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PATH INTEGRAL FOR STAR EXPONENTIAL FUNCTIONS OF QUADRATIC FORMS

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Abstract. The Moyal product is considered on the complex plane $\mathbb{C}^2$. Path integral representation of $*$-exponential function is given for a quadratic form on $\mathbb{C}^2$, $H = ax^2 + 2bxy + cy^2$ for $(x, y) \in \mathbb{C}^2$, where $a, b, c \in \mathbb{C}$.

1. Introduction

The formal deformation quantization theory was started by Bayen, Flato, Fronsdal, Lichnerowicz and Sternheimer [1] and the general theory is well designed studying the existence, classification and applications (cf. [2–4, 7–9]). However, if we take as a deformation parameter a number, we have no general theory of non-formal deformation quantization at present (see e. g. [5]).

In this note, we consider a deformation quantization with a deformation parameter $\hbar > 0$. The star product is given by the Moyal product formula. As is well known, any formal star product is locally isomorphic to the Moyal star product, hence we deal with local theory in this sense, but non-formal.

Let $H = ax^2 + 2bxy + cy^2$ be a quadratic form on $\mathbb{C}^2$ with $a, b, c \in \mathbb{C}$. We consider the Moyal product $*_0$ given in Definition 1 below. We will study the $*$-exponential function $\mathrm{e}_*^{tH/\mathrm{i}\hbar}$. In [6], the star exponential function is defined by solving a certain differential equation which characterizes the $*$-exponential function. The purpose of this paper is to give a path integral description of this $*$-exponential function (see Theorem 2 below).