THE IMPACT OF DEVELOPING TEACHER CONCEPTUAL KNOWLEDGE ON STUDENTS' KNOWLEDGE OF DIVISION

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This study investigated children's knowledge of division and its relationship to their teacher's conceptual understanding of division following Professional Development. A paper and pencil test was administered to 47 year 7 students and 2 teachers over 2 phases. Following the testing, six students and the teacher from each phase were interviewed. Results from this study indicate that most Phase 1 students rely on following a procedure with limited understanding. Their teacher displayed some conceptual understanding, however she too demonstrated a bias for procedural knowledge. This contrasts with Phase 2 teacher and students who demonstrated conceptual knowledge both in the test and the interviews.

INTRODUCTION

Division is required for many of the processes used in everyday situations, ranging from finding averages to determining rates and proportions involved with fuel use, cost and selling prices, and budgeting. It is also critical in problem solving and as part of the thinking underlying determination of area, volume and probabilities. A secure understanding of division, along with other forms of multiplicative thinking, is essential for work with fractions, ratios, algebra and further mathematics that marks the transition from the arithmetical thinking of the primary school to the more advanced thinking of the secondary curriculum and beyond (Booker 2003). Yet, many students experience considerable difficulties with division and it is often viewed as the most difficult of the four operations to learn by students and teachers.

A major source of these difficulties are the rote procedures that have often dominated its teaching and the language associated with them that provide little insight into the steps to be followed or their link to the underlying multiplication notions that allow division to be carried out (Booker et al 2004). Frequently, students have little understanding of the division concept in terms of sharing that provides a link to the multiplication facts needed at each step of the division process and are unable to read or interpret division statements or results (Greer 1995; Simon 1993).

Whether the teaching and learning of mathematics should focus on conceptual understanding or procedural competency has been the topic of research over many years (Ma 1999, Hiebert 1987). However conceptual understanding does not appear to be achieved by rote learning procedures in isolation, nor does simply knowing the steps necessarily indicate conceptual understanding of a mathematical process such as division (Silver 1987). Rather, the concepts and processes of mathematics need to grow hand-in-hand so that students can see connections between concepts and processes, interpret mathematics from multiple perspectives, be aware of the basic ideas fundamental to mathematics and possess an ability to reflect on previously learned concepts to give coherence to their knowledge (Ma 1999:122).

Ma’s (1999) study compared conceptual understanding and procedural competency of experienced U.S. and Chinese primary teachers across a range of mathematical topics. She concluded that the major influence on students’ learning of the fundamental ideas on which all future learning and problem solving depended was the strength of their teacher’s conceptual understanding of mathematics and their capacity to use this to generate meaningful examples, explanations and processes.

In contrast, many students are expected to embrace division without the concept development stages that occurred with the other three operations because they have already had considerable experience with symbolic representations (Anghileri 1996; NCTM 2000). This has led to considerable difficulties in reading division where they are often unaware of the importance of order when using division expressions (Anghileri 1999) and in interpreting remainders in calculations (Silver et al 1993). Insufficiently developed understanding of the division concept also leads students to calculate unrealistic answers and then not question their results (Simon 1993).

An investigation of the division understanding of teachers and their students might then provide insights into the reasons for students’ difficulties with division and its applications, similar to those Ma has reported for other mathematical topics.

THE STUDY

Forty-seven students and their teachers from two year 7 classes participated in a pencil and paper test containing 6 division questions made up of ten items and were asked to solve them without any imposed time limit. Following the testing 6students from each class and their teachers were interviewed on their test responses and their understanding of division. The two classes participated a year apart during which time professional development sessions were attended by most staff at the school.

Prior to the testing teachers used a process called *Divide Multiply Subtract bring down* commonly called *DMS bring down* to teach the division algorithm. Following Phase 1 professional development was conducted in the school where a focus on the development of conceptual understand by the use of games, and concrete representations of the sharing of division. This paper reports on the responses provided to 2 of these 6 questions aimed at identifying an ability to solve word problems through interpreting the problem, completing calculations involving internal zeros and interpreting their result to produce answers to the problems.

