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PATH INTEGRALS ON RIEMANNIAN MANIFOLDS WITH SYMMETRY AND STRATIFIED GAUGE STRUCTURE

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Abstract. We study a quantum system in a Riemannian manifold M on which a Lie group G acts isometrically. The path integral on M is decomposed into a family of path integrals on quotient space $Q = M/G$ and the reduced path integrals are completely classified by irreducible unitary representations of G. It is not necessary to assume that the action of G on M is either free or transitive. Hence the quotient space M/G may have orbifold singularities. Stratification geometry, which is a generalization of the concept of principal fiber bundle, is necessarily introduced to describe the path integral on M/G. Using it we show that the reduced path integral is expressed as a product of three factors; the rotational energy amplitude, the vibrational energy amplitude, and the holonomy factor.

1. Basic Observations and the Questions

Let us consider the usual quantum mechanics of a free particle in the one-dimensional space $\mathbb{R}$. A solution for the initial-value problem of the Schrödinger equation

$$\mathrm{i}\frac{\partial}{\partial t}\phi(x,t) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\phi(x,t) = \frac{1}{2}\Delta\phi(x,t) \tag{1.1}$$

is given by

$$\phi(x,t) = \int_{-\infty}^{\infty} \mathrm{d}y\, K(x,y;t)\phi(y,0) \tag{1.2}$$