturer develops product *A*'s renewal product, *B*, and sells it with product *A* in market via the retailer.

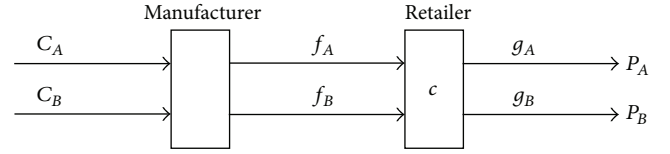In order to fully describe the model of the SC, we state the following hypotheses.

FIGURE 1: The structure of renewal products SC.

 (i) The retailer is in a monopoly position; that is, only the retailer sells *A* and *B* in market.

 (ii) Suppose that the consumer's repurchase is possible and each time one customer only buys one product. When there is an RP, some customers of OP may shift to buying RP in other words, there is repetition purchase for the products studied in this paper, such as toys, and small electric appliance equipment.

(iii) There is no stochastic demand in the market, so the demand is only decided by price.

(iv) Inventory is not considered.

 (v) The manufacturer's productivity is sufficient enough to satisfy demand.

(vi) There is no fixed cost for unit product, and margin production cost is an invariant for manufacturer.

(vii) The product renewal generally causes its performance promotion.

The SC model of this paper is as follows: there is one manufacturer and one retailer in SC. Suppose that the manufacturer produces products *A* and *B* at marginal costs C_A and C_B, respectively, and then distributes them at the wholesale price of f_A and f_B to retailer. Based on the wholesale price, a price markup g_A and g_B is, respectively, added by retailer to product *A* and *B* to form the market retail price: $P_i = g_i + f_i$. Retailer's selling cost is *c* per unit. The product renewal SC including one manufacturer and one retailer that sells combination products is shown in Figure 1.

2.2. Market Model. Product renewal rate is used to measure the change of product performance caused by product renewal. Combined with price and performance price ratio ($E_{ij} = kP_j/P_i$), product renewal rate can be used to forecast the market share change. In fact, what is focused on is not performance value, but the ratio of OP's performance value to RP's [15], namely renewal rate *k* ($k > 1$).

The RP marketing process can be divided into two stages:

In Stage 1. there is only OP in market. Supposing the OP price of *A* is P_A, according to [15], the market demand for *A* is

$$T = a - bP_A. \tag{1}$$

In expression (1), *a* means market scale, the absolute value of *b* denotes price flexibility, and $b > 0$.

In Stage 2. RP comes onto the market and competes with OP. If RP comes onto the market, it will cause

several changes in the market: (1) RP will stimulate new consumption demand because of its appended or upgraded functions, which makes the latent customer quantity of the total products market enlarge, and the increment is ΔT. ΔT is related to original market demand T and a decreasing function of RP price, while an increasing function of renewal rate k. (2) Customers have the tendency to purchase RP. In addition, the performance price ratio of RP is generally higher than that of OP. So some OP's customers will shift to buying RP. The number of shift customers is marked as T_s. (3) This will cause OP's price to be adjusted to market.

Definition 1. There exists an OP A and its RP B in the market. Suppose that the demand of A is T, and the product renewal rate is k. Then ΔT satisfies the following function:

$$\Delta T = w_1 T \left(k - w_2 P_B \right), \tag{2}$$

where w_1 is the influence coefficient of T and $0 < w_1 < 1$. w_2 is the influence coefficient of RP's price on its market increment, and $w_2 \ll 1$.

From $\Delta T = 0$ and $T = 0$, $P_{B1} = k/w_2$ and $P_{A1} = a/b$ can be got. If $P_A > P_{A1}$, OP's demand will be zero, which leads ΔT to be zero. If $P_B > P_{B1}$, no matter how great OP's demand is, demand increment ΔT will be equal to zero.

Theorem 2. *The market shift function T_s relates to OP price, RP price, and OP demand. T_s has the following features.*

  (i) *If $P_B \leq P_A$, $T_s = T$, that is, all of the OP demand will shift to RP. So OP will have to exit from the market, and RP will hold the market.*

  (ii) *Existing $P_{B0} > PA$, if $P_B \geq P_{B0}$, $T_s = 0$. P_{B0} can be treated as the zero point of market shift.*

  (iii) *If $P_A < P_B < P_{B0}$, T_s will be a monotone decreasing function of P_B, namely, $\partial T_s / \partial P_B < 0$.*

  (iv) *If $P_A < P_B < P_{B0}$, T_s is to be a monotone increasing function of P_A, namely, $\partial T_s / \partial P_A > 0$.*

  (v) *With the increase of k, P_{B0} increases correspondingly. T_s is an increasing function of k, namely, $\partial T_s / \partial k > 0$.*

  (vi) *Let $\Delta T = 0$, and the corresponding price is P_{B1}, which is the upper limit of RP price.*

  (vii) *Let $T_s = 0$, and there is still demand increment caused by RP; that is to say, on any condition, $P_{B0} < P_{B1}$.*

 (viii) *T_s is a direct proportion increasing function with T.*

From Theorem 2, given P_A, the laws of changes of T_s and ΔT can be got with the variety of P_B, as shown in Figure 2. The specific shape of this figure is changing with the variety of P_A.

