

Local nonfactorization of functions on locally compact groups

By

H. G. FEICHTINGER and W. SCHACHERMAYER

0. Introduction. The meanwhile well-known Cohen-Hewitt factorization theorem ([13], §32) states that for any locally compact group G and $1 \leq p < \infty$ one has $L^1(G) * L^p(G) = L^p(G)$, i.e. any $g \in L^p$ can be factorized into a product, $g = f * g_1$, with $f \in L^1(G)$ and $g_1 \in L^p(G)$. For discrete groups this result is trivial, because in that case the Dirac measure belongs to $L^1(G)$. For non-discrete groups, however, it is a natural question to ask whether there exists a single factor or a suitable set W in $L^1(G)$ such that the first factor may be chosen in W . For a single factor this comes down to the question, whether a convolution operator $T_f: g \mapsto f * g$ on $L^p(G)$ may happen to be surjective. This question has been already answered in the negative by J. Dieudonné ([4], Corollary 1) for arbitrary non-discrete groups and reproved by other authors, citing only the abelian versions of this result ([1], [14]). In the first part of this note we shall improve on this result (going back to the arguments of Dieudonné and to related compactness arguments), showing that, for example, a weakly compact subset $W \subseteq L^1(G)$ (in particular, a (convergent) sequence in $L^1(G)$) does not suffice in order to obtain $L^p(G)$ through factorization, not even locally. The clearest evidence concerning the actual situation, however, comes from another, much stronger result (and some of its variants) showing among others that $f * L^1(G)$ (and related sets) does not even contain $\mathcal{K}(G)$, the space of continuous functions with compact support.

1. Notations and Preliminaries. Throughout G denotes a non-discrete locally compact (l.c.) group. For $1 \leq p \leq \infty$ ($L^p, \| \cdot \|_p$) denotes the usual Lebesgue space with respect to a fixed left invariant Haar measure dx . For a measurable subset A of G its measure is denoted by $|A|$. Writing L_y for the left translation operator given by $L_y f(x) := f(y^{-1}x)$ we note that $y \mapsto L_y f$ is a bounded, continuous function from G to $L^p(G)$ for any $f \in L^p(G)$, if $1 \leq p < \infty$. This is easily verified using the density of the space $\mathcal{K}(G)$ of continuous, complex-valued functions on G with compact support. We write $C^b(G)$ for the space of continuous, bounded functions on G , and $C^0(G)$ for the subspace of functions vanishing at infinity (which coincides with the closure of $\mathcal{K}(G)$ in $L^\infty(G)$).

Let us recall that a subset $M \subseteq L^p(G)$ is called *equicontinuous* (in $L^p(G)$) if for $\varepsilon > 0$ there exists a neighborhood U of $e \in G$ such that $\|L_y f - f\|_p < \varepsilon$ for all $y \in U, f \in M$. Finally let us mention that a bounded set $M \subseteq L^p(G)$ is known to be relatively (norm-)compact in $L^p(G)$ if and only if it is equicontinuous and tight, i.e. for $\varepsilon > 0$ there exists some compact set $\mathcal{K}' \subseteq G$ such that $\|f - f\chi_{\mathcal{K}'}\|_p < \varepsilon$ for all $f \in M$, where $\chi_{\mathcal{K}'}$ stands for the

indicator function of K (cf. Weil [18], p. 52, [6], [8] or [9] for a general version of this criterion).

It will simplify the presentation to use the following notations: $B_p(\alpha) := \{f \mid f \in L^p(G), \|f\|_p \leq \alpha\}$, and $B_M(\alpha)$ for the α -ball in $M(G)$, and B_p, B_M for $B_p(1)$ and $B_M(1)$ respectively. Whenever it is convenient $L^1(G)$ is considered as a subspace of $M(G)$, the dual of $(C^0(G), \|\cdot\|_\infty)$.

2. Weak local nonfactorization results. Our first nonfactorization result will be a consequence of the compactness criterion. For its proof the following variant will be useful:

Lemma 1. *For any bounded equicontinuous subset $W \subseteq L^1(G)$ and $h \in L^\infty(G)$ with compact support $h(W * B_p)$ is relatively compact in $L^p(G)$ for any $p \in [1, \infty)$. Even $h(W * B_M)$ is relatively compact in $L^1(G)$.*

Proof. In order to check equicontinuity of $W * B_p$, observe that $\|L_y(f * g) - f * g\|_p \leq \|L_y f - f\|_1 \|g\|_p \leq \|L_y f - f\|_1 \rightarrow 0$, uniformly for $f \in W$. Since evidently equicontinuity in L^p is preserved under multiplication with any bounded uniformly left-continuous function it follows that $h_1(W * B_p)$ is relatively norm-compact in L^p for any $h_1 \in \mathcal{X}(G)$. Choosing h_1 in such a way that $h = h h_1$ it then follows that $h(W * B_p)$ is relatively compact (as the continuous image of a rel. compact set). The additional assertion follows from the well known fact that $L^1(G)$ is a closed ideal in $M(G)$.

