SOME UNDERGRADUATES' EXPERIENCES OF LEARNING MATHEMATICS

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One theme of current research about higher education students of mathematics concerns those who fail. At our institution, some of the entrants are students who have previously failed in mathematics; others come to us with a comparatively weak mathematical background. Most of these students go on to become confident and effective mathematicians, some even achieving first class honours. We believe that understanding something of their perceptions of this experience may contribute to the current debate about who succeeds and who fails in higher education mathematics study and why.

INTRODUCTION

This paper contributes to the body of research into the teaching of mathematics undergraduates (for example, Jaworski, 2001) and specifically relates to that concerned with success and failure in learning mathematics in higher education (for example, Leder *et al*, 1998); our particular concern is with previously failing students who become successful. Our interest in researching this topic was prompted by recent research into students' experience of undergraduate mathematics at two traditional English university – 'Marmion' and 'Waverley' - universities which are among the elite institutions in England for undergraduate mathematics (see, for example, Rodd, 2002; Macrae, Brown, Bartholomew and Rodd, 2003).

Attending a seminar reporting aspects of the research and particularly hearing some of the tales of failure of individual students discussed, we were struck by how little the characterization of the teaching and learning of mathematics at these institutions matched our characterization of our own. On some of our courses, many of the students come to us with weak mathematical backgrounds[1] - either through non-traditional routes or an experience of previous failure in higher education or both - but most of them go on to become confident, engaged and successful mathematicians. We believed that hearing the voices of some of these students talking about themselves as mathematicians and their thoughts and feelings about learning mathematics would provide a useful contribution to the current debate about who fails mathematics in higher education and why. Thus, our research questions were: what is the student experience of learning mathematics in our institution? how is it likely to be different from that at 'Marmion' and 'Waverley'? and what ways of

[1] For those familiar with the system of qualifications in England, entry might be perhaps through an Access Course, perhaps through a Foundation Course, occasionally perhaps with only a grade E at Advance Level.

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understanding themselves as mathematicians, what mathematical identities, are available to and claimed by our students (but not, perhaps, those failing students at 'Marmion' and 'Waverley')?

CONTEXT OF THE STUDY AND DATA COLLECTION AND ANALYSIS

The cohort of students involved in this piece of research is following one of the longer routes into secondary mathematics teaching. For this cohort, a pattern of a weak entry profile followed by success in mathematics was clearly evident. On their course, they study undergraduate mathematics for two years within the context of a Mathematics Education Centre rather than in a university mathematics department; (this is followed by a professional year). Their mathematical studies comprise the equivalent of three quarters of a first year of undergraduate mathematics followed by the equivalent of half a second year and half a third year; there almost no options. (The remainder of their studies relates to the teaching of mathematics and other educational and professional studies.) In other words, they study mathematics to honours degree level but within a narrower range than would a single honours mathematics student.

We interviewed seven of the students, sometimes alone and sometimes in pairs. The interviews were relatively open and unstructured with just a few prompts. We encouraged them to talk about

- whether or not and in what way(s) they thought their relationship with mathematics had changed and developed during their current studies;
- whether or not they thought they had changed as mathematicians;
- and whether what they thought about mathematics itself had changed.

Each interview lasted between an hour and an hour and a half. We taped and transcribed the interviews (with occasional editing for clarity) and then began working with these texts in a familiar way. We each read and re-read the transcripts, immersing ourselves in the data and searching for themes. Separately, we each derived some themes from the data and coded the transcripts accordingly. Next we met to discuss our themes and our coding and to re-work the analysis, subsequently returning repeatedly to the transcripts to check out and evidence our developing ideas, seeking to keep them grounded in the data.

Elsewhere, we have presented much fuller portraits of individual students (Povey and Angier, 2003; Angier and Povey, 2004). Here we adopt a more conventional style, closer to a thematic analysis. We have some methodological difficulties with this approach and are sceptical about the sense of certainty of representation it may generate. We share Margaret Walshaw's doubts about 'the possibility of true and accurate research findings and, moreover, about the very possibility of knowing others and telling their stories' (2002: 349). We do not claim that these are the only or true accounts about the students' experiences of learning mathematics with us, nor that the interviews represent the students unproblematically 'telling it like it is'

(Hollway and Jefferson 2000: 10). We came to regard the transcripts as, in part, stories through which the students were able to construct - for us and for themselves - an account of themselves as mathematicians. The interviews were places where they did 'identity work[2]' (Mendick, 2002: 336), where they spoke into being mathematical identities. So, in the rest of the paper, we first draw on the students' descriptions of their experience of learning mathematics with us, contrasting this very briefly with what they might have encountered elsewhere, and then connect these with the mathematical identities which these experiences and the available discursive frameworks allowed them to produce.