According to Figure 2, the shift function T_s in $[0, P_A]$ is

$$T_{s1} = T. \tag{3}$$
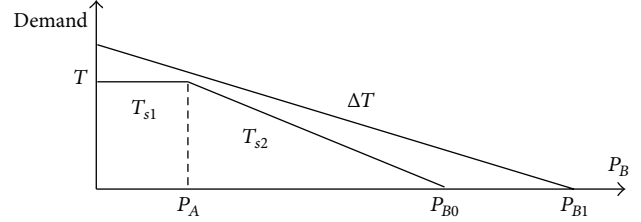

Figure 2: The changes of T_s and ΔT with the variety of P_B.

Definition 3. According to Figure 2, in $[P_A, P_{B0}]$, there exist an OP A and its RP B; suppose the prices of A, B are P_A, P_B, respectively, and the demand of A is T. Then, T_{s2} satisfies the following function:

$$T_{s2} \left(P_A, P_B \right) = w_3 T \left(k w_4 P_A - P_B \right), \tag{4}$$

where w_3 is the influence coefficient of P_A and P_B, $0 < w_1 < 1$, w_4 is the influence coefficient of P_A, and $w_4 > 1/k$.

The functions (3) and (4) meet eight features listed previewed, and can be used as the paper's model base. From the previous analysis, the shift function includes two portions, so it can be expressed as follows:

$$T_s = \begin{cases} T & P_B \leq P_A, \\ \left[w_3 T \left(k w_4 P_A - P_B \right) \right]^+ & P_B > P_A. \end{cases} \tag{5}$$

Letting $T_s = 0$ and solve (5), the zero point of market shift function P_{B0} can be got:

$$P_{B0} = k w_4 P_A. \tag{6}$$

With the increase of k, P_{B0} obviously becomes higher. Therefore, when RP comes onto the market, the total demand will change. The market share of RP B is

$$T_B = \Delta T + T_s, \tag{7}$$

and OP A is

$$T_A = T - T_s. \tag{8}$$

In addition, product renewal will make marginal production cost change from C_A to C_B.

Figure 2 is a variety chart of shift demand T_s in the situation that P_A is given, so the shift demand T_s actually depends on P_A and P_B simultaneously. Accordingly, a three-dimensional coordinate can be built, P_A-P_B-T_s, as shown in Figure 3, and axis T_s is shift demand. Plane P_A-P_B is divided into the following four areas by six lines in Figure 3:

Area I: the area surrounded by axis P_A and lines $P_A = P_B$ and $P_A = a/b$,

Area II: the area surrounded by lines $P_A = P_B$, $P_A = a/b$ and $P_B = k w_4 P_A$,
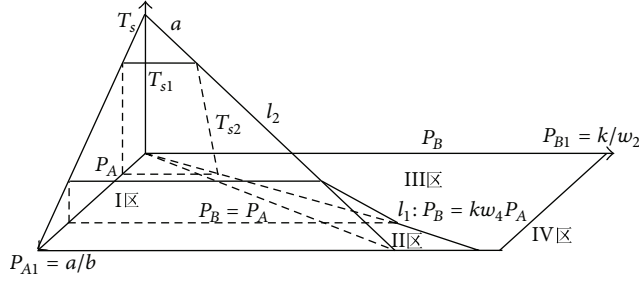
Figure 3: The change of T_s with the variety of P_B and P_A.

Table 1: The table of area's shift function and increment function.

	$T_s(P_A, P_B)$	$\Delta T(P_A, P_B)$
Area I	$T(P_A)$	$w_1 T(P_A)(k - w_2 P_B)$
Area II	$w_3 T(P_A)(kw_4 P_A - P_B)$	$w_1 T(P_A)(k - w_2 P_B)$
Area III	0	$w_1 T(P_A)(k - w_2 P_B)$
Area IV	0	0

Area III: the area surrounded by axis P_B and lines $P_B = kw_4 P_A$, $P_A = a/b$ and $P_B = k/w_2$,

Area IV: the areas excepting area I, II, and III.

In area I, Letting $P_A = 0$, T_s's maximum value a can be obtained from (1) and (2). Given P_A, the line T_{s1} will be parallel to plane P_A-P_B, and it is clear that the slope of line T_{s1} is $k_{Ts1} = 0$. From (3), with the increasing of P_A, T will become small. Meanwhile, T_{s1} will become small. If $P_A = a/b$, $T = T_{s1} = 0$. Therefore, T_s will be a sloped plane in area I.

In area II, given P_A, $k_{Ts2} = -w_3 T < 0$ can be obtained from (5). The line T_{s2} will become smoother and smoother with the increase of P_A and k_{Ts2}. If $P_A = a/b$, $k_{Ts2} = T_{s2} = 0$. Letting $T_{s2} = 0$ in (4), a line l_1 : $P_B = kw_4 P_A$ in P_A-P_B plane can be got. The line linking maximal T_s point $(0, 0, a)$ to point $(a/b, a/b, 0)$ is a T_s line named l_2. The lines linking the points at which line l_1 or l_2 intersects with T_{s2} constitute T_s complex curving surface; in other words, T_s in area II is a complex curving surface.

