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GEOMETRICALLY IMPLIED NONLINEARITIES IN MECHANICS AND FIELD THEORY

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Abstract. Here we discuss the concept of essential nonlinearity, i.e., one which cannot be meaningfully decomposed into well-defined linear background and "small" nonlinear correction. Therefore, the traditional perturbative techniques and asymptotic methods are non-effective then. Two well-established classes of essentially nonlinear field theories exist in the market: 1) The General Relativity and other generally covariant schemes, 2) The Born–Infeld type nonlinearity in traditional and generalized sense. The essential nonlinearity of 1) is intimately connected with the invariance under the very huge group $\mathrm{Diff}(M)$ of all space–time diffeomorphisms. The Born–Infeld scheme 2) is also geometrically motivated by the theory of scalar densities in manifolds. But there is no explicitly seen relationship between these two types of nonlinearities. Below we show that there exists however some hidden link between general covariance and Born–Infeld mechanism. The structure of the group of internal symmetries (target space symmetries) is also relevant. Roughly speaking, "huge" symmetry groups are intimately connected with essential strong nonlinearities. It is so even in finite-dimensional analytical mechanics. Let us remind our affinely-invariant models in mechanics of homogeneously deformable bodies [18–21,23,25–29]. There is no systematic theory, nevertheless some rough although convincing arguments do exist. This essay is just concentrated around the study of the interplay between (high) symmetries and (essential) nonlinearities. The examples quoted below confirm the idea and exhibit a kinship between general covariance and Born–Infeld paradigm. The special stress is laid on models which, by abuse of language, resemble the structured continua with affine geometry of degrees of freedom. These models are based on the bundles $\mathrm{L}M = T^1_1 M$ over the "space–time" manifold M, and FM, the principal bundle of linear frames. And the special stress is laid on scalar multiplets (trivial bundles over M).