FACILITATING PEER INTERACTIONS IN LEARNING MATHEMATICS: TEACHERS' PRACTICAL KNOWLEDGE

Olive Chapman
University of Calgary

This paper reports on teachers' practical knowledge [PK] about peer interactions [PI] in learning mathematics. The focus is on high school teachers who consistently engaged students PI in their teaching. Data consisted of interviews and classroom observations. Findings indicate that these teachers have PK of students' roles in PI and learning activities and teacher's behaviors to support PI that creates a meaningful classroom culture to facilitate PI in learning mathematics. Their classroom experiences and their conceptions of mathematics and learning played an important role in the PK. Their PK offers insights into pedagogical strategies that can be effective in facilitating PI. PK is a basis for teachers' sense making and can play an import role in teacher education.

There are reform recommendations in mathematics education that assign a significant role to peer interactions in teaching and learning mathematics. For example, the National Council for Teachers of Mathematics [NCTM] standards advocate:

> Whether working in small or large groups, they [students] should be the audience for one another's comments – that is, they should speak to one another, aiming to convince or to question peers [NCTM 1991, p.45].

However, whether or how peer interactions get implemented in the classroom will likely depend on the teacher. Thus, the study in this paper focused on understanding peer interactions through the teacher. In particular, it investigated teachers' practical knowledge of peer interactions in learning mathematics in terms of how the teachers made sense of peer interactions and when and how they incorporated it in their teaching.

RELATED LITERATURE AND THEORETICAL PERSPECTIVE

Studies on mathematics teachers have examined their content knowledge, beliefs, conceptions, classroom practices, learning, professional development and change (e.g., Chapman, 1997; Fennema & Nelson, 1997; Lampert & Ball, 1998; Leder et al., 2003; Schifter, 1998; Thompson, 1992). These studies have provided us with insights on, for example, the relationship between beliefs/conceptions and teaching, deficiencies in teachers' content knowledge, and the challenges of teacher education and teacher change. In particular, research on the mathematics teacher suggests that an understanding of teachers' thinking and actions are important to improve the teaching of mathematics. Boaler (2003) argued that researchers need to study classroom practices in order to understand the relationship between teaching and learning. The teacher is the determining factor of how the mathematics curriculum is interpreted and taught. Thus, it is important to learn from teachers what they do and how they make sense of what they do in the classroom. In this paper, the focus is on the teacher's perspective of peer interactions in the teaching and learning of mathematics.

The research literature provides a lot of theory on group or cooperative learning (e.g.,

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Davidson, 1990; Slavin, 1995), but there is little attention to the teacher's perspective of it. Davidson provides several examples and "practical strategies" for using cooperative groups in mathematics teaching and learning. It is not clear what conceptions underlie teachers' thinking that will help or hinder their integration of such approaches in their teaching. Investigating their practical knowledge is one way of making this explicit.

Theoretically, the study is framed in a social or interactive perspective of learning and a practical knowledge perspective of teacher thinking, each of which is briefly described here. An interactive perspective on teaching and learning has been discussed by several people including Bauersfeld, 1979; Dewey, 1916; Lave & Wenger, 1991; and Vygotsky, 1978. Lave and Wenger conceived of learning in terms of participation. Dewey emphasized learning through active personal experience and learning as a social process. In his view, purposeful activity in social settings is the key to genuine learning. Vygotsky claimed that individual development and learning are influenced by communication with others in social settings. In his view, interacting with peers in cooperative social settings gives the learner ample opportunity to observe, imitate, and subsequently develop higher mental functions. Specific to mathematics, Bauersfeld (1979) explained:

> Teaching and learning mathematics is realized through *human interaction*. It is a kind of mutual influencing, an interdependence of the actions of both teacher and student on many levels. … The student's reconstruction of meaning is a construction via social negotiation about what is meant and about which performance of meaning gets the teacher's (or peer's) sanction. [p.25]

This theoretical perspective, then, promotes the position that learning takes place in a social setting and emphasizes human interactions as a key factor to facilitate learning. One way in which this perspective has been conceptualized in relation to the classroom is in terms of cooperative groups. Such groups have been promoted as having three key goals: to distribute classroom talk more widely, encouraging students to talk, to share their ideas and to become more actively engaged; to specify the social processes to help students to work cooperatively; and a way to develop their social and collaborative skills. However, like any tool, how these goals are interpreted and applied by teachers may or may not fit the interactive perspective beyond an instrumental level. In this study, peer interactions are considered to be classroom situations where students talk with students directly to learn mathematics. This includes small-group or whole-class situations but excludes situations where the teacher mediates a discussion, e.g., the teacher asks a student to react to another student's response or a student initiates the response but directs it to the teacher who acts as a bridge between the students.