These questions are

Q5: When Movie World opened the Wild, Wild west ride, 6445 people went on the ride on the first day. If each wagon holds 7 people, how many full wagons could there have been?

Q6: A Birch Carroll and Coyle cinema needed 9238 packets of skittles to stack their shelves. If 4 packets are contained in each box how many boxes would need to be ordered?

RESULTS AND DISCUSSION

Student and teacher test results and interview responses are discussed in terms of the overall results and interview responses, error analysis of test responses, and the manner in which the remainder is interpreted to finally solve the problem.

Overall results

	Algorithm with recording	Algorithm without recording	Algorithm not recorded: answer only	No answer	Algorithm correct recording	Algorithm incorrect recording	% correct with calculation shown
Q5							
Phase 1 n=24	12	7	3	2	3	1	21%
Phase 2 n=23	20	1	2	-	16	1	80%
Q6							
Phase 1 n=24	12	7	2	3	6	Zero	31%
Phase 2 n=23	21	-	2	-	16	-	76%

Table 1: Comparison of Phase 1 and Phase 2 results

Table 1 shows the impact that professional development had on student results between Phase 1 and Phase 2. Phase 1 testing shows that only 21% of students were able to correctly complete the division algorithm for question 5 and 31%, for question 6. The year 7 students the following year (Phase 2) demonstrated far greater understanding, with 80% correctly calculating question 5 and 76%, question 6

Interview analysis

There was a noticeable shift towards conceptual explanations given by the six interviewed students in Phase 2 when compared to the interviewed responses for students in Phase 1. During the interviews the students where asked

> 'Please complete this division for me, $6\overline{)1845}$ and explain exactly what you are doing so that I can understand'.

Phase 1 students gave procedural descriptions. Some showed all their working and some chose to simply write in the quotient. Jimmy for example, wrote and said:

$$\begin{array}{r} 37 \text{ r}3 \\ 6\overline{)1845} \end{array}$$

> 6 doesn't go into 1 so 6 into 18 goes 3 times. 6 doesn't go into 4 so 6 into 45 goes 7 with remainder 3. Easy.

Beth also exhibited considerable confusion although she remained very confident throughout the interview. Her working below is supported by the following description of her calculation.

$$\begin{array}{r} 2\,10\,7\,r2 \\ 6\overline{)1845} \end{array} \qquad 217$$

> Beth: Well 6 divided by 18 goes in 2 times with 6 left over. 64 divided by 6 equals 10 with 4 left over. 45 divided by 6 which is (counting on her fingers) 7 remainder 2.
>
> Interviewer: So what is your answer?

Beth: 217 (Beth replied as she wrote the number beside her calculation.)

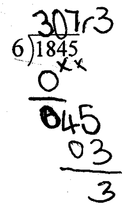
Interviewer: So is that a zero in there or ten?

Beth: You don't have to worry about the zero. The answer is just 217r2. (Pointing at her calculation)

Although Beth said *'6 divided by 18'* she calculated 18 divided by 6. She then went on to correctly say *'45 divided by 6.'* The ease with which she vacillates between the two descriptions of her calculation indicates her limited conceptual understanding of the division operation. She failed to recognise that the remaining 6 could have been shared with her initial calculation, that a 2digit number cannot sit in one place, that a zero marks a place and cannot simply be omitted and that 45 divided by 6 is not 7 remainder 2.

Leea correctly completed the algorithm, however her description is very procedural.

Leea: 6 into 1 can't go. 6 into 18, 3. Bring down the 4. 6 into 4 can't go so bring down the 5. 6 into 45, 7 remainder 3.

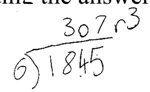


Leea has adhered to the *DMS bring down* process very strictly and has produced a correct answer. However this has not fostered a deeper understanding of the division operation nor the importance of place value.