In area III, $P_B > kw_4 P_A$, $T_s = 0$ can be got from (5).

Considering T_s function's feature (vii), it is obvious that market increment $\Delta T > 0$ in area I, II, and III.

In area IV, $P_A > a/b$ or $P_B > k/w_2$, $\Delta T = T_s = 0$. OP and RP do not come onto the market in this area.

The shift functions and increment functions in different areas are shown in Table 1.

According to the feature (vii) of shift function, the zero point of increment function is bigger than that of shift function, namely, $P_{B1} > P_{B0} = kw_4 P_A$. From expression (1), it is known that the maximum of P_A is a/b. So it requires $b > aw_2 w_4$ to meet the seventh feature of shift function. Based on the previous analysis, the retailer's and manufacturer's profits including the profits of OP and RP are, respectively, as follows:

$$\pi_r = (T_s + \Delta T)(g_B - c) + (T - T_s)(g_A - c),$$
$$\pi_m = (\Delta T + T_s)(f_B - C_B) + (T - T_s)(f_A - C_A) - C_g. \tag{9}$$

Where C_g is the fixed cost of the manufacturer. According to the price relationship between RP and OP in different areas, the profit functions can be shown as follows.

(i) In area I, $P_B \leq P_A$, $\Delta T > 0$ and $T_s = T > 0$. The market share of RP B is $\Delta T + T_s$, and OP A is zero. The profits of retailer and manufacturer are

$$
\begin{aligned}
\pi_r(g_A, g_B) &= T(1 + w_1(k - w_2 P_B))(g_B - c) \\
&= (a - b(f_A + g_A)) \\
&\quad \times (1 + w_1(k - w_2(f_B + g_B)))(g_B - c), \\
\pi_m(f_A, f_B) &= T(1 + w_1(k - w_2 P_B))(f_B - C_B) - C_g \\
&= (a - b(f_A + g_A)) \\
&\quad \times (1 + w_1(k - w_2(f_B + g_B)))(f_B - C_B) - C_g.
\end{aligned}
\tag{10}
$$

(ii) In area II, $P_A \leq P_B \leq P_{B0}$, $\Delta T > 0$ and $T_s > T > 0$. The market share of RP B is $\Delta T + T_s$, and OP A is $T - T_s$. The profits of retailer and manufacturer are

$$
\begin{aligned}
\pi_r(g_A, g_B) &= T\{(w_1(k - w_2 P_B) + w_3(kw_4 P_A - P_B)) \\
&\quad \times (g_B - c) + (1 - w_3(kw_4 P_A - P_B)) \\
&\quad \times (g_A - c)\} \\
&= (a - b(f_A + g_A)) \\
&\quad \times \{(w_1(k - w_2(f_B + g_B)) \\
&\quad + w_3(kw_4(f_A + g_A) \\
&\quad - (f_B + g_B)))(g_B - c) \\
&\quad + (1 - w_3(kw_4(f_A + g_A) - (f_B + g_B))) \\
&\quad \times (g_A - c)\}, \\
\pi_m(f_A, f_B) &= T\{(w_1(k - w_2 P_B) + w_3(kw_4 P_A - P_B))(f_B - C_B) \\
&\quad + (1 - w_3(kw_4 P_A - P_B))(f_A - C_A)\} - C_g \\
&= (a - b(f_A + g_A)) \\
&\quad \times \{(w_1(k - w_2(f_B + g_B)) \\
&\quad + w_3(kw_4(f_A + g_A) \\
&\quad - (f_B + g_B)))(f_B - C_B) \\
&\quad + (1 - w_3(kw_4(f_A + g_A) - (f_B + g_B))) \\
&\quad \times (f_A - C_A)\} - C_g.
\end{aligned}
\tag{11}
$$

(iii) In area III, $P_{B0} \le P_B \le P_{B1}$, $\Delta T > 0$ and $T_s = 0$. RP B's market share is ΔT, and OP A's market share is T. The profits of retailer and manufacturer are

$$
\begin{aligned}
\pi_r(g_A, g_B) &= T\{w_1(k - w_2 P_B)(g_B - c) + g_A - c\} \\
&= (a - b(f_A + g_A)) \\
&\quad \times \{w_1(k - w_2(f_B + g_B))(g_B - c) + g_A - c\}, \\
\pi_m(f_A, f_B) &= T\{w_1(k - w_2 P_B)(f_B - C_B) + f_A - C_A\} - C_g \\
&= (a - b(f_A + g_A)) \\
&\quad \times \{w_1(k - w_2(f_B + g_B)) \\
&\quad \times (f_B - C_B) + f_A - C_A\} - C_g.
\end{aligned}
\tag{12}
$$

(iv) In area IV, $P_B > P_{B1}$ or $P_A > P_{A1}$, $\Delta T = T_s = 0$. The prices of OP and RP are so high that it could not come onto the market. It is not necessary to consider this situation.

