Geometry, Integrability and Quantization
September 1–10, 1999, Varna, Bulgaria
Ivaïlo M. Mladenov and Gregory L. Naber, Editors
Coral Press, Sofia 2000, pp 95-104

GEOMETRIC QUANTIZATION, COHOMOLOGY GROUPS AND INTERTWINING OPERATORS

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Abstract. Higher Dolbeaut cohomology groups of the flag manifolds are explicitly constructed using the technique of the intertwining operators. The integral representation for the higher order harmonic forms is obtained.

1. Introduction

Borel–Weil theory [1] gives realization of the compact group unitary representations in the space of sections of the induced linear fiber bundle over the full flag manifold. It could be viewed as direct ancestor of the geometric quantization of Kostant and Souriau [12], [4]. Generalization of the Borel–Weil theorem — famous theorem of Bott [2] served as a basis for the well known hypothesis of Landglands about the geometric realization of discrete series representations of semisimple Lie groups.

Necessity to pass to the higher cohomology groups of quantized phase space arises also in geometric quantization [5], [6]. It is connected with the notion of polarization [7] which is important technical tool that provides irreducibility of the space of quantum states. More precisely, the space of quantum states of the standard geometric quantization [7] coincides with the space of polarization-invariant sections of the line bundle over symplectic manifold. But, in some cases, the space of such sections may be equal to zero. To obtain space of quantum states in such a cases one has to pass to higher Dolbeaut cohomology groups [6]. For the reasonably wide class of situations they all but one are trivial. In the space of non trivial cohomology group the desired space of quantum space lies.

One of the cases when it is necessary to pass to the higher Dolbeaut cohomology groups is the case when the phase space coincides with the orbit O_χ of the