ABSTRACT. A vector $\mathbf{v} = (v_1, v_2, \ldots, v_k)$ in $\mathbb{R}^k$ is ϵ-*badly approximable* if for all m, and for $1 \leq j \leq k$, the distance $\|mv_j\|$ from mv_j to the nearest integer satisfies $\|mv_j\| > \epsilon m^{-1/k}$. A badly approximable vector is a vector that is ϵ-badly approximable for some $\epsilon > 0$. For the case of $k = 1$, these are just the badly approximable numbers, that is, the ones with a continued fraction expansion for which the partial quotients are bounded. One main result is that if $\mathbf{v}$ is a badly approximable vector in R^k then as $x \to \infty$ there is a lattice $\Lambda(\mathbf{v}, x)$, said lattice not too terribly far from cubic, so that most of the multiples $k\mathbf{v}$ mod 1, $1 \leq k \leq x$, of $\mathbf{v}$ fall into one of $O(x^{1/(k+1)})$ translates of $\Lambda(\mathbf{v}, x)$. Each translate of this lattice has on the order of $x^{k/(k+1)}$ of these elements. The lattice has a basis in which the basis vectors each have length comparable to $x^{-1/(k+1)}$, and can be listed in order so that the angle between each, and the subspace spanned by those prior to it in the list, is bounded below by a constant, so that the determinant of $\Lambda(\mathbf{v}, x)$ is comparable to $x^{-k/(k+1)}$.

A second main result is that given a badly approximable vector $\mathbf{v} = (v_1, v_2, \ldots, v_k)$, for all sufficiently large x there exist integer vectors $\mathbf{n}_j, 1 \leq j \leq k+1 \in \mathbb{Z}^{k+1}$ with euclidean norms comparable to x, so that the angle, between each $\mathbf{n}_j$ and the span of the $\mathbf{n}_i$ with $i < j$, is comparable to $x^{-1-1/k}$, and the angle between $(v_1, v_2, \ldots, v_k, 1)$ and each $\mathbf{n}_j$ is likewise comparable to $x^{-1-1/k}$. The determinant of of the matrix with rows $\mathbf{n}_j, 1 \leq j \leq k+1$ is bounded. This is analogous to what is known for badly approximable numbers α but for the case $k = 1$ we can arrange that the determinant be always 1.