STUDENTS' WAYS OF INTERPRETING ASPECTS OF CHANCE EMBEDDED IN A DICE GAME

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In this paper seventh-grade pupils' ways of handling aspects of probability have been investigated. The aspects in question were embedded in a dice game, based on the total of two dice. Four different set-ups of dice were included in the situation in which they were up to explore optimal strategies for winning the game. How children understand concepts is regarded from the perspective of how the pupils' understanding varies with their interpretation of the situation, in which the concepts are embedded. Empirical data have been analyzed with intentional analysis, a method by which we regard pupils' act as intentional. The results show approaches of extremes and of a number model, as consequences of how the pupils process and bring to the fore information in the situation.

BACKGROUND

Two research perspectives are seen in the area of chance encounters. First there is the psychology/cognitive perspective including the works by Kahneman and Tversky, quoted and developed in Gilovich *et al*. (2002), with focus on analyzing patterns in order to identify misconceptions and judgmental heuristics. The second perspective is that of mathematicians and mathematics educators, with focus more on learning probability from a mathematical point of view (Shaughnessy, 1992).

The results of numerous psychological studies are reflected in the research of mathematics educators, in that the psychologists have provided a theoretical framework in considering judgmental heuristics. An extensive mapping based on the psychologist approach, and from related studies of misconceptions, can be identified in the literature, including *representativeness and availability* (Gilovich *et al.*, 2002), the *outcome approach* (Shaughnessy, 1992) as well as the *equiprobability bias* (Lecoutre, 1992).

The bias in focus here is equiprobability. Regarding the total of two dice this notion implies that all sums are equally likely to appear. Based on her results Lecoutre (1992) argues that this bias mainly stems from a conceptualization of the random experiment as being only a matter of chance. Confronting pupils with the total of two dice in a computer-based environment Pratt (2000) also identifies responses in accordance with equiprobability. He concludes that the participants base their decisions in random experiments on different types of resources, external as well as internal. Regarding equiprobability the four local internal recourses *unpredictability, irregularity, unsteerability* and *fairness* are mainly in play.

In a study of factors affecting probabilistic judgments Fischbein *et al*. (1991) argue that there seems to be no natural intuition regarding the order of the two dice. This implies that children are not aware of all possible combinations when they are comparing different totals of the dice. In connection with this Keren (1984) also found evidence that it is important to identify the sample space decisions are based on, "the knowledge of the sample space used by the students is crucial for understanding their responses" (*ibid.*, pp. 127).

Synthesizing the discussion so far, results indicate that pupils' responses in situations of uncertainty are affected by what resources they make available, and how they choose to make use of them, in a given situation. Their responses can be seen as a question of how they process and bring to the fore information for their judgments. This, I believe, challenges the view of misconceptions and, particularly, the equiprobability bias. The problem of chance encounters is rather a question of how pupils interpret and organise the situation as a whole. In the theoretical considerations I will discuss such processes from a constructivist perspective, in terms of contextualization and differentiation.

THEORETICAL CONSIDERATIONS

Regarding personal resources, Fischbein (1975) argues that intuitions play a prominent role. He distinguishes between primary intuitions as cognitive acquisitions, derived from individual experiences, without systematic instruction, and secondary intuitions as formed by education and linked to formal knowledge. Resources in general and intuitions in particular bear a strong likeness to what is commonly called *alternative frameworks* (Driver, 1981). Such frameworks, as intuitions established in every-day life, are in the same way related to teaching objects as primary intuitions are related to secondary intuitions. Considering learning from such an experience-based standpoint, the constructivist tradition usually regards this as a process in which naive, alternative, conceptions are abandoned in favour of more scientifically based knowledge. But since a constructivist approach to learning also presupposes the two basic principles continuity and functionality such a learning model is difficult to accept. First of all, and in relation to continuity: How can new formal knowledge be constructed on the basis of naive conceptions, if the two forms of knowledge are inconsistent with each other? The large mapping of different kinds of misconceptions in probability emphasizes this problem. Another problem is that knowledge does not always seem to be stable between similar situations. These issues illustrate the problem of transfer that the constructivist perspective struggles with, and which is emphasized by a model of learning as a process of abandoning naive, alternative notions in favour of more scientifically based knowledge.

