

ON THE GEOMETRY OF RED BLOOD CELL

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Abstract. The differential geometry of a normal red blood cell is treated using the Cassinian oval for modelling its profile. In this connection an explicit parametrization via Jacobian elliptic functions of the usual polar coordinates is found. The first and the second fundamental forms, and correspondingly, the Gaussian, mean, and principal curvatures, are derived. The integrals determining the volume, area, cross-section area, and circumference of a red blood cell are evaluated analytically and expressed in a form relevant to the sphere geometry via some correction factors. The free elastic energy U , associated with the outer bilayer membrane of the cell is integrated and its scale dependence is established. A relation between U and the surface area correction factor is determined. Approximate formulae, using elementary functions, that should be directly applicable to experimental data are developed.

Plots of these dimensionless parts of volume, area, cross-section area, and circumference are obtained. The sphericity index, homogeneity index, and volume/area ratio associated with the red cell geometry are derived in approximate forms as well.

1. Introduction

Red blood cells, or erythrocytes, serve to deliver oxygen to cells via the protein hemoglobin. The description of the form of the Red Blood Cell (RBC) is a subject of a research interest for a long time (cf. Funaki [6] for the early history of the subject). The most precise description now available is that given by the equation of the oval of Cassini (see e. g. Funaki [6], Vayo [20], Canham [1], and Figs 1 and 2). The main difficulty that remain unsolved is that related to the complexity of the mathematical expressions describing quantities of geometric interest. Some preliminary results in this field are provided by Funaki [6] and Vayo [20]. For a researcher that is not prepared