LEARNING UNDERGRADUATE MATHEMATICS

We start with what seems to us to be an accurate picture of what most undergraduate mathematics is like. It is a description provided by Melissa Rodd in the context of writing about 'Marmion' and 'Waverley'.

> [B]eginning university mathematics is invariably presented as an abstract subject, *without fuzziness or debateable results*, which is *assessed through individuals' timed exam performance*. Such assessment arrangements are personal and adrenalin-producing yet *the assessment's mathematics does not express any personal view*. There is nothing to hide behind in mathematics: *no experiment, no interpretation of evidence, no comparison of criticisms*. The students are relatively *more exposed* – intellectually and emotionally – than in other subjects. (Rodd, 2002: 2; our emphasis)

Much of this description, however, is contrary to our own 'common sense' about the nature of mathematics itself; and consequently about what is entailed in the learning and teaching of the subject at undergraduate level. We define ourselves as mathematics educators at least as much as mathematicians. Our understandings place us firmly in the now well-established alternative tradition of 'inquiry mathematics' (Cobb *et al*, 1992) where learning is co-constructed; classroom practice is 'discussion orientated' (Boaler, 2002: 116); and an agentic and authoritative epistemology supports coming to know (Povey, 1995). We draw on such earlier theoretical frameworks in the analysis which follows. Unsurprisingly, our students evoke a very different picture of undergraduate mathematics from that given above. We argue that these different experiences enable them to construct their identities as mathematicians and therefore to succeed. Below we cluster the things they said around four interconnected themes. The first three - mathematics is negotiable, a subject to explore; assessment in mathematics can be personal; and learning is social, supported, collaborative – offer a clear contrast to the description of undergraduate

[2] 'Identities are unstable, contradictory and multiple. Identities are the way we have of talking about ourselves, and are constantly being produced in our actions and our interactions with others; identities are always in process and never attained. However, the word "identity" suggests coherence and completeness so I have decided instead to use the phrase "identity work".' (Mendick, 2002: 336)

mathematics at traditional universities given above. The fourth relates to their authoritative and agentic identities as makers of mathematics.

Mathematics is negotiable, a subject to explore

We hear these students describing mathematics as a subject to explore, which is negotiable, where there is productive space for difference, where there is room for fuzziness and debate, for experiment and interpretation.

> You go through your entire A levels and you have got very set ways of doing things, you're kind of trained to do things by the book and then suddenly here… being forced to think quite laterally and actually solving problems again meant that you were stretched… We've been introduced to different ways of approaching things as well, not just here's a method of doing this which we used to get… you have to think sort of sideways… And of course when you discuss with other people and you see that they have done in a different way to you, you don't think "Oh I've done it wrong because they are obviously getting further than me" you think "They've done it a different way to me". (Myra[3])

> If I was approached with a problem now I'd know a whole host of different ways to try and tackle it because we've seen them all and I've tried looking laterally at things that aren't necessarily set problems. I think when you first come to university you're used to working through questions at school and you'd either get them right or wrong… [Here you say] "Have I got the right answer?" and the person at the front goes "Well but there isn't actually an answer" or "There are some other things which – " and you think well, how could I achieve that? I think now we are able to just like not be too worried about getting the correct answer at the end of the day. (Michael)

> I've got my work from previous degrees where a big NO written in the margin all over the place and you can't be wrong. Whereas here you can be wrong or you can explore and it's taken as that's part and parcel of the whole thing… All the things that are supposedly proved and are correct mathematically all came from dead ends and so on. All the great mathematicians made mistakes and said well "That didn't work." You don't see it any more because it's all been polished up into the thing that is correct but there are so many mistakes that are quite valid and certainly things come from them sometimes. (Geoff)

> That's what it is like here, that the way that we have been encouraged to think about maths is so different from what I have done before and it makes it more interesting because there is room to think about it and to kind of look at different possibilities... the fact that maths isn't right or wrong, I think that is probably the biggest difference, that there's not always a right answer. (Anna)

It is interesting how frequently the issue of 'right and wrong' came up during the interviews. It is a commonplace that, unlike other subjects, 'maths is right or wrong'; yet this truism seems to mis-describe both mathematics and, by implication, other subjects too. Clearly, in some sense it is very much possible to be right and to be wrong in mathematics (and history and French and…). So what are the students claiming by using this alternative discourse? A number of different things: amongst

[3] Pseudonyms are used throughout.

others, that employing different methods can provide different productive insights; that looking at things from different angles uncovers new aspects of a problem; that mistakes can be productive; and that both history and personal experience show that mathematical progress is a messy business. Crucially, however, we believe they are claiming a relationship of author/ity (Povey 1995) with the subject. We can hear this in the way in which they describe the work they produce for formal assessment.