The second aspect of the theoretical perspective framing this study is the construct of practical knowledge [PK]. PK has been used in research of the teacher to describe knowledge that guides teachers' actions in practice (Johnston, 1992). Whereas scientific or formal knowledge is abstract and propositional, PK is experiential, procedural, situational, particularistic, and implicit (Carter, 1990, Fenstermacher, 1994). It corresponds with positions teachers take. It refers to teachers' knowledge of classroom situations and the practical dilemmas they face in carrying out purposeful action in these settings (Carter, 1990). Experience is an important source of PK.

PK in this study is based on Sternberg & Caruso's (1985) theory of it. In their view, PK is procedural information that is useful to one's everyday life [which includes teaching, in the case of the teacher]. It is used in three main forms of interaction with the everyday world – adaptation, shaping and selection. For example, a teacher would use PK to adapt to situations in the classroom, or to shape situations in the classroom or to make selections when choices are available. PK is stored as or conveyed by statements that embody "condition-actions sequences"(p. 135), i.e., if I do A (the condition), B (the action) will happen. In the case of teaching, such statements describe the procedure or condition that will bring about a certain action or performance state in students. For example: I know that students will not improve their achievement in mathematics if I use groups. PK can be probabilistic in nature. For example: If I use groups, it probably wouldn't make a difference. Sternberg & Caruso explain, "Knowledge becomes practical only by virtue of its relation to the knower and the knower's environment (p. 136)." This implies that a teacher's PK is relevant to his or her personal context or classroom context. The goal of this study, then, is to identify conditions and actions for peer interactions that are common to a sample of teachers to understand how this process makes sense to them.

RESEARCH PROCESS

The data for this study is based on a larger study on teacher thinking in teaching word problems framed in a phenomenological research perspective (Creswell, 1998) that focuses on participants' meaning, what they value, and how they make sense of their experiences. The participants were 22 elementary, junior high and senior high school mathematics teachers from local schools. The main criterion for selecting the teachers was willingness to participate. However, most of the ten high school teachers were considered to be exemplary teachers in their school systems. Some had won teaching awards. All of the participants were articulate and open about their thinking and experiences with word problems and mathematics.

The main sources of data for the study were open-ended interviews and classroom observations. Interview questions were framed in a phenomenological context to allow the teachers to share *their* way of thinking and to describe their behaviors as lived experiences (i.e., stories of actual events). The interviews focused on their thinking/ experiences with word problems in three contexts: (i) past experiences, as both students and teachers, focusing on teacher and student presage characteristics, task features, classroom processes and contextual conditions, (ii) current practice with particular emphasis on classroom processes, planning and intentions, and (iii) future practice, i.e., expectations. Questions were often in the form of open situations, e.g., telling stories of memorable, liked and disliked classes involving word problems that they taught, giving a presentation on word problems at a teacher conference, and having a conversation with a preservice teacher about word problems. Because of the open-endedness of the questions, their responses extended beyond word problems to their teaching of mathematics in general. Classroom observations over a 2-week period for each teacher, focused on the teachers' actual instructional behaviors during lessons involving/related to word problems. Special attention was given to what the teachers and students did during instruction and how their actions interacted. Post-observation interviews with each teacher focused on clarifying her/his thinking in relation to her/his actions.

The analysis began with open-ended coding (Strauss & Corbin, 1998) of the audio-taped transcripts of interviews. The coding was done by the researcher and two research assistants working independently to identify attributes of the teachers' thinking and actions that were characteristic of their perspective of teaching word problems. The focus was on significant statements and actions that reflected judgments, intentions, expectations, and values of the teachers regarding their teaching that occurred on several occasions in different contexts. The coding was followed by a review of the field notes and audio-taped transcripts of classroom observations to triangulate the findings from the interviews and add and clarify situations. This was followed by comparison of the findings by the three coders and revisions made where needed. The coded information for each participant was then sorted into themes that conveyed the significant features of his/her thinking and teaching. Peer interaction was one of the themes that emerged. In order to elaborate on this theme, the researcher and two assistants returned to the data to obtain details of all situations that conveyed it and to identify the conditions and actions that constituted the teachers' PK about peer interactions. Verification procedures, then, included triangulation, using data from a variety of sources, cross checks by research team, and elimination of initial assumptions/themes based on disconfirming evidence.

