SENSITIVITY FOR THE COMPLEXITY OF PROBLEM ORIENTED MATHEMATICS INSTRUCTION – A CHALLENGE TO TEACHER EDUCATION

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Teaching can be understood as acting and deciding in a complex system. On that problem oriented mathematics instruction (POMI) can be characterized as very high complex particularly regarding to mathematical-cognitive aspects. To cope with resulting demands in the long run, the teacher has to be sensitive for this complexity. But what does this mean? How can you get some clues for one's degree of sensitivity for complexity of POMI? Is it possible to sensitize teachers or teacher students for this complexity?

POMI AS A HIGH COMPLEX SYSTEM

Requests for a stronger problem orientation in mathematics instruction (e.g. NCTM 2000) has been raised worldwide for a long time. However, international investigations indicated a quite resistance of reality of mathematics instruction particularly to this requests. There are different reasons for this and many constraints are often quoted by teachers (e.g. Zimmermann 1991). In my opinion *one* reason are additional demands of POMI on the teacher concerning mathematical aspects and in particular designing and implementing a corresponding lesson plan.

In didactic, pedagogic and psychological literature teaching is very often described as a complex system (e.g. Arends 1997, Bromme 1992, Clark & Peterson 1986, Davis & Simmt 2003, Kießwetter 1994, Lambert, Loewenberg Ball 1998, Leinhardt, Greeno 1986; Fritzlar 2004 for more references). This view should be specified. A possible and suitable specification can be found in topical research work of cognitive psychologists (e.g. Dörner, Kreuzig, Reither & Stäudel 1983, see also Frensch & Funke 1995). They depict systems or embedded problems on several dimensions, and elementary and complex systems or problems are on the opposite ends of the scales. Partially based on the initial research work of the German psychologist Dörner the dimensions "comprehensiveness", "connectivity", "dynamic" and "low transparency" are often used (e.g. Kotkamp 1999):

Comprehensiveness: This dimension represents the quantity of information to be considered for an appropriate work on the problem. Therefore the extent of comprehensiveness also depends on the particular agent and his model of the situation. If the quantity of information goes beyond the processing capacity of the agent, he must try to reduce comprehensiveness (in an appropriate way).

Connectivity: This dimension represents the amount of change of system elements on account of modification of other elements. It describes, how close connected the system is. Because of the connectivity of a system, an agent cannot do one thing without many others. That's why he has to consider side and long term effects of his decisions too.
Dynamic: This dimension represents to what extent system elements change without intervention from outside. Dynamic systems do not wait for the agent, and so he often feels time pressure. In addition in dynamic systems an agent has not only to consider the present state, but its development in time too.
Low transparency: This dimension represents to what extent states of the system, its elements and connections between them can be observed. If necessary the agent has to complement his knowledge through active information gathering.
So complex systems can be characterized as comprehensive, close connected, high dynamic and hardly transparent systems, and there is absolutely no doubt that teaching is complex in this sense.

POMI is complex like conventional instruction in regard to the so called "classroom management" (Doyle 1986). But beyond this it is characterized by an additional high complexity concerning math-cognitive aspects: Several, partly different nevertheless coinciding problem solving processes of pupils appear during a lesson, which should be watched and supported by the teacher if necessary. This processes are influenced by numerous anthropogenic or socio-cultural conditions and especially by own, sometimes inconspicuous teaching-decisions in many ways. So a huge network of interactions between conditions, decisions and features of the lesson emerges, which can hardly be overlooked. From there unexpected matters occur very often and the teacher has to free oneself of or at least question own views. In addition, these processes are high dynamic, normally a teacher has only few seconds to find a suitable reaction. So he cannot take into account all available information or exhaust his whole knowledge. Teaching orientated to independent problem solving processes is very low transparent because the teacher cannot look into pupils' minds. In addition he is no longer the only (and authoritarian) source of information and he has to give the pupils more scope for doing mathematics. So in some ways POMI can hardly be under control or be planned.

POSSIBLE CONCLUSIONS

Complexity of teaching outlined above may be hardly disputed, but in my opinion it is considered too little or not the right way in teacher education. Especially complexity of POMI ought to be made a subject of discussion also in math teacher education at university, and – as a first step – educators ought to try to sensitize teacher students in this direction. (In this way teacher education could also make a further contribution to a stronger problem orientation of mathematics instruction.)