As the constructivist perspective focuses on the individual construction of a learning object, criticism has also been raised against its low priority of the situated interaction between individual and environmental aspects (Säljö, 2000).

Discussing conceptual change, Caravita and Halldén (1994) argue that a more appropriate way to conceptualize learning, in accordance with a constructivist approach, would be to describe it in terms of thinking strategies, such as an expanded repertoire of them as well as a refined organization of and between them; "Learning is then a process of decentering, in the Piagetian sense, rather than the acquisition of more embracing logical or conceptual systems replacing earlier less potent ones" (pp. 106).

Based on this reasoning, learning can be looked upon as a problem of differentiating between contexts for interpretations. But in accordance with a constructivist view, context here refers to students' personal constructions. If we let the *conceptual* context denote personal constructions of concepts embedded in a study situation, as well as the *situational* context denotes interpretations of the setting in which learning occurs, and the *cultural* context refers to constructions of discursive rules and patterns of behavior in the society, we can talk about students' ways of appropriating new conceptions as a problem of *contextualization* (Halldén, 1999; Wistedt & Brattström, in press).

Halldén (1999) stresses that these different kinds of contexts are in play simultaneously as we are trying to solve a task. Depending on how we interpret the situation, by focusing certain aspects, they get different priorities in the contextualization process. In studies of learning conceptual structures are of certain interest, why the conceptual context is in focus in analyzing learning situations.

Such a meaning-making process is more in tune with the principles of continuity and functionality; old ideas are combined (and recombined) with other old ideas and new ideas, with respect to personal interpretations of a phenomenon or event.

In line with the contextualization approach I will in this paper describe seventh-grade pupils' ways of handling aspects of probability as a problem of their different ways of contextualizing tasks that bring forward such aspects.

METHOD

By confronting students, during a game situation, with a mathematical content not presented to them before in school I was hoping to create a situation in which a variety of contextualizations would appear. The content in focus was probability, a subject that in terms of conceptualization has shown to be interesting in relation to mathematics as well as to every-day life.

Eight participants were divided into four groups, with two students in each group. In the group discussions, which were tape-recorded and fully transcribed, they were up to explore optimal strategies for winning a dice game, based on the sum of two dice. The dice were designed to bring to the fore several aspects of probability and simultaneously give the students the opportunity of encountering small differences in mathematical structure between different situations. Each team had a board with areas marked 1-12. They also got a set of markers, which they were asked to distribute as they liked among the 12 areas. In the moment of play, which was videotaped, two teams played against each other. Here they took turns on rolling the dice. If one team or both of them had at least one marker in the area, which was marked with the sum of the dice, they removed exactly one marker from this area.

The team who first removed all its markers from the board won. The situation included four different set-ups of dice presented to the students in the following order.

1. The yellow setting – Here the faces were marked with one and two eyes, distributed as (111 222) and (111 222). The set-up was aimed to model the well-known experiment of throwing two coins.
2. The red setting – Included two different dice, each with a distribution of two outcomes among the faces as (222 444) and (333 555). This design implied an interaction in which the order of the dice didn't have to be taken into account.
3. The blue setting – Similar to the yellow with the difference that there were now four sides marked one and two sides marked two, that is (1111 22) and (1111 22).
4. The white setting – Similar to the red setting but now with the distribution shifted towards the lower numbers as (2222 44) and (3333 55).

A central purpose with the third and fourth settings was to stimulate the process of contextualization, with respect to combinations and proportionality.

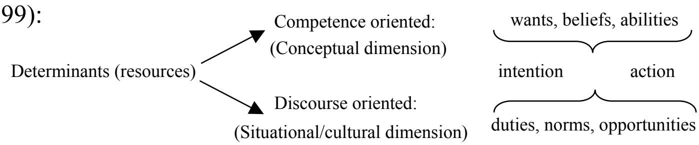
In the analysis I followed the principles of intentional analysis (von Wright, 1971; Halldén, 1999). This means, in order to understand a sequence of activities, that behaviour has been regarded as intentional. The intention in question gives meaning to the behaviour. One way of structuring such an intentional explanation is, according to von Wright, in the form of a practical syllogism, of which the following can serve as an illustration:

P1 A person P intends to bring about x.
P2 P believes that to bring about x would require the fulfillment of y.
C Thus: P does y.

