Factors limiting the operation frequency of magnetic refrigerators

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1. Introduction (why heat-exchange fluid is indispensable for fast transfer of heat)
2. Estimating f_{max}
3. Discussion:
   - ways of raising f
   - some features of optimal refrigerator design
   - implications for magnetic refrigerants

- currently built prototype devices operate at $f \sim 1$ Hz

but it is speculated that f could be made as high as hundreds of Hz or even several kHz

- do we need such a high f ?

cooling power = $f \times$ RCP

- is there a universal f_{max}?
- how high is it?
- which factors determine f_{max}?

thermal conductivity over long distances is prohibitively slow:

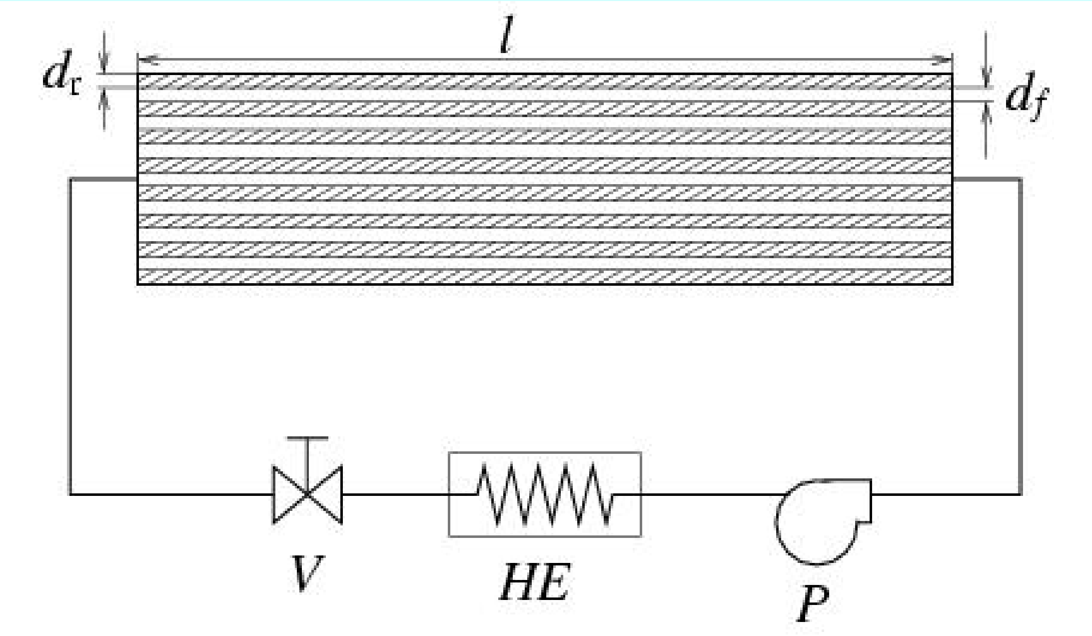
$$\tau_{\mathrm{tc}} \sim \frac{\rho\, C}{\kappa} d^2$$

taking $d = 1$ cm,
one gets for Gd
$\tau_{\mathrm{tc}} \sim 2 \times 10^1$ s

Way out: use forced convection of heat-exchange fluid

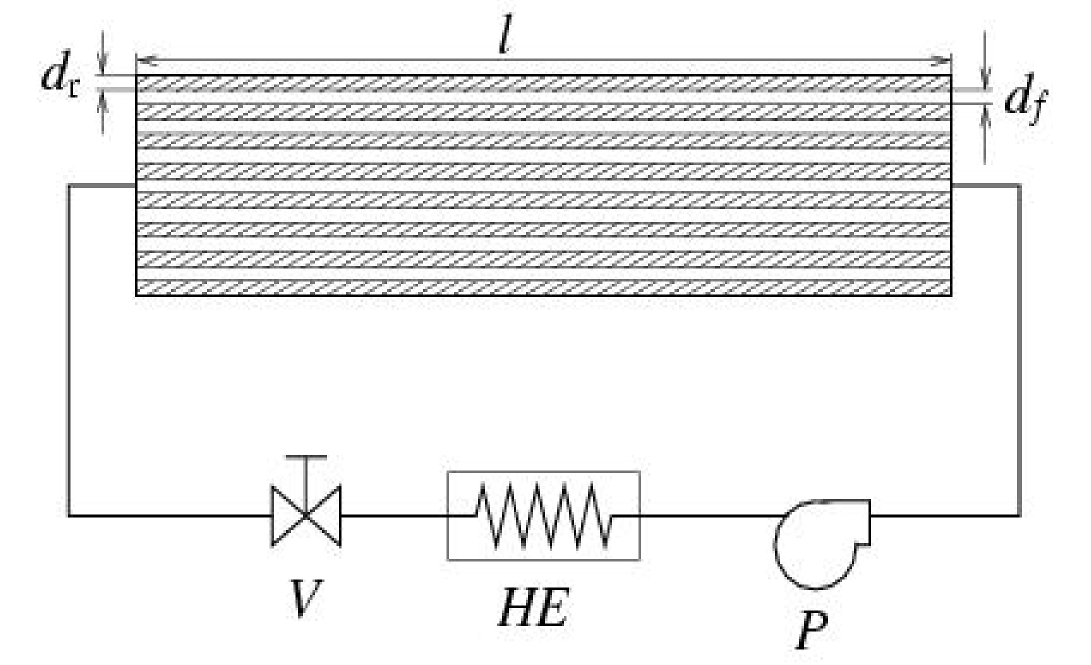
$v \sim 10^1$ m/s, $d \sim 10$ cm, $\tau_{fc} = d/v \sim 10^{-2}$ s

