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EFFECTIVE SOLUTIONS OF AN INTEGRABLE CASE OF THE HÉNON–HEILES SYSTEM

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Abstract. We solve in two-dimensional theta functions the integrable case $\ddot{r} = -ar + 2zr$, $\ddot{z} = -bz + 6z^2 + r^2$ (a and b are constant parameters) of the generalizied Hénon–Heiles system. The general solution depends on six arbitrary constants, called algebraic–geometric coordinates. Three of them are coordinates on the degree two (and dimension three) Siegel upper half-plane and define two-dimensional tori $\mathbb{T}^2$. Each trajectory of the Hénon–Heiles system lies on certain torus $\mathbb{T}^2$. Next two arbitrary constants define the initial position on $\mathbb{T}^2$. The speed of the flow depends multiplicatively on the last arbitrary constant.

Consider a galaxy which gravitational potential U_{gr} is time-independent and has an axis of symmetry. We are interested in the motion of a star in such a potential field.

Let us introduce a system of cylindrical coordinates (r, ψ, z): Oz is the axis of symmetry, z is the height of the star, $r := \sqrt{x^2 + y^2}$ is the distance between the star and the axis Oz, $\psi := \arctan \frac{y}{x}$ is the polar angle.

Two conservation laws (integrals) of the stellar motion are known:

$$I_1 = U_{gr}(r, z) + \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\psi}^2 + \dot{z}^2\right) = \text{total energy}\,,$$

$$I_2 = mr^2\dot{\psi} = \text{angular momentum of the star around } Oz \text{ axis}\,,$$

m is the mass of the star, $\dot{} = \dfrac{\mathrm{d}}{\mathrm{d}t}$ is the derivative with respect to the time t.

With the help of the second integral I_2 we reduce the dynamics of the star on