3. Stackelberg-Nash Game

Product renewal process can be regarded as two stages. First, manufacturer and retailer make their optimal decisions for OP A, respectively, and synchronously. After OP A comes onto market, RP B is developed by the manufacturer and comes onto market, too. In this condition, both OP A and RP B are sold in the same market. Then the manufacturer and retailer make their optimal decisions for RP B, respectively, and synchronously. The optimal decision-making process is also divided into two stages. The two-stage games constitute a Stackelberg-Nash game, which is the real process of product renewal game.

> In Stage 1: There is a decision for product A. The manufacturer and the retailer play a Nash game. The manufacturer makes a strategic decision of f_A, and the retailer makes a strategic decision of g_A.

> In Stage 2: There is a decision for product B. The manufacturer and the retailer also play a Nash game. The manufacturer makes a strategic decision of f_B, and the retailer makes a strategic decision of g_B.

The game solving process is divided into three steps. Following three steps, the optimal solution of product renewal Stackelberg-Nash game can be derived.

> In Step 1, given f_A and g_A, according to the Nash equilibrium of the manufacturer and retailer profit functions, the optimal reaction function of $f_B(f_A, g_A)$, $g_B(f_A, g_A)$ or the optimal value of g_B and f_B can be obtained from $\partial \pi_r / \partial g_B = 0$, $\partial \pi_m / \partial f_B = 0$.

> In Step 2, put the optimal reaction function or the optimal value into the profit functions of Step 1, and the optimal solution $f_A{}^*$ and $g_A{}^*$ can be obtained from the Nash equilibrium of the manufacturer and retailer profit functions in Step 1.

In Step 3, letting $f_A{}^*$ and $g_A{}^*$ substitute f_A and g_A in the $f_B(f_A, g_A)$ and $g_B(f_A, g_A)$ of Step 2, the optimal value of $f_B{}^*$ and $g_B{}^*$ can be obtained.

Theorem 4. *In Stackelberg-Nash game, in areas I, II, and III, given f_A and g_A, π_r is a convex function of g_B, and π_m is convex functions of f_B.*

Proof. In area I

$$
\begin{aligned}
\frac{\partial \pi_r(g_A, g_B)}{\partial g_B} &= (a - b(f_A + g_A)) \\
&\quad \times (-2w_1 w_2 g_B + w_1 w_2(c - f_B) + w_1 k + 1), \\
\frac{\partial \pi_m(f_A, f_B)}{\partial f_B} &= (a - b(f_A + g_A)) \\
&\quad \times (-2w_1 w_2 f_B + w_1 w_2(C_B - g_B) + w_1 k + 1), \\
\frac{\partial \pi_r^2(g_A, g_B)}{\partial g_B^2} &= -2w_1 w_2(a - b(f_A + g_A)) < 0, \\
\frac{\partial \pi_m^2(f_A, f_B)}{\partial f_B^2} &= -2w_1 w_2(a - b(f_A + g_A)) < 0.
\end{aligned}
\tag{13}
$$

Therefore, π_r is a convex function of g_B, and π_m is a convex function of f_B. The proofs in II and III are the same as area I. □

Theorem 5. *The Stackelberg-Nash decision-making process can be divided into the following three situations.*

(i) *In area I, there is no Stackelberg-Nash equilibrium.*

(ii) *In area II, there exists only one optimal pricing strategy for Stackelberg-Nash game.*

(iii) *In area III, there exists only one optimal pricing strategy for Stackelberg-Nash game.*

Proof. (i) In area I, according to (10), based on Theorem 4 and Step 1, the optimal value of g_B and f_B can be derived as follows:

$$
\begin{aligned}
g_B{}^* &= \frac{(1 + w_1 k + 3w_1 w_2(2c - C_B))}{(3w_1 w_2)}, \\
f_B{}^* &= \frac{(1 + w_1 k + 3w_1 w_2(2C_B - c))}{(3w_1 w_2)}.
\end{aligned}
\tag{14}
$$

From the supply chain's structure, it is obvious that $g_B{}^* + f_B{}^* = P_B{}^*$. Because $P_B{}^*$ is greater than P_{A1} (from Theorem 6), considering the condition of area I, $P_B \le P_A$, the conclusion that $P_B{}^*$ is out of area I can be drawn. Accordingly, there is no Nash equilibrium in area I.

