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CONSTRUCTION OF ELLIPTIC SOLUTIONS TO THE QUINTIC COMPLEX ONE-DIMENSIONAL GINZBURG–LANDAU EQUATION

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Abstract. The Conte–Musette method has been modified for the search of only elliptic solutions to systems of differential equations. A key idea of this a priory restriction is to simplify calculations by means of the use of a few Laurent series solutions instead of one and the use of the residue theorem. The application of our approach to the quintic complex one-dimensional Ginzburg–Landau equation (CGLE5) allows us to find elliptic solutions in the wave form. We also find restrictions on coefficients, which are necessary conditions for the existence of elliptic solutions to the CGLE5.

1. Introduction

At present time the methods for construction of special solutions of nonintegrable systems in terms of elementary (more precisely, degenerated elliptic) and elliptic functions are still actively developed (see [12, 23] and references therein).

Elliptic and degenerate elliptic functions are single-valued functions, therefore, the necessary condition for the existence of such solutions of a nonintegrable system is the existence of the Laurent series solutions of it. Such local solutions can be constructed by means of the Ablowitz–Ramani–Segur algorithm of the Painlevé test [1] (see also [11, 12, 17]). Moreover for a wide class of dynamical systems using this method one can find all possible Laurent series expansions of solutions. In this way one obtains solutions only as formal series, that is sufficient, because really only a finite number of coefficients of these series are used. Examples of construction of such solutions are given in [13] and [18]. The Laurent series solutions give the information about the global behavior of differential systems and assist to look for exact solutions [14]. The Laurent series solutions can be used to prove the nonexistence of elliptic solutions [10, 22] as well.