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CURRENT ASPECTS OF DEFORMATION QUANTIZATION

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Abstract. After a short introduction to the program of deformation quantization we indicate why this program is of current interest. One of the reasons is Kontsevich's generalization of the Moyal product of phase-space functions to the case of general Poisson manifolds. We discuss this generalization, including the graphical calculus for presenting the result. We then illustrate the techniques of deformation quantization for quantum mechanical problems by considering the case of the simple harmonic oscillator. We indicate the relations to more conventional approaches, including the formalisms involving operators in Hilbert space and path integrals. Finally, we sketch some new results for relativistic quantum field theories.

1. Introduction

Deformation quantization is an approach to quantum mechanics which uses phase-space techniques from the early days of quantum mechanics [16, 15, 14], and which was formulated as an autonomous theory by Bayen et al [1] in 1978. Using general mathematical techniques it provides a continuous deformation of the commutative algebra of classical observables to the non-commutative algebra of quantum mechanical observables, where the deformation parameter is $\hbar$. The main tool used is the **star product** of functions on phase-space, which corresponds to the product of operators in Hilbert space used in the conventional formulation of quantum mechanics.

For many years the only explicit example of a star product was the **Moyal product** [11] of functions on $\mathbb{R}^n$. It was only comparatively recently that Kontsevich [9] succeeded in constructing a star product on general Poisson manifolds. This development has caused a resurgence of interest in the deformation quantization program, both for quantum mechanics and for relativistic