A STRUCTURAL MODEL FOR PROBLEM POSING

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Based on a synthesis of the literature, a model for problem posing cognitive processes was formulated, and validated. The major constructs incorporated in this framework were the situations in which problem posing occurs. For each situation, four cognitive processes were established: the editing of problems based on iconic or symbolic stimuli, the filtering of important and critical information, the comprehending of the structural relations in quantitative information, and the translating on the quantitative information from one mode to another. The data suggested that all four cognitive processes contributed to problem posing abilities with the filtering and editing having a heavier role than the comprehending and translating processes.

INTRODUCTION

Problem posing and problem solving have been identified to be central themes in mathematics education. Problem posing involves the generation of new problems about a situation or the reformulation of a given problem (English, 1997a; Silver & Cai, 1996). Recent recommendations for reform in mathematics education suggest the inclusion in instruction of activities in which students generate their own problems in addition to solving pre-formulated problems (NCTM, 2000).

Most of our knowledge about the development of students' cognitive skills involves studies of students engaged in cognitive tasks in which they are provided with problems that are well defined. With the exception of a few studies (English, 1997a; Silver, 1994), problem posing remained unexplored as a tool for studying cognitive processes in the domain of mathematics education. Because problem posing is intellectually a more demanding task than solving problems (Mestre, 2002), in the present study we investigate students' cognitive processes in problem posing by proposing a model that encompasses most of the previous research in the area. This article begins by reviewing two strands of research that have a bearing on this study, and then discusses a theoretical model of the cognitive processes of problem posing.

THEORETICAL CONSIDERATIONS

In this section we describe two kinds of research studies on problem posing in mathematics instruction. The first strand of research describes the development of students' problem posing abilities, and the second strand discusses the classification of problem posing tasks.

The problem posing abilities

Research studies provided evidence that problem posing has a positive influence on students' ability to solve word problems (Leung & Silver, 1997), and provides the opportunity for teachers to get an insight of students' understanding of mathematical concepts and processes (English, 1997a). It was also found that students' experience with problem posing enhances their perception of the subject, and produces excitement and motivation (English, 1998; Silver, 1994). Specifically, English (1997a; 1997b; 1998) asserted that problem posing improves students' thinking, problem solving skills, attitudes and confidence in mathematics and mathematical problem solving, and contributes to a broader understanding of mathematical concepts.

English (1997a, 1997b, 1998) investigated students' abilities in generating problems in three studies with third, fifth and seventh graders, respectively. In the first of these studies, third graders revealed significant difficulties in posing problems both in informal and formal contexts. They were only able to create several change/part-part-whole problems by altering the contexts of the original problems and by focusing on the operational and not the semantic structure of the problems (English, 1998). In the second study, English (1997a) organized a problem posing program through which fifth graders improved their abilities to model a new problem on an existing structure and to diversify the story context of the problem. In contrast to the previous study, fifth graders developed their abilities to perceive the problem structure as independent of a particular context, providing them with greater flexibility in their problem creations. In the third study, English (1997b) proposed a theoretical framework for tracing seventh graders' abilities in problem posing across a range of mathematical situations. This framework encompassed abilities that referred to the knowledge and reasoning of students in problem posing as well as abilities for assessing students' metacognitive processes. In this study, the students who participated in the program exhibited greater facility in creating solvable problems than their counterparts that did not participate. Most of the students in the program created quite sophisticated problems using semantic relations in their problems.

Silver and Cai (1996) conducted a study in which a large number of sixth and seventh grade students were asked to pose questions to given story problems and classified them in terms of mathematical solvability, linguistic and mathematical complexity. Most students in Silver and Cai's study were able to pose appropriate mathematical questions when presented with a story situation as a stimulus for question generation. In addition students were able to generate syntactically and semantically complex mathematical problems.