(ii) In area II, $P_A \le P_B \le P_{B0}$. According to (11), based on Theorem 4 and Step 1, the optimal reaction

$(f_B(f_A, g_A), g_B(f_A, g_A))$ of the manufacturer and retailer can be got as follows:

$$
\begin{aligned}
f_B(f_A, g_A) =\ & (w_3(kw_4 + 2)f_A + w_3(kw_4 - 1)g_A \\
& + w_1k + 2w_3(C_B - C_A) \\
& + w_1w_2(2C_B - c))/(3w_3 + 3w_1w_2), \\
g_B(f_A, g_A) =\ & (w_3(kw_4 - 1)f_A + w_3(kw_4 + 2)g_A \\
& + w_1k + w_3(C_A - C_B) \\
& + w_1w_2(2c - C_B))/(3w_3 + 3w_1w_2).
\end{aligned}
\tag{15}
$$

From Step 2, two binary cubic equations can be got. Because the analytic solution of the equations cannot be found, the numerical method can be used to find the optimal approximate solution $(f_A{}^*, g_A{}^*)$. From Step 3, the optimal of $(f_B{}^*, g_B{}^*)$ can be got.

(iii) In area III, $P_{B0} \le P_B \le P_{B1}$. According to (12), based on Theorem 4 and Step 1, the optimal values of g_B and f_B of π_r and π_m are as follows:

$$
\begin{aligned}
g_B{}^* &= \frac{(k + w_2(2c - C_B))}{(3w_2)}, \\
f_B{}^* &= \frac{(k + w_2(2C_B - c))}{(3w_2)}.
\end{aligned}
\tag{16}
$$

From Step 2, the optimal values $g_A{}^*$ and $f_A{}^*$ of π_r and π_m can be got as follows:

$$
\begin{aligned}
g_A{}^* =\ & -\left(w_1 b(k - w_2 C_B)^2 + 2w_1 w_2 bc(w_2 C_B - 1) + w_1 bc^2 w_2^2 \right. \\
& \left. + 9bw_2(C_A - 2c) - 9000w_2\right)/(27w_1 b), \\
f_A{}^* =\ & -\left(w_1 b(k - w_2 C_B)^2 + 2w_1 w_2 bc(w_2 C_B - 1) + w_1 bc^2 w_2^2 \right. \\
& \left. + 9bw_2(c - 2C_A) - 9000w_2\right)/(27w_1 b).
\end{aligned}
\tag{17}
$$

Theorem 6. *The value of Nah equilibrium for area I, $P_B{}^*$, is out of area I.*

Proof. From the supply chain structure, it is obvious that: $g_B{}^* + f_B{}^* = P_B{}^*$:

$$
P_B{}^* = g_B{}^* + f_B{}^* = \frac{(2 + 2w_1 k)}{(3w_1 w_2)} + C_B - c.
\tag{18}
$$

Because of $0 < w_1 < 1$ and $k > 1$,

$$
P_B{}^* \ge \frac{(2 + 2k)}{(3w_2)} + C_B - c \ge \frac{4}{(3w_2)} + C_B - c,
\tag{19}
$$

whereas $w_2 \ll 1$,

$$
P_B{}^* \ge P_{A1} = \frac{a}{b}.
\tag{20}
$$

It is clear that the value of Nah equilibrium, $P_B{}^*$, is greater than P_{A1}. Since P_{A1} is the maximum of P_A in area I, considering the condition of area I, $P_B \le P_A$, the value of Nah equilibrium, $P_B{}^*$, does not satisfy this condition. So $P_B{}^*$ is out of area I. □

4. Stackelberg-Merger Game

In the decision-making process of Stackelberg-Nash game mentioned previously, if the manufacturer and retailer constitute a manufacturing and sales league to maximize the SC profit, that is, there is no retailer, and manufacturer sells its product in market directly, the product renewal Stackelberg-Nash decision-making process will become a Stackelberg-merger decision-making process. This process can be divided into two stages, and this process will play a Stackelberg-merger game. In this condition, the league which consists of the manufacturer and the retailer can be regarded as a enterprise, because the league's decision variables are the retail price P_i of OP and RP (f_i and g_i can be regarded as the transfer price of the manufacturer and retailer profits distribution).

> In Stage 1, the league sells OP A and makes a strategic decision of P_A.
>
> In Stage 2, the league sells OP A and RP B at the same time, and the league makes strategic decisions of P_B and P_A.

The solving process of Stackelberg-merger game is divided into three steps, and by three steps the optimal solution of product renewal Stackelberg-merger game can be derived.

> In Step 1, OP retail price P_A is given; the optimal reaction function $P_B(P_A)$ or the optimal value of P_B can be obtained from $\partial\pi/\partial P_B = 0$.
>
> In Step 2, put the optimal reaction function $P_B(P_A)$ or the optimal value $P_B{}^*$ into the first stage; the optimal pricing $P_A{}^*$ can be obtained from $\partial\pi/\partial P_A = 0$.
>
> In Step 3, put $P_A{}^*$ into the optimal reaction function $P_B(P_A)$; the optimal pricing $P_B{}^*$ can be derived.