The year 7 teacher from Phase 1 expressed concern during her interview that she did not have sufficient time to *'teach for understanding'* and that *'if the students just do as I tell them they will get it right.'* She said she liked structured teaching, which she classified as *'sit up and shut up.'* This style of teaching fits very well with the procedural explanation she gave for the division process:

The students just have to learn to deal with one number at a time, apply the learned process of DMS bring down, and they will get it right every time.

Of the 6 students interviewed in Phase 2, Peter was the only student who chose to calculate his answer by writing the answer without any working:



He was also the only student to explain his calculation from a procedural perspective.

Peter: How many 6's can I put into one? I can't. How many 6's can I put into 18? 3. Write it up here. How many 6's can I put into 4? Can't, so I put down the zero.

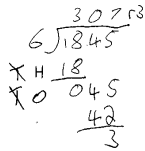
Interviewer: Why did you put a zero here? Can you explain?

Peter: You just do. Now, how many 6's can I put into 45? (counting quietly) 42, so 7 goes up here, remainder 3. The answer is 307r3.

Not only is Peter's explanation procedural, limited conceptual knowledge is evidenced by his inability to explain why the zero was placed in his answer.

Katie's work and explanation was typical of the remaining 5 interviewed students.

> Can I share 1 thousand among 6? No, so I rename as 18 hundreds. 18 shared among 6, which is 3. 3 times 6 equals 18. 18 minus 18. Then you bring the 4 down. Can I share out 4 tens among 3? I can't so I put zero here. Then I bring down the 5 and rename as 45 ones. 7, 6 times 7 equals 42 so 45 take away 42 is 3. I now have to rename to tenths. 6 into 30 tenths equals 5 with nothing left over.



Katie is aware of place value, that renaming is occurring and that division involves sharing. She is still using the language of 'bring down' as she renames however it could be argued that she understands exactly what she is doing and why.

Following the Professional Development, the Phase 2 year 7 teacher introduced her classroom to an approach to teaching mathematics where concrete materials were used and discussions encouraged to explain the concepts being developed. When asked how this approach had impacted on her teaching of division she replied:

> Once the kids get into the upper (primary) school they stop using concrete materials (for the operations) and the kids have a real reluctance to go back to using them. Maybe that is why DMS bring down was taught But with division being introduced in the upper school, and I have used games and other materials it is now OK to use concrete materials, in their eyes. I have found that by using concrete materials that kids get it, and I am talking about kids who haven't got it mathematically in the past. I think it is because we now do so much concrete stuff and they can all see it together and talk about it. You don't have to be able to work in the abstract to get it.

Error Analysis

When the errors made by the students are analysed further insight into the varying levels of student conceptual understanding is evident between Phase 1 and Phase2 students and their teachers. While the number of students who chose not to attempt the questions or to show any of their working has reduced from Phase 1 to Phase 2, the area of most change is in the process errors. Table 2 documents the student errors:

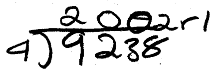
	Correct calculation	% Correct	Incorrect calculation	No attempt or no working	Internal zero error	Process error	Fact error	Misc errors
Q5								
Phase1 n=24	4	21%	15	5	5	9	4	1
Phase2 n=23	18	80%	4	1	3	-	1	-

Q6								
Phase1 n=24	6	31%	13	5	4	7	5	-
Phase2 n=23	16	76%	5	2	2	-	1	2

Table 2 Error Analysis

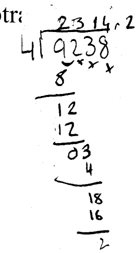
Phase 2 students did not make any errors that have been classified as process errors in contrast to the large numbers of process areas for questions 5 and 6 in Phase 1. However, internal zero and fact errors have continued.

In Phase 1 there were a variety degrees of process errors which provide good insight into the limited conceptual understanding these students possess. For example David wrote:



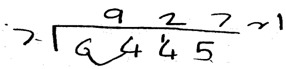
He has learned that when you cannot share a number in a place, you put a zero to indicate that no sharing has occurred. However, he has failed to share 230 and the remainder 1 is possibly the remainder from the sharing of 9 thousands.