Classification of problem posing tasks

The second strand of research discusses the classification of problem posing tasks. Stoyanova (2000) identified three categories of problem posing experiences that can increase students' awareness of different situations to generate and solve mathematical problems: (a) free situations, (b) semi-structured situations, and (c) structured problem-posing situations. In the free situations students pose problems without any restriction. An example of the free problem posing situation are the tasks where students are encouraged to write problems for friends to solve or write problems for mathematical Olympiads. Semi-structured problem posing situations refer to situations where students are asked to write problems, which are similar to given problems or to write problems based on specific pictures and diagrams. Structured problem posing situations refer to situations where students pose problems by reformulating already solved problems or by varying the conditions or questions of given problems.

Silver (1994) classified problem posing according to whether it takes place before (presolution), during (within-solution) or after problem solving (post-solution). He argued that problem posing could occur (a) prior to problem solving when problems are being generated from particular presented stimulus such as a story, a picture, a diagram, a representation, etc., (b) during problem solving when students intentionally change the goals and conditions of problems, (c) after solving a problem when experiences from the problem solving context are applied to new situations.

Stoyanova (2000) and Silver (1994) classified problem posing tasks in terms of the situations and experiences which provide opportunities for students to engage in mathematical activity. Both classifications involve five categories of problem posing tasks, which were used throughout the studies so far: Tasks that merely require students to pose (a) a problem in general (free situations), (b) a problem with a given answer, (c) a problem that contains certain information, (d) questions for a problem situation, and (e) a problem that fits a given calculation.

It is acknowledged that there are a variety of ways to analyze problem posing tasks and each may give a different understanding of the process. However, there is a need for a framework that can be used on responses from a wide range of tasks and from different age groups so that inter-task study and development of problem posing behavior can be investigated. The model proposed in the present study synthesizes most of the ideas articulated in previous studies, including a classification scheme of cognitive processes. The focus of the proposed model is on students' ability to pose their own two-step addition and subtraction problems, but the model can be applied to many other areas of mathematics.

THE PROPOSED MODEL AND THE PURPOSE OF THE STUDY

Notwithstanding the extent of research into students' thinking in problem posing, recent research has not investigated systematically the quantitative information of the problem posing tasks in combination with the cognitive processes used in each task. Accordingly, the literature does not provide the kind of coherent picture of students' problem posing thinking that is desirable for current approaches to instruction. In this paper, we propose a model, which may enable young students' problem posing thinking to be described across four cognitive processes. As it is highlighted in Figure 1, the cognitive processes that are postulated to occur when a person engages in problem posing refer to filtering quantitative information, translating quantitative information from one form to another, comprehending and organizing quantitative information by giving it meaning or creating relations between provided information, and editing quantitative information from given stimulus.

We speculate that the cognitive processes correspond to specific problem solving tasks presented in iconic, tabular or symbolic forms. It is possible for a cognitive process to correspond to more than one task, but for clarity and simplicity purposes, we incorporate in the model the most prominent cognitive process for each task. It is also hypothesized that each cognitive process emerges and develops in a way that incorporates the continuing development of cognitive processes. Editing quantitative information is mostly associated with tasks that require students to pose a problem without any restriction from provided information, stories or prompts (Mamona-Downs, 1993). Filtering quantitative information is associated with tasks that require students to pose problems or questions, which are appropriate to specific, given answers. The given answer functions as a restriction, making filtering a more demanding process than editing. Comprehending quantitative information refers to tasks that students pose problems from given mathematical equations or calculations. Comprehending problem posing tasks require the understanding of the structural context of problems and the relations between the provided information. Translating quantitative information requires students to pose appropriate problems or questions from graphs, diagrams or tables.

In order to capture the nature of problem posing, our model (Figure 1) incorporates forms of semi-structured and structured situations (Stoyanova, 2000) in which students are asked to generate problems from a presented stimulus (presolution phase). The stimulus situations involve quantitative information, which contain representations either in the iconic or in the symbolic form. For example, students posing problems based on a picture are handling information in iconic form. Similarly, students are handling quantitative information in iconic form if they are given graphs and diagrams. Students posing problems based on words or phrases or calculations are handling quantitative information in symbolic form. Examples of the tasks that correspond to each cognitive process are shown in Table 1.