Theorem 7. *In Stackelberg-merger game, in areas I, II, and III, given P_A, π is a convex function of P_B.*

Proof. In area I

$$
\begin{aligned}
\frac{\partial\pi(P_A, P_B)}{\partial P_B} = &\ (a - bP_A) \\
& \times (-2w_1 w_2 P_B + w_1 w_2(C_B + c) + w_1 k + 1),
\end{aligned}
\tag{21}
$$

$$
\frac{\partial\pi^2(P_A, P_B)}{\partial P_B^2} = -2w_1 w_2(a - bP_A) < 0.
\tag{22}
$$

Therefore, π is a convex function of P_B. The proofs in II and III are the same as those of area I. □

Theorem 8. *The process of the Stackelberg-merger game is divided into three situations according to areas I, II, III. There exists only one optimal pricing strategy for Stackelberg-merger game in each situation.*

Proof. (i) In area I, $P_B \leq P_A$. the total profit of the SC can be got from the sum of (10):

$$\pi(P_A, P_B) = (a - bP_A)(w_1(k - w_2 P_B) + 1) \times (P_B - C_B - c) - C_g. \tag{23}$$

Base on Theorem 7 and Step 1, the optimal value P_B of π can be obtained as follows:

$$P_B^* = \frac{(1 + w_1 k)}{(2w_1 w_2)} + \frac{(C_B + c)}{2}. \tag{24}$$

The value of w_2 is so small that the value of P_B^* is greater than the maximum value of P_{A1}. Considering $P_B \leq P_A$ and Theorem 7, the maximum value of π can be obtained in the condition that $P_B = P_A$, and then (21) can be expressed as follows:

$$\pi(P_A) = (a - bP_A)(w_1(k - w_2 P_A) + 1)(P_A - C_B - c) - C_g. \tag{25}$$

From $\partial \pi / \partial P_A = 0$, the two values of P_A can be got. One solution is so small that it cannot match the situation, and it is cast out. The other is the optimal value of P_A. So there is only one solution for area I (from Theorem 9).

(ii) In area II, the total profit of SC can be got from the sum of (11):

$$\pi(P_A, P_B) = (a - bP_A) \\ \times \{(w_3(kw_4 P_A - P_B) + w_1(k - w_2 P_B)) \\ \times (P_B - C_B - c) \\ + (1 - w_3(kw_4 P_A - P_B))(P_A - C_A - c)\} - C_g. \tag{26}$$

Based on Theorem 7 and Step 1, the optimal reaction $P_B(P_A)$ of π can be obtained as follows:

$$P_B(P_A) \\ = \frac{(w_3(kw_4 + 1)P_A + w_1 w_2(C_B + c) + w_1 k + w_3 C_A)}{2(w_1 w_2 + w_3)}. \tag{27}$$

From Step 2, P_A^* can be got, and the expression of P_A^* is so long that it could not be written down here.

From Step 3, the optimal value of P_B for π can be obtained.

(iii) In area III, the total profit of SC can be derived from the sum of (12):

$$\pi(P_A, P_B) = (a - bP_A) \\ \times (w_1(k - w_2 P_B)(P_B - C_B - c) + P_A - C_A - c). \tag{28}$$

Based on Theorem 7 and Step 1, the optimal value of P_B for π can be obtained as follows:

$$P_B^* = \frac{k}{2w_2} + \frac{(C_B + c)}{2}. \tag{29}$$

From Step 2, the optimal of P_A for π can be obtained as follows:

$$P_A^* = \frac{a}{2b} \\ - \frac{(w_1(k - (C_B + c))(k - w_2(C_B + c)) - 4(C_A + c))}{8}. \tag{30}$$

□

Theorem 9. *In area I, the optimal value of P_A is the only one valid solution, which matches the real situation.*

Proof. In area I, from $\partial \pi(P_A)/\partial P_A = 0$,

$$3bw_1 w_2 P_A^2 - (2w_1 w_2 a + b(C_B + c)(w_1 w_2)) \\ + (w_1 k + 1)(a - b(C_B + c)) + w_1 w_2 a(C_B + c) = 0. \tag{31}$$

Two P_A can be got from (31):

$$P_A = \frac{((2w_1 w_2 a + b(C_B + c)(w_1 w_2 - 1)) \pm \sqrt{F})}{(6bw_1 w_2)}, \tag{32}$$

where

$$F = (2w_1 w_2 a + b(C_B + c)(w_1 w_2))^2 \\ - 12bw_1 w_2 \{(w_1 k + 1)(a - b(C_B + c)) + w_1 w_2 a(C_B + c)\}. \tag{33}$$

In (32), P_A with the negative value of sqrt F is

$$P_A^1 = \frac{((2w_1 w_2 a + b(C_B + c)(w_1 w_2 - 1)) - \sqrt{F})}{(6bw_1 w_2)} \tag{34}$$

$$\approx \frac{a}{(3b)} + \frac{(C_B + c)}{6} - \frac{1}{(6w_1 w_2)} - \frac{\sqrt{F}}{(6bw_1 w_2)},$$

where

$$\frac{\sqrt{F}}{(6bw_1 w_2)} \approx \frac{a}{(3b)} + \frac{(C_B + c)}{6} - \frac{1}{(6w_1 w_2)}. \tag{35}$$

Substituting (35) in (36),

$$P_A^1 \approx 0 < C_B + c. \tag{36}$$

It is obvious that the solution cannot match the real situation, and it was cast out. Therefore, the optimal value of P_A is the only one valid solution, which matches the real situation.