But to my knowledge no experiences concerning such orientation of teacher education exist. From there groundwork has to be done first!
Important conditions for a corresponding supplementation of teacher education are

- some information about students' initial situation regarding sensitivity for complexity, and
- possibilities to evaluate new elements of education to what extent they can help sensitize teacher students in this direction.

So within my research I created a diagnostic instrument and tested in a first explorative study, to what extent it can provide clues for the degree of sensitivity of teacher students for the complexity of math-cognitive aspects of POMI. For this I designate (in sense of a provisional definition for working) an agent as sensitive in this regard, if he is aware of the complexity of POMI, of special demands arising from it and of limits of his possibilities to decide and to act in an appropriate way.

Main element of the created diagnostic instrument is an interactive realistic computer scenario, which models selected aspects of the complexity with an appropriate example. In the following I want to report some details about the subject of the scenario, its important features, about the explorative study and main results.

AN ATTEMPT TO ANALYZE SENSITIVITY FOR COMPLEXITY

A mathematical problem – the "Faltproblem"

Subject of the computer scenario is the use of the following problem in a math lesson:

> The *Faltproblem* (folding paper-problem): A sheet of usual rectangular typing paper is halved by folding it parallel to the shorter edge. The resulting double sheet can be halved again by folding parallel to the shorter edge and so on.
> After n foldings the corners of the resulting stack of paper sheets are cut off. By opening the paper, it will be detectable that (for $n>1$) a mat with holes has resulted. Find out and explain a connection between the number n of foldings and the number $A(n)$ of folding-cutting-operations.[1]

Teachers, students and the author tried out this problem in about 50 lessons mainly in fourth and fifth grades of different school types. This experiences showed special potentials of the *Faltproblem* for POMI, which can only be listed here: the problem can be understood very easily (also by young pupils), nevertheless it is not at all mathematical simple; many possibilities to come to terms allow a differentiated work on the problem; very often pupils can evolve presumptions, search for explanations and, more generally, work heuristically; the problem is a motivating challenge for pupils,

[1] Used formulation of the problem and goal of working on are intended for the teacher. This problem was developed by Kießwetter (e.g. Kießwetter & Nolte 1996) to use in an entrance examination of the university of Hamburg.

generally they enjoy working on it; there are many possibilities for communication and cooperation; the problem is open in regard to ways and also to goals of working on; it has many points of contact to several mathematical subject areas and other mathematical problems; and many variations and extensions are possible.[2]

But the use of these potentials leads to a higher complexity and to additional demands on the teacher like outlined above.

The computer scenario

By analyzing the lessons about the *Faltproblem* I collected important aspects of the underlying network of conditions, decisions and features of pupils' problem solving processes. Based on this, on interviews of students, teachers and teacher educators and on theoretical literature I created a descriptive model and implemented it in an interactive computer scenario, which confronts the user with decision-situations connected to the use of the *Faltproblem* in a fifth grade's lesson. In this scenario the *Faltproblem* can be virtually taught in three different classes. For that the user takes the part of the teacher. At first he can decide about the lesson goals and how to begin the work on the problem. Then the scenario models some possible and probable reactions of the pupils, especially their working processes, ideas and results, and the user has to react again. But he can also go back and correct his former decisions or give some additional alternatives for reaction.[3] At the end of the lesson the user is told an assessment of his decisions particularly with regard to his lesson goals. From here he can also go back to former decision-situations or start again. (The figure on the next page illustrates these possibilities of interaction with the scenario.)

The scenario makes it possible for the user to vary his decisions systematically in the same class or to check effects of his decisions in different classes. So he can explore a large number of possible and probable lesson courses. Especially in this combination of realism, interactivity and the intimated possibilities of investigating the scenario I see its potentials regarding to the research goals:

- By realistic modeling of appropriate decision-situations through the scenario some special demands of POMI on the teacher can be simulated.
- A scenario gives a chance to sufficient complex modeling with concentrating on very important but often more or less ignored math-cognitive aspects.
- A scenario enables an interactive investigation. In this way arises a complex network of decision-situations comparable to real teaching.
- Decision-situations can be explored repeatedly (as often as the user want) and without time pressure. In addition for the user it doesn't make any difference, if

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[2] For more details see FRITZLAR (2004). I want express my thanks to all pupils, students, teachers and school administrators involved in the investigations.