What you see as an observer is the conclusion, i.e. a person P doing y. By ascribing P the intention x we can find reasons for P doing y, under those circumstances that are implicated in premise 2. In terms of von Wright (1971):

> Behaviour gets its intentional character from being seen by the agent himself or by an outside observer in a wider perspective, from being set in a context of aims and cognitions. This is what happens when we construe a practical inference to match it, as premises match a given conclusion. (pp. 115)

Premise 2 should be seen as a mental stage, connecting the intention x with the verbal or non-verbal behaviour in C. von Wright describes this relation in terms of internal and external determinants. Based on principles for this an educational approach has been worked out, which is related to the previously referred perspective of contextualization, in order to explain the relations in the practical syllogism (Halldén, 1999):



RESULTS AND ANALYSIS

Rounds 1 and 2 – A question of finding possible outcomes

In the group activities, there seems to be no doubt that the major intention for the participants is to win the game. During the two first play-rounds they interpret this issue as a matter of differentiating possible outcomes from impossible ones. They focus possible sums, regarding the dice in question. However, what does not seem to be in focus in such a contextualization is the various ways sums can be represented. Regarding the first design this was not surprising as I had reasons to believe that the pupils would not be able to take into account the order of the dice, i.e. the difference between the outcomes (1, 2) and (2, 1). But, still focusing only on possible sums, none of the groups reflect on the distinct different ways 2+5 and 4+3 either, representing the sum of seven in the second setting. Actually, two of the groups, using an approach of extremes, ended up with a sample space in the second setting as {5, 6, 7, 8, 9}, including the impossible outcomes 6 and 8. The approach of extremes was a strategy in which they started by identifying the smallest and largest values five and nine and after that ascribing possible outcome to all sums within these extremes. This approach even more implies that the pupils are only focusing resulting sums, with little or no considerations regarding underlying processes generating the sums.

In order to understand the pupils' ways of ascribing probabilities to the sums, in terms of distributing markers within the identified sample space, I argue that we have to keep in mind their interpretation of the situation; that the experiment is a question of finding possible sums. Since their action, regarding distribution of markers, in great details are similar during the two first settings the first round may serve as an illustration. Referring to the sample space {2, 3, 4}, Sabina in group A suggests a solution to the distribution of markers as:

> Sabina: How many on each? 24 divided by 3…it will be 8, doesn't it?

In group D we identify a similar approach. After that Lars and Petra also have identified the same sample space {2, 3, 4} Petra moves on with:

> Petra: We are going to have 24 [referring to the number of markers]…it will be 8 on each. If we have 8 on each there will be 3 times 8, which is 24.

These responses could be explained in terms of Lecoutre: The pupils just consider the situation as a matter of chance and therefore place equiprobable. I could agree with that. But since it seems to be that they just bring to the fore, and base their judgments on, the single outcomes 2, 3, and 4, such responses seem to be reasonable and not clear biases. By this I mean that an equiprobability response, based on an idea that everything is just a matter of chance, may be determined by the reason that they only have made available, that is, are aware of three different outcomes. As no further information is processed by the pupils it seems reasonable for them to conclude that each outcome is equally likely to appear, a notion which is in line with the classical definition of probability. This implies that the pupils in some sense may connect sample space with probability for events within this sample space.

From the utterances above, there are also reasons to believe that their behaviour has been determined by conceptions concerning norms and expectations according to school mathematics. They find a solution to the task presented by application of tools that are relevant for the culture of education; that is, the numbers 3, 8, and 24, included and the computational devices of multiplication and division (cf. Säljö, 1991). Since the Swedish expression "delat med", used above by Sabina, could be interpreted as either "divided by" or "distributed among" it could also be argued that an imprecise language in the activity plays a crucial role as well.