The key of the Stackelberg-merger game is how to distribute the total profit between manufacturer and retailer. Using Nash bargaining model can coordinate profit of each entity [16], and the study of this field needs to go further. □

TABLE 2: The effect of RP cost changing in Stackelberg-Nash game ($k = 1.2$).

Case result	$k = 1.2 \quad C_B = 220$			$k = 1.2 \quad C_B = 250$		
	$P_B \leq P_A$	$P_A < P_B < P_{B0}$	$P_{B0} < P_B < P_{B1}$	$P_B \leq P_A$	$P_A < P_B < P_{B0}$	$P_{B0} < P_B < P_{B1}$
f_A		284.3316	266.0267		286.2994	267.7067
g_A		104.3316	86.0267		106.2994	87.7067
f_B		371.5171	540.0000		393.8308	560.0000
g_B		171.5171	340.0000		163.8308	330.0000
P_A	365.9493	388.6632	352.0533	381.8052	392.5987	355.4133
P_B	365.9493	543.0343	880.0000	381.8052	557.6615	890.0000
T	268.1014	222.6737	295.8933	236.3896	214.8025	289.1733
ΔT	178.8881	117.0312	75.7487	154.7302	110.3807	71.7150
T_s	268.1014	29.6436	0	236.3896	13.1986	0
T_A	0	193.0301	295.8933	0	201.6040	289.1733
T_B	536.2027	252.3172	295.8933	472.7791	228.0011	289.1733
π_r		38502	43776		35173	41811
π_m		28502	33776		25173	31811
π	46298	67005	77553	33729	60346	73621

TABLE 3: The effect of RP cost changing in Stackelberg-Nash decision ($k = 1.4$).

Case result	$k = 1.4 \quad C_B = 220$			$k = 1.4 \quad C_B = 250$		
	$P_B \leq P_A$	$P_A < P_B < P_{B0}$	$P_{B0} < P_B < P_{B1}$	$P_B \leq P_A$	$P_A < P_B < P_{B0}$	$P_{B0} < P_B < P_{B1}$
f_A		279.2893	253.4637		281.5733	255.4993
g_A		99.2893	73.4637		101.5733	75.4993
f_B		399.1822	606.6667		422.1999	626.6667
g_B		199.1822	406.6667		192.1999	396.6667
P_A	366.3029	378.5785	326.9000	382.0862	383.1467	331.0000
P_B	366.3029	598.3644	1013.3000	382.0862	614.3999	1023.3000
T	267.3943	242.8429	346.1452	232.8276	233.7066	338.0030
ΔT	221.1238	155.7372	107.0742	192.0417	146.8800	101.8516
T_s	267.3943	73.1396	0	235.8276	54.7557	00
T_A	0	169.7034	346.1452	0	178.9509	338.0030
T_B	534.7886	315.9825	346.1452	471.6552	288.4624	338.0030
π_r		54466	59910		49319	57120
π_m		44466	49910		39319	47120
π	51701	98933	109820	37958	88639	104250

5. Numerical Simulation

5.1. Stackelberg-Nash Game Simulation. Look at mobile phone, for example. There are two types of mobile phone A and B in the same market, and B is the renewal product of A. The related parameters' values are as follows: $a = 1000, b = 2, w_1 = 0.8, w_2 = 0.001, w_3 = 0.008, w_4 = 1.2, C_A = 200$. Table 2 shows the effect of the change of RP cost on price, demand, and profits of both products in the condition $k = 1.2$. Table 3 shows the effect in the condition $k = 1.4$. From Tables 2 and 3, the effect of renewal rate k on price, demand, and profits can be concluded.

From numerical simulation in Tables 2 and 3, the following conclusions can be drawn.

(1) There is no Stackelberg-Nash equilibrium in area I, and so Stackelberg-merger is used to simulate this situation. The total profit in this situation is the optimal profit of area I. Obviously, the optimal total profit of area I is less than those of areas II and III. Therefore, area I ($P_B = P_A$) is not the optimal area of Stackelberg-Nash game. The rational manufacturer and retailer could not select area I.

(2) For all parameter conditions, the profits of manufacturer, retailer, and the total profit of SC in area III are greater than those of area II (in area II, there is a little difference between the price of RP and OP). Because the manufacturer and retailer are rational, the final decision-making result (P_A, P_B) of the whole SC should be in area III. In this condition, OP is set at a low price (break-even sales), while RP is priced at a high level (the price differential between RP and OP is huge); that is to say, RP comes onto the market at a high price, and OP comes onto the market at a low price, which leads to larger market

TABLE 4: The effect of RP cost changing in Stackelberg-merger model ($k = 1.2$).