[3] The scenario cannot react on alternatives given by users. But they were automatically collected and can be used for further development of the scenario.

he is able to execute his plans. (This could be important specially for teaching novices.) Altogether a scenario can model decision-situations realistically, and it enables ways of analysis of these situations, which do not exist in reality but whose use can provide some clues for the degree of sensitivity for complexity.

- Modeled situations can be varied systematically. By this the user can experience complexity of teaching in a special way and the teacher educator can analyze his examination of this complexity.
- As many students as wanted can work with the scenario, and it can be handled in an easy way.

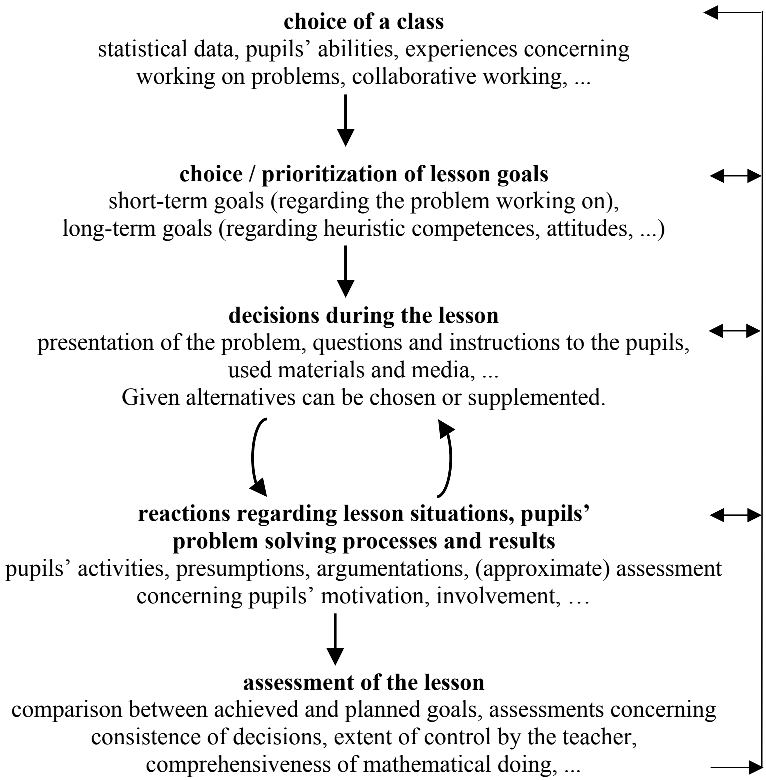

Figure 1: Work on the computer scenario

An explorative study

Within an explorative study I investigated how teacher students work with the scenario (without extensive instructions) to get some clues for their degree of sensitivity for complexity. Therefore I needed some indications for sensitivity, which could only be partially deduced from preliminary theoretical considerations. On account of the innovative character of the study I had to get indications from empirical data (characteristic features of students' investigation of the scenario) too. Based on these I created a fourdimensional "sensitivity for complexity (sfc) – vector", which describes the investigation of the scenario by the user and with it gives some clues for his degree of sensitivity for complexity. The "sfc – vector" has the following components:

Exploratory behavior*:* A sensitive agent is expected to try to get to know an unknown system, to come off own conceptions, to scrutinize and to correct own decisions if necessary. Therefore the exploratory behavior of the user can indicate to his degree of sensitivity for complexity, and consequently this component represents quantitative aspects (number of loops and jumps back within the program) and qualitative aspects (e.g. number of different modes of representation the problem) of exploring the scenario by the user.[4]

Context sensitivity: A sensitive agent is expected to analyze modeled situations in a detailed manner and to check offered alternatives for possible (side and long-term) effects. He is expected to consider (detailed modeled) math-cognitive aspects in particular, because these are so very important for POMI. Consequently this component represents to what extent the user referred in decision-situations to problem solving processes of pupils, aspects of the mathematical content, or more social aspects (motivation, teaching methods, …) of the lesson.