a model refrigerator



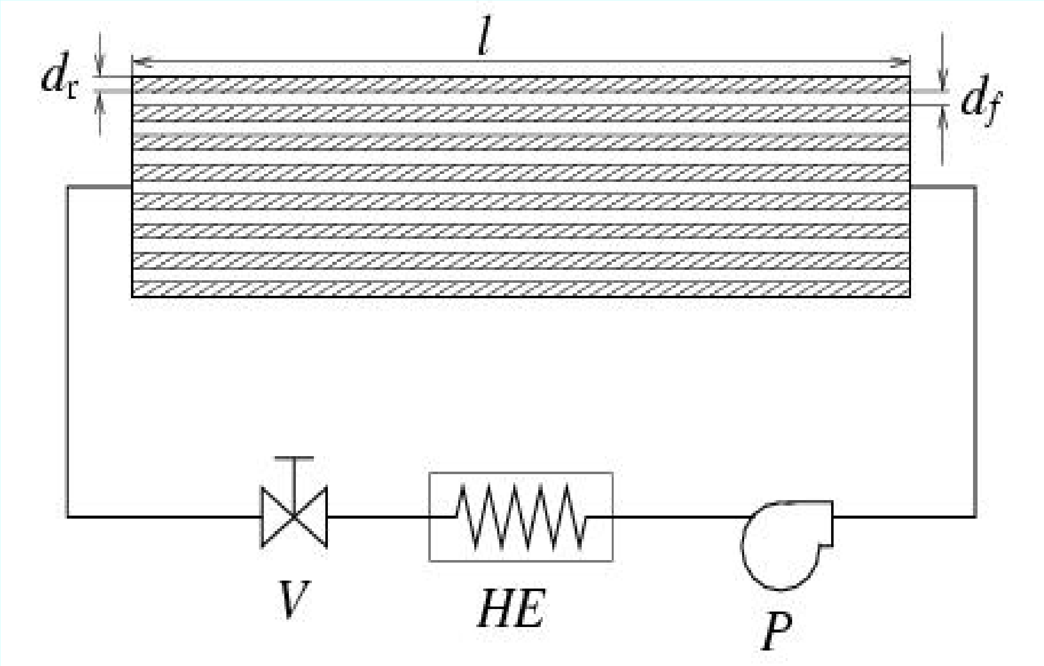
$$\tau_{\mathrm{cycle}} = \cancel{\tau_{\mathrm{magnrelax}}} + \tau_{\mathrm{tc}} + \tau_{\mathrm{fc}}$$

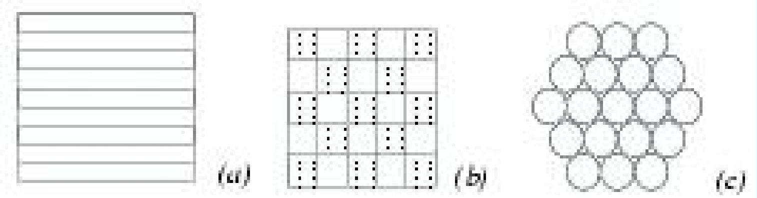
$$\tau_{\mathrm{magnrelax}} \lesssim 1\ \mathrm{ms}$$



$$\tau_{\text{cycle}} = \tau_{\text{tc}} + \tau_{\text{fc}}$$

$$\tau_{\text{tc}} = \frac{\rho_r C_r}{\kappa_r} d_r^2$$





$$\tau_{fc} = \tau_{bed} + \tau_{outer}$$

$$\tau_{bed} \sim 32 \frac{\eta l^2}{\Delta P} \frac{1}{d_f^2} \qquad \text{(Hagen-Poiseuille)}$$

why not reduce τ_{bed} by raising ΔP?