Case result	$k = 1.2$ $C_B = 220$			$k = 1.2$ $C_B = 250$		
	$P_B \leq P_A$	$P_A < P_B < P_{B0}$	$P_{B0} < P_B < P_{B1}$	$P_B \leq P_A$	$P_A < P_B < P_{B0}$	$P_{B0} < P_B < P_{B1}$
P_A	365.9493	346.2141	267.8400	381.8052	346.2141	273.5100
P_B	365.9493	458.5284	720.0000	381.8052	473.5284	735.0000
T	268.1014	307.5718	464.3200	236.3896	307.5718	452.9800
ΔT	178.8881	182.4446	178.2989	154.7302	178.7538	168.5086
T_s	268.1014	98.4720	0	236.3896	61.5634	0
T_A	0	209.0998	464.3200	0	246.0084	452.9800
T_B	536.2027	406.0438	464.3200	472.7791	369.1352	452.9800
π	46298	87780	107800	33729	79981	102600

TABLE 5: The effect of RP cost changing in Stackelberg-merger model ($k = 1.4$).

Case result	$k = 1.4$ $C_B = 220$			$k = 1.4$ $C_B = 250$		
	$P_B \leq P_A$	$P_A < P_B < P_{B0}$	$P_{B0} < P_B < P_{B1}$	$P_B \leq P_A$	$P_A < P_B < P_{B0}$	$P_{B0} < P_B < P_{B1}$
P_A	366.3029	346.2141	225.4400	382.0862	346.2141	232.3100
P_B	366.3029	505.3881	820.0000	382.0862	520.3881	835.0000
T	267.3943	307.5718	549.1200	232.8276	307.5718	535.3800
ΔT	221.1238	220.1259	254.7917	192.0417	216.4351	241.9918
T_s	267.3943	187.6227	0	235.8276	150.7141	0
T_A	0	119.491	549.1200	0	156.8577	535.3800
T_B	534.7886	495.1945	549.1200	471.6552	458.2859	535.3800
π	51701	123350	150770	37958	111730	143320

scale and wider influence on product. In this case, even if RP is set at a high price, the shift demand and the increment demand of market are proportional to the size of OP scale, so high price has less influence on market demand. But the profit of unit renewal product rises substantially with its price increase. In this situation, RP comes onto the market with a great price differential, and so the profits of all products are the highest; that is, area III is the optimal pricing area of supply chain.

(3) The profit of retailer in areas II and III is greater than that of manufacturer, mainly because the retailer is a direct participator of market, and it reacts very rapidly when the demand of market changes. Therefore, the retailer can adjust the process of decision making in the fastest time to increase revenue and reduce losses. The manufacturer's reaction to the market relies on the retailer decision making information's transmission, and there is a delay in the transmission process. Accordingly, the manufacturer's profit is less than that of the retailer.

(4) With the cost of RP increasing, the profits of the manufacturer, retailer, and total will decrease.

(5) If the cost of RP is constant, with product renewal rate k increasing, the profits of the manufacturer, retailer and total will increase.

5.2. Stackelberg-Merger Game Simulation. The parameters are the same as Section 5.1. With the changing of product

renewal rate or the cost of RP, the variety of price, demand, and profits are shown in Tables 4 and 5.

From numerical simulation in Tables 4 and 5, the following conclusions can be drawn:

(i) The profits in areas II and III of Stackelberg-merger game are greater than those in area I, so the Stackelberg-merger pricing strategy would not be in area I. The situation is consistent with Stackelberg-Nash game.

(ii) If OP adopts low-price strategy, and RP adopts high-price strategy, the total profit will be maximum value; in other words, area III is the optimal merger pricing area for the SC. The situation and its reason are the same as those of Stackelberg-Nash game.

(iii) If the cost of RP is constant, with the product renewal rate increasing, the total profit will increase obviously.

(iv) With the cost of RP increasing, the total profit will decrease.

From the analysis of Sections 5.1 and 5.2, the following can be drawn: (i) Both two kinds of decision-making processes are consistent with each other in profits, cost, and product renewal rate changing (with the cost of RP increasing, profits decrease, with product renewal rate increasing, profits increase). (ii) Both cases obtain the optimal profits in area III; in other words, the RP which comes onto the market has a great price differential with OP. (iii) Compared with Stackelberg-Nash game, the price in Stackelberg-merger

game is lower while the profits are more, so Stackelberg-merger decision making is virtually a kind of win-win situation for enterprises and market.

6. Conclusion

This paper studied the process of product renewal. A market shift model of RP was built on incremental function and shift function. Based on the model, Stackelberg-Nash game model and Stackelberg-merger game model for RP in SC were built, and their theoretical analysis was carried out. The following conclusions can be drawn. (i) In both two models, the increase of RP cost will make participants' profits and total profit decrease, while higher renewal rate will make the profits increase at the same margin cost. (ii) Manufacturer and retailer obtain the optimal profits in area III; in other words, the way of the optimal decision making in SC is that RP comes onto the market with a great price differential with OP. (iii) Compared with Stackelberg-Nash game model, Stackelberg-merger game model's pricing is lower and the profits are higher, which is actually a kind of win-win situation for enterprises and market.

In this paper, a part of premise conditions of the model was built on the basis of some rational hypothesis, and the model parameters need to be confirmed by real statistics data. These problems need to be further studied.

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