Inconsistence*:* A sensitive agent is expected to react consistently with modeled features of the lesson and (linked to them) his previous decisions. Consequently this component represents the percentage of decisions of the user, which are interpreted to be not consistent with modeled aspects of the lesson, particularly with features of pupils' problem solving processes.

Reflectivity: A sensitive agent is expected to try to create an appropriate mental model of connections between different aspects of the lesson. He is expected to question the quality of modeling by the computer program, to create additional alternatives in decision-situations if necessary and to reflect own decisions and his decision behavior. Consequently this component represents the degree of (critical) reflectivity on a rating scale.[5]For more details of the empirical investigation and its results I have to refer to Fritzlar (2004). For a first impression I want to indicate main results and give as examples coarsened results of two "extreme" experimental subjects of the study:

- Differentiated clues for the degree of sensitivity for complexity can be gained by analyzing the investigation of the scenario.
- There are no objections against the independence of the components of the "sfc-vector".
- I could not find specific sensitivity types in the experimental group.

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[4] Of course quantitative and qualitative aspects are not completely independent from each other.

[5] The scale is related to the relative differences between the subjects in this respect.

- Most of the involved students had only a low degree of sensitivity for complexity (regarding to the sfc-components). Generally the scenario was not much explored by the students. Possibilities for systematic testing of teaching-decisions also on different conditions were hardly used. The scenario focuses on math-cognitive aspects of POMI. This was realized by many students, nevertheless features of pupils' problem solving processes were considered only superficially and to a very small extent. Reflectivity of the scenario investigation was low in general. Users hardly reflected on connections between conditions of and decisions during the lesson and pupils' problem solving processes. Also from there arose only few motives for exploration. Multidimensionality of numerous decisions was rarely taken into account, meta-cognition was hardly perceptible.

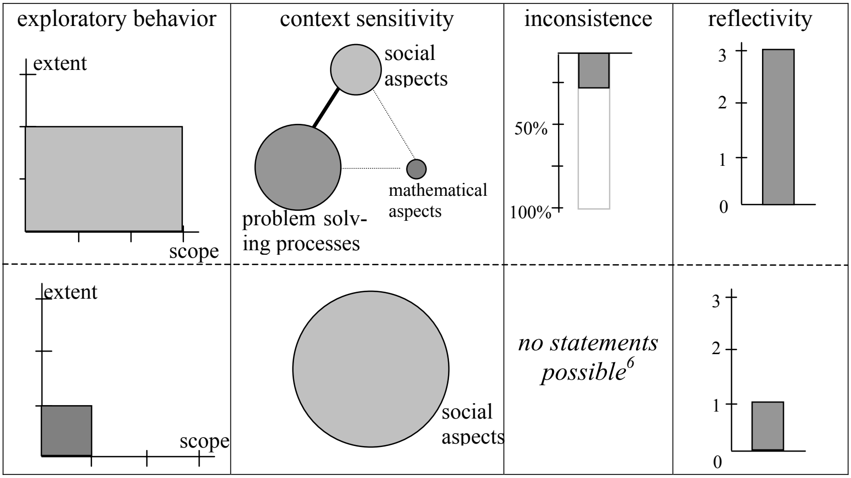

Figure 2: Coarsened results of two teacher students

The study presented here is new in regard to used methods and research goals. That's why I consider it appropriate to work exploratively and related to a concrete example. For several parts of the study almost inevitably arise limits and beginnings for improvements and supplementations. But beyond this the reported computer scenario could also be suitable to contribute to *sensitize* math teacher students for the complexity of POMI. For this I see the following possible potentials of the scenario:

- The program models selected parts of the network of interaction composed from conditions of the lesson, decisions during the lesson and features of pupils' problem solving processes and connected aspects. It can be easily handled and investigated by the user on his own.

[6] The student worked on too few decision-situations.

- Unavoidable reductions were made in view of my goal to enable experiencing specific aspects of the complexity of POMI by working on the program.
- The scenario enables ways of investigation, which do not exist in reality but allow conclusions about it. So it allows experiences, which could hardly or only tediously be gathered otherwise.
- The scenario could widen the "horizon of learning" by confronting the user with possible effects of his decisions directly, hardly delayed and in time-lapse.
- Working on the program could encourage the user to critical reflection of modeled structures and processes, among others in regard to a possible transferability to real situations.

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