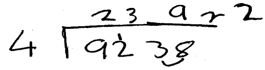
While Bella's work superficially appears to indicate substantially more understanding than the previous examples, on closer examination she demonstrates processing errors with both division and subtra



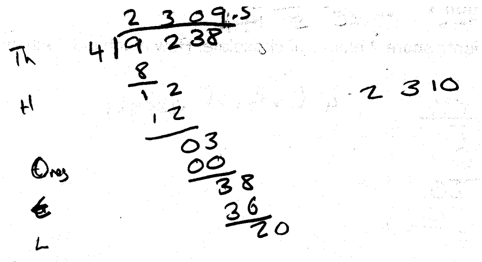
When Bella could not divide 3 tens between 4 she changed the divisor to 3 wrote in the quotient that 3 was shared once and that she had used all 4 tens. She then takes the 4 tens from 3 tens leaving her with one ten. Examples such as this indicate how superficial her understanding of the other operations may be, yet could be so very easily marked wrong without considerations to what aspect was incorrect particularly when her setting out looks like she has learned the process.

The results from both phases of this study indicate that a zero in the ones place in the quotient is more difficult to master than a zero in tens place. Phase 1 student, Daniel, has difficulty with internal zeros in both questions.





Divesh's work is a typical example of the 16 students in Phase 2 who correctly calculated the answers to questions 5 and 6. He clearly documents what place is being shared indicating place value knowledge and a thorough understanding of the concept of division. These children have had experience using games to demonstrate the sharing that is division, they then moved on to sharing MAB on a place value chart and then documenting the sharing in the form of the algorithm seen by the following example.



Interpretation of Remainder

The final aspect of these two questions to be analysed in this paper is the question of interpretation of the remainder to solve the problem.

	Correct calculation Remainder interpreted correctly	Difficulty Interpreting Remainder -calculation correct to this stage	No attempt to interpret remainder – calculation correct to this stage.	Difficulty Interpreting Remainder - calculation incorrect	No attempt to interpret remainder- Calculation incorrect
Q5					
Phase 1 n=24	0	1	3	1	14
Phase 2 n=23	8	1	8	1	3
Q6					
Phase 1 n=24	0	0	6	2	11
Phase 2 n=23	4	3	9	1	4

Table 3 Interpretation of Calculation

The example above shows that Divesh has interpreted the remainder in his calculation in order to arrive at his final answer of 2310 boxes. However, this was not commonplace as Table 3 shows.

No student in Phase 1 completed problems 5 and 6 successfully. There has been an improvement in students correctly completing both questions in Phase 2 but this remains at a low level, 47% for question 5 and 25% for question 6. When teacher responses are taken into consideration Phase I students can not be blamed for their low priority to interpret their calculation and solve the problem when their teacher does not have this aspect of problem solving as a priority and only interpreted one of the calculations herself.

Teacher Phase 1

$$4 \overline{)9238} \quad 2309 + 2$$

$$7 \overline{)6445} \quad 920 + (5)$$

2 310 boxes requred

On the other hand, while the Phase 2 teacher gives the correct solution to both problems, she does not appear to have prioritised this to her students judging by their poor results on this aspect of problem solving.

CONCLUSION

The responses given by both students and teachers demonstrated considerable growth in conceptual understanding of division. Prior to PD both teachers taught the process known as *DMS bring down* when teaching the division operation. Following PD the Phase 2 teacher introduced games, use of concrete materials as representations and then mirrored these representations with documented working of the algorithm This allowed children to understand the sharing that occurred with greater respect for place value. The students did not have to learn a process they did not understand nor were they expected to work in the abstract. They were able to see exactly what was being shared and how this impacted on the final result. Phase 2 students outperformed Phase 1 students, their error rate was lower, their conceptual understanding of division was greater and they had began to consider interpreting their calculation to solve the problem.

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