Rounds 3 and 4 – Contextualizing in terms of a number model

In a similar way as the first two rounds were related by a common contextualization the activities were in accordance between the last two settings as well. Now focus was shifted from the resulting sums towards a more detailed exploration of the situation. But for such an approach to happen the participants had to be aware of the differences in design between the first and the third setting of the dice. Two of the four groups did not recognize this by themselves. Instead they contextualized in the same manner as earlier and therefore again respond in terms of equiprobability. Being aware of that, the observer made a choice to intervene in the situation, in that he made them aware of the differences between the designs.

Observer: Are these the same as the yellow dice?

Tom: Yes, they are the same.

Observer: Okay. So you don't see any differences between the blue dice and the yellow dice?

Tom: Wait! There is some difference between the twos.

Louise: There are fewer twos.

Observer: Will that matter?

Tom: Yes, we shall place little more on two… and not so many on four. Look, there are many, many ones and not so many twos!

Tom's answer on the first question emphasizes that the sums have been and still are in focus at that particular moment. However, being stimulated regarding differences in design the last utterance implies that he has made the information, offered by the observer, explicit for himself. Hence we can assume that all groups, from this point on, are aware of the situational determinant, regarding contrast between designs.

Overall, this awareness affects the pupils' ways of interpreting the aspect of chance in the situation in similar ways as is evident in Tom's last utterance above. In group C – ending up with a linear model of 16 markers on two, 12 on three and 8 on four in the third round – this was approach by:

Sabina: There are more ones then twos, so it is twice as big chance…that it will be two then it will be…

Peter: Four!

This type of reasoning is in several aspects in accordance with what Lecoutre (1992) calls the number model and Pratt (2000) relates to as the (structural) fairness resource. This means that the pupils now, in a more detailed way, focusing structural features underlying the random process, by interpret the situation as a question of mirroring the structure of individual dice in the structure of the total. Regarding such a contextualization, which in turn emphasizes that they take in consideration the number of ways they can represent each sum, the pupils again make use of extremes. However, this time the approach of extremes is more directed towards smallest and largest chance of the sums. Starting by focusing the most likely sum to appear, this strategy was exhibited during the fourth setting by Tom in group A as:

Tom: We should have many fives. Look here, I got 4 of twos and you got 4 of threes.

In group C Sabina emphasizes this further, in that she concludes:

Sabina: It should be most at five since there are most of threes and of twos… and least on nine, since there are few high numbers.

With respect to such approach the sums three and seven respectively were only reflected upon as being in between the two identified extremes. That means that the pupils do not take into account either the order of the dice or the distinct different ways of representing the sum of seven in the fourth setting.

CONCLUDING DISCUSSION

The aim of the paper has been to describe how chance encounters can be viewed as a problem of contextualizing. By ascribing intentions to sequences of activities we could find reasons for such meaning-making processes, under circumstances that may be described in terms of determinants.

In the results above I have argued that two main contextualizations appeared during the activities. During the two first set-ups it became evident that the pupils interpreted the game as a question of finding possible outcomes. I have argued for how such an approach, affected by situational and cultural conditions, restricted their strategy of distributing markers in that they only were aware of three single outcomes. Considering the last two settings, the pupils interpreted the situation as a question of mirroring the structure of individual dice in the structure of the total. Regarding such an interpretation, which in turn emphasizes that they reflect upon the number of ways they can represent each sum, an approach of extremes was used. An approach of extremes was used in the two former rounds as well, as a device for ascribing possible sums. However, in the two latter settings the approach was more directed towards smallest and largest chance of the sums.

Even if it could be argue that learning has taken place, in that the pupils deviate from equiprobability, the results indicate as well the crucial importance for pupils to make appropriate contextualizations. What is obvious is that neither of their interpretations activates a more systematical approach regarding possible outcomes. By this I mean that the sample space in focus, the sample space which they are aware of, is of great importance for their responses. They seem to have a natural intuition regarding proportionality, but as their contextualisations does not stimulate them to bring to the fore all representations of each sum, they base their decisions on a limited amount of information. Thus, to explore chance encounters, I claim that it is of crucial importance to take into account how the pupils interpret a phenomenon, an event or a situation as a whole.

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