PRACTICAL KNOWLEDGE OF PEER INTERACTIONS

The findings reported here focus only on those teachers who consistently provided opportunities for students to engage in peer interactions. These were the eight high school teachers who were identified as exemplary in teaching mathematics. There was more depth and scope to their PK than that of the other teachers. Their PK was influenced by their experience as a teacher, for e.g., they recognized that teachers and students vary in their explanations of the same content. For example,

> I don't see things the way kids see things, and I don't solve problems the way kids solve problems, … If you're explaining something, they can sit and look at the board and you can tell they don't get it … so you ask somebody else in class and they might say exactly the same thing I've said …and then the kid will go, yes that's right, I understand, and you're there, like, but I just said that. … Somehow they know how to relate it to each other, and many times they can express things in different ways that I haven't thought of.

Their PK was also influenced by their own experience with peer interactions, for e.g.,

> I need to talk and often when I talk some of my best ideas come out. Writing about it doesn't always do it for me, so I think the peer interaction can do that.

Conditions and actions for four themes characterized the teachers' PK of peer interactions: conceptions [conditions] that support a social perspective; students' behaviors/outcomes [actions] for/from peer interactions; learning activities [conditions] to support peer interactions; and teacher's behaviors [actions] to support peer interactions. Each is discussed with examples of representative quotes from the teachers.

Conceptions That Support a Social Perspective: The teachers' PK included a view of mathematics and learning that supported a social perspective of mathematics education and embodied the need for peer interactions in their teaching. For example:

- Math is primarily a problem solving activity and problem solving is a social activity.
- Mathematics to me is a shared experience.
- I believe math is a language and to be able to articulate it is a very important part of the learning process.
- Learning comes through talk, comes through discussion.
- Mathematics learning occurs when the learner understands and can explain the concept that has been presented in their own words … and knows it sufficiently to teach to someone else, talk about it to someone else.

Students' Behaviors/Outcomes for/from Peer Interactions: The teachers' PK indicated that through peer interactions, students learn mathematics from and with each other as they engage in/achieve the following seven behaviors/outcomes.

Compare experiences: This allows students to learn about learning. For example,

Getting information from your peers helps you understand that there are people who are experiencing the same difficulties as you or have a perspective that you can share.

Share ideas: This allows students to collaborate and expand their thinking. For e.g.:

- They can bring something to this situation that you may not have thought of.
- Sometimes a student just doesn't understand, and then another student will go well what about such and such, and together they can formulate a conclusion. Whereas, individually, they would have been stuck.
- Just by people sharing each other's views, they help refine each other's views.

Articulate mathematics: This involves students orally expressing mathematics in words or describing and explaining it (e.g., concepts) in a meaningful way. For e.g.:

- They have to explain in plain English what something means.
- They talk math language … they use the language and each other understands more than they do with the teacher.
- By having the students explain, they put it in words or they put it in a context that is more meaningful than I have done.

Pose questions: This involves students "asking each other questions" and allowing the rest of their group to question them and to debate with them whether or not what they've told the group is valid or whether or not that's taking them down the garden path.

Be motivated and gain confidence: For example,

- They motivate each other and help build confidence.
- They lend support to each other and … motivate each other to get their work done.

Gain autonomy: This involves students depending less on the teacher's thinking. For e.g

They won't always look to the teacher for solutions, they'll look to each other, … but also they get to interact more with each other, and can use each other more to enhance their own learning.

Test understanding: For example,

- It's only through peer interaction that you get to test out your thoughts, your ideas, that you get to formulate your own perception of the mathematics that confirms for you that you understand it.

- It's by saying, "Well now, how did you get that?" My answer doesn't look like that. It's with peers beside you that they start to discuss and compare their work, compare the answers, compare the steps and they promote each other's learning and understanding.

Learning Activities That Support Peer Interactions: The teachers' PK indicated that by engaging students in the following five learning activities/experiences, they will have opportunities to engage in peer interaction. The focus, as one teacher noted, is to "encourage students to investigate and solve mathematical problems with each other."

Inquiry of the problem-solving process: The teachers allowed students to work in groups to learn about problem solving. For example, some teachers presented students with a problem to solve on their own. More importantly, they were required to analyze their process by considering, for e.g., what they did, why used that method, why it worked, how they made sense of the problem and the solution. This is followed by group sharing and whole-class discussions. The process is repeated for a few problems to establish a model for solving problems. One teacher had students work in groups to solve the problem, but one student in each group assumed the role of observer and "looks at the process that is going on to solve the problem". Each student got a turn at being observer. A whole-class sharing and comparison of findings followed each round of observations. This process eventually led to the development of a problem-solving model.