The purpose of the present study was twofold: First, to validate the proposed model, i.e., to confirm that problem posing consists of the proposed cognitive processes, and second to search for a possible developmental trend in students' abilities to pose problems based on the editing, filtering, comprehending, and translating cognitive processes and to find out meaningful differences in students' thinking in generating problems. However, in this paper, due to space limitations, we present the results of the first aim of the study.

METHOD

Subjects

The sample for this study consisted of 143 Grade 6 students from six classes at elementary schools in an urban district in Cyprus. Seventy-nine students were males and sixty-four females. The school sample is representative of a broad spectrum of socioeconomic backgrounds. Prior to the start of this study, none of the children had been exposed to problem posing instruction.

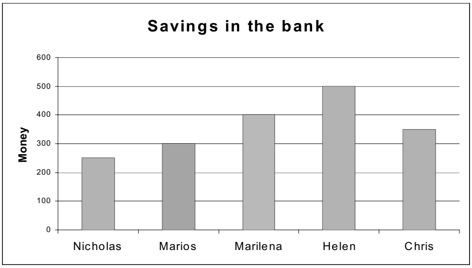
Cognitive Process	Tasks
Filtering	Write a question to the story so that the answer is "285 stamps". "Chris has 135 stamps while Helen has 15 stamps more than Chris"
Translating	Write a problem based on the following diagram: 
Comprehending	Write an appropriate problem for (1300+2100)-790=n
Editing	Write a problem based on the following picture: 

Table 1: Tasks examples corresponding to each cognitive process

Instruments

Each student completed four problem posing tests, which contained situations that help students to perceive mathematical context in diverse ways. Test 1 consisted of three tasks in which students were required to complete the problems with the missing question as to correspond to the provided answer. Test 2 involved three tasks, which required from students to write problems that fit to given equations. Test 3 consisted of four tasks, which presented pictures, and diagrams with mathematical information. Students were asked to use information from the pictures and diagrams to write problems whose solutions would require or not specific operations, i.e., two additions or one addition and one subtraction. In Test 4, which involved three tasks, students had to pose problems based on interesting stories. For these 13 tasks, students were required not only to pose questions or problems but also to justify their answers by writing the mathematical solutions of the constructed problems or the mathematical equation, which corresponded to their own problems.

The tests were administered to the students by the researchers in five 20-minute sessions. Prior to the administration of the test, which lasted ten working days, one researcher visited the classes involved in the project and worked with the students on problem solving for approximately 40 minutes.

Data Analysis

The goal of the analysis was to estimate the relative strength of the proposed model. Because we proposed a theoretically driven model about the components of problem posing cognitive processes, our first interest was in the assessment of fit of the hypothesized a priori model to the data. The assessment of the proposed model was based on confirmatory factor analysis, which is part of a more general class of approaches called structural equation modeling. One of the most widely used structural equation modeling computer programs, MPLUS, was used to test for model fitting. In order to evaluate model fit, three fit indices were computed: The chi-square to its degrees of freedom ratio (x^2/df), the comparative fit index (CFI), and the root mean-square error of approximation (RMSEA). These three indices recognized that the following needed to hold true in order to support model fit (Marcoulides & Schumacker, 1996): The observed values for χ^2/df should be less than 2, the values for CFI should be higher than .9, and the RMSEA values should be lower than .08.

RESULTS

In this section, we refer to the results of the analysis, establishing the validity of the latent factors and the viability of the structure of the hypothesized latent factors. In this study, we posited an a-priori structure of the proposed model and tested the ability of a solution based on this structure to fit the data.