Assessment in mathematics can be personal

Most of the assessments that the students chose to talk about were by coursework rather than by examination. (This also reflects the pattern of assessment on the course.) There is a strong sense of the personal, of authorship and of a claim to constructing knowledge. For the coursework tasks they are asked to complete, a broader range of assessment criteria is used than would be encountered in a more traditional course. For example, mathematical imagination, originality and creativity are prized and mathematical communication and mathematical thinking are valued.

> The fact that you are doing coursework and can investigate – it's not about getting the right answer a lot of the time so that the whole work we've been asked to do it's just so completely different, I can't really relate it [to my previous degree course] at all. You're taught, you do an exam and you either pass or fail whereas here it's like you go and find out something or you work something out for yourself. We have done a couple of assignments where you start without looking at any reference material at all, it's just your own, you're given a starting point and go off and work it out for yourself. (Geoff)

> The work what we've been given, sort of a topic or a title, you just go and find things out about it and just do it your own way instead of having structured work like "First do this, then that, follow on to that, as long as you put this you are okay". You can start with something and by going through some kind of procedure and doing something completely different, it's because you've actually looked into it so well and you've thought about it and you've involved so much mathematical things, it sort of spurs me on to do, to keep going at it. I can never - like when I'm doing an assignment in maths - I can never say "Right, that's it. I'm going to end it here". 'Cos I think "Well, what if, instead of when I were doing that, but instead of going that way I did that way" and I end up doing another so many pages about that thing because I can't stop thinking about it, that kind of task... for me I like these kind of assignments where you're given "This is the unit, this is what we've talked about, find something interesting about it and work on it and see where you get or where you don't get and what you find out". (Naomi)

> You had to work through it, you had to think, you had to draw things together, you had to understand why certain equations worked and how they worked and be able to put them into another context to get some more results, do a little bit more and you know it was just this ongoing thing and I found that really enjoyable… being able to do your own thing and work through it and nothing was wrong or right and I think that's a very good of working. You don't start off with "You should know this before you do this". (Joanne)

> [An assignment that requires reflection] helps me concentrate what I think because it means that I am thinking about - like instead of just thinking in a big jumble I have to sit

there and untangle what I am thinking, and say "Well I think this about this because - " and things like that, more than just things going round and round in my head. (Anna)

It seems, then, that these assessments do allow the presentation of a personal view and that this is a real source of pleasure and affirmation of the self for these students.

Learning is social, supported, collaborative

From the beginning, the creation of a collaborative classroom is prioritised and unhelpful habits of competitiveness are challenged. Great emphasis is placed on the social and intellectual ground rules of the classroom and discussion is a central pedagogical device. This, coupled with a reconstruction of the learners' relationship to knowledge, has reduced the risk of intellectual and emotional exposure.

It's probably to do with how I learn maths, I like to be able to bounce things off people. I like to, if I find something interesting, it's nice to be able to find somebody to talk to about it, whereas in a big lecture approach you don't, you go to a lecture, you go home, you write up the lecture and that's it really. So the fact that both in the sessions and outside the sessions you can talk to people in a small group, people that you know well and that helps me I think… talking to other people on the course, they're able to put links in for you and you are able to put links in for them you know they might not have noticed. (Geoff)

We were bouncing off each other like one of us would have a good idea and the other would try and implement it because we had different strengths that we could bring in to what we were doing as well and I think that's important… I think it's been good being able to discuss things and then go away and do your own work... talking about it and then going away - I think talking about it, it gives you more ideas that you can then go away and develop on your own and then you come back and you talk a bit more and I think that's how it develops. (Myra)

Sometimes I need some space and some time for it to sink in... and I suppose in the way that I work better on my own than with other people just so I can sort out what I am thinking… But like often… what would happen is that the people that want to work together and want to discuss ideas they'll do that and I can just stay sitting in my corner thinking about it. And that's fine and, you know, I can listen to their ideas and, if I've got something to say, say it. (Anna)

If you're stuck, I can easily go and speak to somebody and they can point out little things what will trigger something and you suddenly understand it. And the other way round as well. There are other people what you can help... Now it's more like trying to get peers to help the others in the class. (Naomi)

These comments are in contrast to the discourse of undergraduate mathematics, reported from another elite university, as ' "a kind of competition that you train for"' (Mann, 2003: 19) with 'a "performance" route' (Mann, 2003: 20) to success.

Makers of mathematics

We do not claim that *all* our students are successful. Some fail; and some pass but without ever gaining a real sense of themselves as successful creators of

mathematics. Ray, for example, has grown tremendously as a mathematician and is set to gain a lower classed honours degree; but he has never felt confident about his learning or his mathematics – although it can sometimes be a challenge to be enjoyed, it is still something external to be forced in from the outside.

