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A PLATEAU PROBLEM FOR COMPLETE SURFACES IN THE DE-SITTER THREE-SPACE

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Abstract. In this paper we establish some existence and uniqueness theorems for a Plateau problem at infinity for complete spacelike surfaces in $\mathbb{S}^3_1$ whose mean and Gauss–Kronecker curvatures verify the linear relationship $2\varepsilon(H-1)-(\varepsilon+1)(K-1)=0$ for $-\varepsilon \in \mathbb{R}^+$.

1. Introduction

The global approach to surfaces with a constant curvature is a subject of many studies in Submanifolds Geometry, especially of that ones whose structure equations are integrable in terms of holomorphic data, because it represent a powerful tool in the study of these surfaces. Some representative examples are the Enneper–Weierstrass representation for minimal surfaces in $\mathbb{R}^3$ [13] and the McNertney–Kobayashi one for maximal surfaces in $\mathbb{L}^3$ presented in [9].

In this paper we will deal with spacelike surfaces in $\mathbb{S}^3_1$, a topic developed in the recent years. For instance, in the compact case Ramanathan [14] proved that every compact spacelike surface in $\mathbb{S}^3_1$ with constant mean curvature is totally umbilical. On the other hand, Li [10] showed that every compact spacelike surface in $\mathbb{S}^3_1$ with constant Gaussian curvature is totally umbilical.

As a natural generalization of Ramanathan and Li results, Aledo and Gálvez [2] characterized the totally umbilical round spheres of $\mathbb{S}^3_1$ as the only compact linear Weingarten spacelike surfaces.

In this work we study a special case of linear Weingarten surfaces of Bianchi type, in short BLW-surfaces, studied in [3]. We center our attention on BLW-surfaces whose mean and Gauss–Kronecker curvatures verify the linear relationship

$$2\varepsilon(H-1)-(1+\varepsilon)(K-1)=0, \qquad -\varepsilon \in \mathbb{R}^+. \tag{1}$$