The proposed model consists of four first-order factors and one second-order factor. The first-order factors represent the cognitive processes: the editing (F1), the filtering (F2), the comprehending (F3), and the translating (F4). The editing, the filtering and the comprehending factors were measured by three tasks each, while translating was measured by four tasks. F1, F2, F3, and F4 were hypothesized to construct a second order factor "problem posing abilities", which was hypothesized to account for any correlation or covariance between the first order factors. Figure 1 makes easy the conceptualization of how the various components of problem posing cognitive processes relate to each other. The descriptive-fit measures indicated support for the hypothesized first and second order latent factors (χ^2/df=1.45, CFI=.965, and RMSEA=.056). The parameter estimates were reasonable in that all factor loadings were large and statistically significant (see Figure 1). The r-squares (shown in the parentheses in Figure 1) also illustrate that modest to large amounts of variance are accounted for all tasks corresponding to each cognitive process and suggest that editing and filtering explained the shared variance of their corresponding tasks much better than did translating, and comprehending.

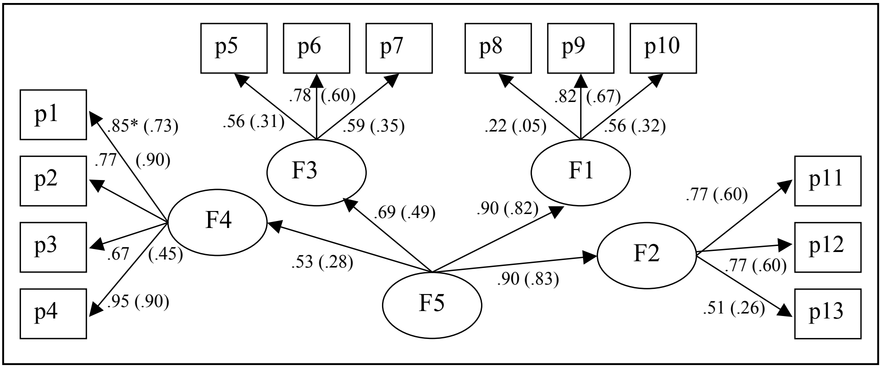


Figure 1: A structural model of problem posing cognitive processes

Note: F1=Editing, F2=Filtering, F3=Comprehending, F4=Translating and F5=Problem posing abilities, p1-p13 refer to the problems assigned to students.

* The first number indicates factor loading and the number in parenthesis indicates the corresponding r^2.

The main focus in this study was to address the fact that the cognitive processes of editing, filtering, comprehending and translating constitute the students' problem posing abilities. In the context of good model fit, the effects of each cognitive process on the problem posing abilities of students were investigated. The structure of the proposed model also addresses the differential predictions of the four cognitive processes for the problem posing abilities. Considering the effects among the cognitive processes reveals that the filtering and the editing cognitive processes were the primary source explaining students' abilities to generate problems (r^2=.83 and r^2=.82, respectively). The translating cognitive process had a small significant effect (r^2=.28), while comprehending had moderate effects on students' abilities to pose problems (r^2=.49).

DISCUSSION

Problem posing is currently discussed as a function of complex and concomitant growth in a knowledge base, strategies, motivation, and metacognition (English, 1998). It was argued in this study that few models exist to help educators explain how problem posing actually develops. Hence, the goal of this study was to articulate and empirically test a theoretical model to help educators build new understandings about the cognitive processes required by students in generating problems. The model integrated most of the abilities and tasks from existing problem posing research (Silver & Cai, 1996; English, 1997a) and extended the literature in a way that cognitive processes are recognized as important components of developing problem posing abilities. The model proved to be consistent with the data leading to the conclusion that the four cognitive processes (filtering, editing, comprehending, and translating) mediate the ability to pose problems. Specifically, it was found that the four cognitive processes contribute to the students' abilities to pose problems with the filtering and editing cognitive processes being more important than comprehending and translating in generating problems. This particular finding suggests that students' abilities to filter and edit problems are highly related to pose problems.

The model used in this study offers teachers and researchers a means to examine the complexity and sophistication of problem posing. From the perspective of teachers, the model may be used in order to include in their instruction the development of the four cognitive processes. From the prospective of researchers, it is likely that the model could be useful as a prototype for further analyses of the cognitive processes of problem posing.

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