Inquiry of a new concept: The teachers used peer interactions in a variety of ways in introducing a new concept. The most common approach was to allow students to explore a situation in small groups before the teacher provided any explanations or led a discussion on it. The situation could involve "just looking for patterns and relationships" to make sense of a concept or trying to solve a problem to understand a procedure. For example, in introducing systems of equations, one teacher presented a scenario involving weights of large and small cats by drawing a picture of it on the board and asking students to work in their groups to find the weights of the cats. She gave no other direction. After students shared and compared their approaches, she connected them to the formal approaches. Another teacher assigned each group a different method of solving systems of equations and required that they analyzed solved examples to understand the method. Each group then taught the method they explored to the other groups, trying to convince each other that their method was the best approach.

Whole-class presentation: The teachers encouraged students to interact during whole-class presentation. The common approach was that, students' did not only present and justify their solutions, but, for example:

> They have to encourage others, okay now what did you use, and how did it work, how did you do it and which method do you think you like better, and why.

One teacher, as an introduction on systems of equations, had students collect pictures of real-life situations of graphs that intersect and take turns to lead a discussion about

> What the graph represents, the significance of the graph, what it means when graphs intersect, what the intersection shows, why the intersection is important, why anyone

would want to find the intersection in the first place.

Practicing problem solving: Students practiced problems individually or collaboratively, but always had the opportunity to discuss their work with peers.

Investigations/projects: Students planned and conducted group projects that included the use of technology, use of art and outdoor activities, e.g., finding heights of tall objects.

Teacher's Behaviors to Support Peer Interactions: The teachers' PK indicated that the following four behaviors of the teacher will facilitate or support peer interactions that promote learning.

Listens and observes: This provides the teacher with feedback to determine when and how to intervene. For example:

> I circulate and I listen to the kinds of conversations that go on in the groups and how are they processing the information, how are they developing the strategies that they are going to use and try and get some clues on what they understand.

Questions and prompts: This involves the teacher using questions to extend concept development or check for understanding or prompts when students are stuck. For e.g.:

> - I ask them questions if they're stuck but that's about it. … I will try to come up with a question that will allow them to continue but I will not give them the answer at any time.
> - My role is to ask questions, not to give answers. … It is by how you ask the questions that they gain access to the answer that they wanted you to just tell them.

Supports students' thinking: For example,

> - I make sure that I tell them that I don't care how they solve it, but they have to say exactly how they did this.
> - Make them aware that they have tools, mathematical tools, then give them the freedom to use whatever they want to solve these problems.
> - I always tell students I am not the only expert in the classroom and I learn from you just as well as you learn from me.

Models questioning: This involves the teacher using a questioning approach during whole-class instruction that students then mirror. For example: I notice they ask themselves the same questions when they're group working on problems.

Promotes good peer relations: As one teacher explained, "If you can facilitate good peer relations then you can have some really healthy dialogue." Some ways in which this occurred were through shared questions, seating plan, voluntary grouping, and peer observations:

> - You can only ask me a question when all of you [in a group] share the same question.
> - I sit them beside each other in groups. There's no time that they're ever in isolated rows or isolated desks. …I don't force the groups because I think that just creates a social conflict when you're trying to deal with the math conflict.

A unique approach used by one teacher was having each student take turns observing his/her group behaviors as the group solved genuine mathematics problems.

> They are looking at how the actual group interaction occurs so they do become a good cooperative learning group and they support … and work with each other.

CONCLUSION

The findings indicate that PK is a basis for the teachers' sense making in using peer interactions [PI] in their teaching. These teachers have PK of PI that facilitates a meaningful classroom culture to support the learning of mathematics. Their experience provides evidence that this instructional approach is a feasible and meaningful way of teaching high school mathematics. The findings also suggest that the teachers' classroom experiences and conceptions of mathematics and learning play an important role in characterizing their PK. This has implications for teacher education. For e.g., positive personal experiences with PI may be necessary for teachers to develop meaningful PK. Exposure to PK of teachers like these participants could open the door to gaining such experience. Their PK offers examples of the way things are or could be. Also, since all teachers, including preservice teachers, have some PK used as a basis of sense making of PI, providing them with theory on PI that conflicts with their PK could be problematic for them if their PK is not explicitly dealt with. The findings of this study offer a structure against which other teachers could examine their PK, either through reaction against or resonance with what is offered, to understand it. Finally, this conception of PK can be used to study other aspects of teaching mathematics.

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