> First day I come, me first day I can remember [the tutor] sticking something on board and having no idea right from the first minute I got in that classroom and I started panicking then and to a point I don't think really I've been in a lesson where I've not, not so much dreaded but felt confident that I know where - everything is supposedly there to challenge us and there is always maybe something - but I've never been able to go in and think I'm sure I am going to understand everything in this lesson, even if it get hammered into me one way or another I'm going to come out and I'm going to know what I've done and I've never been confident of that and I've never done that I don't think. (Ray)

Nevertheless, offering a different pedagogy, one that values agency and authorship, one that places the learning community as central (Boaler and Greeno, 2000), has enabled some failing and some initially weak students to construct authoritative mathematical identities: we argue that these in turn have been instrumental in producing their success.

> I like to explore things. Never before have I sat down in my spare time and just started doodling triangles or something like that, you know proving things which have been proved many times before but I'm just doing it for my own sake, I've never done that before but I am now. (Geoff)

> If you find out something interesting... you can actually discuss it; and you can get ideas off your peers and even the tutor. You can go to them and talk, talk to them about something, even if it's something you don't cover in class but they're interested and then you get sort of nice discussions going on with them about whatever you've found out. (Naomi)

> I think it's mostly confidence to put your own ideas on the table … to realise that what you say might be worth listening to, that every little thing that gets chuck in this pot in the middle can contribute to finding an answer… you were allowed to develop it, isn't it. There were no sort right and wrong answers, you were allowed to put your own input into it and your own direction. (Joanne)

These students have constructed a positive and active 'disciplinary relationship' (Boaler, 2002: 113) which allows them to identify as mathematicians and which is productive of increased capability. They have found opportunities to 'engage in a disciplinary dance' (Boaler, 2002: 119)': that is, their own agency and the agency of the discipline have worked together effectively. The mathematical community of practice thus produced is one to which they can belong, with which identification is possible: they have found a way of making mathematics which is authoritative and self-affirming and also successful in enabling them to come to know. We believe that what they have to say helps us re-envision the undergraduate experience of learning mathematics and has a useful contribution to make to the current debate about who succeeds, who fails and why.

References

Angier, Corinne and Povey, Hilary (2004) ' "I can do it but it'll be a battle": finding her place as an undergraduate mathematician' paper submitted to IOWME at the 10th ICME Conference, Copenhagen

Boaler, Jo (2002) 'The development of disciplinary relationships: knowledge, practice, and identity in mathematics classrooms' in the Proceedings of the 26th PME Conference, Norwich, 2002, (2), 113-120

Boaler, Jo and Greeno, James (2000) 'Identity, agency and knowing in mathematical worlds' in Boaler, Jo (ed) Multiple Perspectives on Mathematics learning and Teaching, Westport, CT: Ablex Publishing

Cobb, Paul, Wood, Terry, Yackel, Erna and McNeal, Betsy (1992) 'Characteristics of classroom mathematics traditions: an interactional analysis' in American Educational Research Journal, (29), **3**, 573-604

Hollway, Wendy and Jefferson, Tony (2000) Doing Qualitative Research Differently: free association, narrative and the interview method, London: Sage

Jaworski, Barbara (2001) 'University mathematics teaching – where is the challenge?' in the Proceedings of the 25th PME Conference, Utrecht, 2001, (3), 193-200

Leder, Gilah, Forgasz, Helen and Brew Christine (1998) 'Who persists with mathematics at the tertiary level: a new reality?' in the Proceedings of the 22nd PME Conference, Stellenbosch, 1998, (3), 183-190

Macrae, Sheila, Brown, Margaret, Bartholomew, Hannah and Rodd, Melissa (2003) 'The tale of the tail: an investigation of failing single honours mathematics students in one university' in Proceedings of the BSRLM Day Conference, Oxford, June 2003, pp. 55- 60

Mann, Christine (2003) Indicators of Academic Performance, retrieved in December, 2003 from http://www.admin.cam.ac.uk/reporter/2003-03/weekly/5913/6.html

Mendick, Heather (2002) ' "Why are we doing this?" A case study of motivational practices in mathematics classrooms' in the Proceedings of the 26th PME Conference, Norwich, 2002, (3), 329-336

Povey, Hilary (1995) 'Ways of knowing of student and beginning mathematics teachers and their relevance to becoming a teacher working for change', unpublished PhD thesis, University of Birmingham

Povey, Hilary and Angier, Corinne (2003) 'Some undergraduates' experiences of learning mathematics: (how) can narrative form enable us to create knowledge?' in the Proceedings of the BSRLM Day Conference, Birmingham, November, 2003

Rodd, Melissa (2002) 'Hot and abstract: emotion and learning mathematics' in the Proceedings of the second International Conference on the Teaching of Mathematics at Undergraduate Level, Crete: John Wiley and the University of Crete

Walshaw, Margaret (2002) 'Telling other people's stories: knowledge production and representation lessons' in the Proceedings of the 26th PME Conference, Norwich, 2002, (4), 345-352