ΔP = energy dissipation per 1m^3 of fluid

$\rho_f C_f \Delta T$ = useful heat transfer

$$\frac{\Delta P_{\max}}{\rho_f C_f \Delta T} = q_v \qquad \text{(viscous loss quotient)}$$

$$\text{hence} \qquad \tau_{\text{bed}} = \frac{32\eta l^2}{q_v \rho_f C_f \Delta T} \; \frac{1}{d_f^2}$$

optimally $\tau_{\text{outer}} \approx \tau_{\text{bed}}$ and therefore

$$\tau_{\text{fc}} = \tau_{\text{bed}} + \tau_{\text{outer}} \approx 2\tau_{\text{bed}} \approx \frac{64\eta l^2}{q_v \rho_f C_f \Delta T} \frac{1}{d_f^2}$$

what prevents us from making τ_{outer} much smaller?

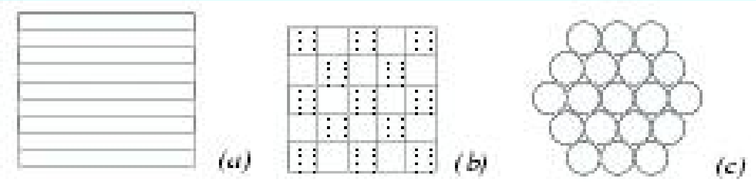
$$q_s = \frac{v_{\text{outer}}^2}{4C_f \Delta T} \qquad \text{(switching loss quotient)}$$

$v_{\text{outer}} \approx 10 v_{\text{bed}} \approx 80$ m/s leads to $q_s \approx 0.16$

thus

$$\tau_{\text{cycle}} = \tau_{\text{tc}} + \tau_{\text{fc}} = \frac{\rho_r C_r}{\kappa_r} d_r^2 + \frac{64\eta l^2}{q_v \rho_f C_f \Delta T} \frac{1}{d_f^2}$$

demanding a possibly low porosity for the bed, we set $d_f \approx ½ d_r$



now using this condition we eliminate d_f from τ_{cycle} above, then minimise τ_{cycle} with respect to d_r

so we get

$$d_{r,opt} \sim 7\left(\frac{\eta\kappa_r l^2}{\rho_r C_r \rho_f C_f \Delta T}\right)^{1/4}$$

$$f_{\max} = \tau_{\min}^{-1} \sim \frac{0.01}{l}\cancel{\sqrt{\frac{\rho_f C_f}{\rho_r C_r}}}\sqrt{\frac{\kappa_r \Delta T}{\eta}}$$

$\ell \approx 1$ cm (length of the bed)

r = Gd:

$\rho_r \approx 7.9\times10^3$ kg/m^3
$C_r \approx 2.4\times10^2$ K/kg K
$\kappa_r \approx 10.5$ W/m K
$\Delta T \approx 2.4$ K (B =1T)

f = water:

$\rho_f \approx 1\times10^3$ kg/m^3
$C_f \approx 4.2\times10^3$ J/kg K
$\eta \approx 1\times10^{-3}$ Pa s

Conclusion 1:

$\Delta P_{max} \sim 10^6$ Pa, or 10 bars

this is not too much.
cf. $P \approx 4$ bars in a bicycle tyre
$P \approx 15$ bars in a domestic espresso machine

Conclusion 2:

$d_{r,\mathrm{opt}} \sim 0.1$ mm

most workers in the field tend to agree on this

Conclusion 3:

$f_{max} \sim 2\times10^2$ Hz

$f \sim$ tens of Hz is a real possibility and a must
$f \sim$ hundreds of Hz is unrealistic

Conclusion 3':

$$\tau_{\min} \sim 100\, l \sqrt{\frac{\eta}{\kappa_r \Delta T}} \sim 5\ \mathrm{ms}$$

must be $\tau_{\mathrm{magnrelax}} \lesssim 1\ \mathrm{ms}$

slower refrigerants may be struck off the list of favorites