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CONSTANT CURVATURE SPACELIKE HYPERSURFACES IN THE LORENTZ–MINKOWSKI SPACE

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Abstract. In this paper we will report on some of our recent results about compact spacelike hypersurfaces with spherical boundary in the Lorentz–Minkowski space $\mathbb{L}^{n+1}$. In particular we will prove that the only compact spacelike hypersurfaces in $\mathbb{L}^{n+1}$ with constant mean curvature and spherical boundary are the hyperplanar balls and the hyperbolic caps. As for the case of the scalar curvature, we will prove that the only compact spacelike hypersurfaces in $\mathbb{L}^{n+1}$ with nonzero constant scalar curvature and spherical boundary are the hyperbolic caps. Our approach is based on the use of several integral formulas, among them there are a volume formula and a flux formula.

1. Introduction and Statement of the Main Results

The study of spacelike hypersurfaces in the Lorentz–Minkowski space has been of increasing interest in recent years from both physical and mathematical points of view. From a physical point of view, that interest is motivated by the role that spacelike hypersurfaces in Lorentzian spacetimes play in different problems of general relativity. For instance, Lichnerowicz [10] showed that maximal hypersurfaces (that is, zero mean curvature spacelike hypersurfaces) are convenient as initial data for solving the Cauchy problem of the Einstein equations. Other reasons justifying their importance in general relativity can be found in [4, 7, 11] and [12], and references therein.

On the other hand, their mathematical interest is also motivated by the fact that spacelike hypersurfaces in the Lorentz–Minkowski space exhibit nice Bernstein-type properties. Let us recall that the Bernstein problem for maximal hypersurfaces in the Lorentz–Minkowski space $\mathbb{L}^{n+1}$ was introduced by Calabi [5], who proposed the study of the maximal hypersurface equation in