This lemma leads us directly to our first result:

Theorem 2. *Let G be a non-discrete l.c. group. Given any equicontinuous subset $W \subseteq L^1(G)$ and any compact set K with positive measure (i.e. $|K| > 0$). Then there exists for any $p \in [1, \infty)$ some $f \in L^p(G)$ such that $f \chi_K \neq \chi_K(w * g)$ for all $g \in L^p(G)$ and $w \in W$.*

Proof. As it is possible to replace W by $\{w/(1 + \|w\|), w \in W\}$, we may suppose (after renormalization) that W is also bounded. Assuming equality $\chi_K L^p(G) = \chi_K(W * L^p)$ it follows from Baire's theorem that there exists $C > 0$ such that $\chi_K(W * B_p(C))^-$ contains some ball, hence $\chi_K(W * B_p(C_1))^- \supseteq \chi_K B_p$ for some $C_1 > 0$. The resulting compactness of $\chi_K B_p$ is of course absurd, G being non-discrete (implying that $\chi_K L^p$ is not finite dimensional).

Corollary 3. *Given a sequence $(f_n)_{n=1}^\infty$ in $L^1(G)$ the inclusion $\bigcup_{n=1}^\infty f_n * L^p(G) \subseteq L^p(G)$ is a proper one, even locally.*

Proof. Without loss of generality one may assume $\|f_n\|_1 \rightarrow 0$.

Remark 1. The above arguments extend to show that $T(L^p(G))$ is (even locally) properly contained in $L^p(G)$ whenever T is a bounded operator on $L^p(G)$ which can be approximated in the operator norm by elementary operators of the form $T_f: g \mapsto f * g$, with $f \in L^1(G)$ (or $\mathcal{X}(G)$).

Indeed, let $p \in [1, \infty)$ and a sequence $(f_n)_{n=1}^\infty$ in $\mathcal{X}(G)$ or $L^1(G)$ be given such that $\|T_{f_n} - T\|_{0,p} < 1/n$. By Lemma 1 $h(f_n * B_p)$ is relatively compact in $L^p(G)$. As

$$h(T(B_p)) \subseteq h(T_{f_n}(B_p) + n^{-1} B_p) \subseteq h(f_n * B_p) + \|h\|_\infty n^{-1} B_p$$

we conclude that $h(T(B_p))$ is relatively compact in $L^p(G)$.

The referee has kindly pointed out another way to verify the above assertion, using the Cohen-Hewitt factorization theorem: The closure of $L^1(G)$ in the space of operators on $L^p(G)$ is an essential $L^1(G)$ -module, consequently given T in this space there exists $f \in L^1(G)$ and T_1 in this space, such that $T = f * T_1$. It follows $T(L^p) = f * T_1(L^p) \subseteq f * L^p$ and we come back to Corollary 3.

Remark 2. The arguments of Theorem 2 apply to much more general spaces (other than the L^p -spaces), for which the compactness criterion is available (cf. [9] for details).

The following kind of results is obviously influenced by the work of Dieudonné [4]:

Theorem 4. a) Let W, B be two $\sigma(M(G), C^0(G))$ -compact subsets of $M(G)$. If W or B is tight in $M(G)$ then $W * B$ is $\sigma(M(G), C^0(G))$ -compact.

b) If, in addition $W * B$ is contained in $L^1(G)$, then $W * B$ is $\sigma(L^1(G), C^0(G))$ -compact in $L^1(G)$. In particular, $W * B$ does not contain any L^1 -ball (not even locally), for non-discrete G .

Proof. a) We formulate our arguments for the case that W is tight. Let $(f_\alpha)_{\alpha \in I}$ resp. $(g_\beta)_{\beta \in J}$ are w^* -convergent nets in W and B with limits f_0 and g_0 respectively. We have to show that the net $f_\alpha * g_\beta$ (with the natural ordering of the index set) is $\sigma(M(G), C^0(G))$ -convergent to $f_0 * g_0 \in W * B$.

In order to show this let us recall that the mapping $f \rightarrow f^*$, given by $f^*(x) := \bar{f}(x^{-1}) \Delta^{-1}(x)$, defines an isometric antiautomorphism of the Banach convolution algebra $L^1(G)$ (extending also to $M(G)$, cf. [17], Chap. 3, § 5), preserving w^* -convergence and satisfying $\langle f^* g, h \rangle = \langle g, f^* * h \rangle$, for $f, g \in M(G)$ and $h \in C^0(G)$. Using this we obtain for $h \in C^0(G)$:

$$\begin{aligned} |\langle f_\alpha * g_\beta - f_0 * g_0, h \rangle| &= |\langle (f_\alpha - f_0) * g_\beta, h \rangle + \langle f_0 * (g_\beta - g_0), h \rangle| \\ &\leq |\langle g_\beta, (f_\alpha - f_0)^* * h \rangle| + |\langle g_\beta - g_0, f_0^* * h \rangle|. \end{aligned}$$

The second term goes to zero by the $\sigma(M(G), C^0(G))$ convergence of the net $(g_\beta)_{\beta \in J}$. That the first one tends to zero is a consequence of the boundedness of $(g_\beta)_{\beta \in J}$ and the norm convergence in C^0 of $(f_\alpha - f_0)^* * h$ (it follows from Theorem 2.2 in [7]).

b) Assume that $W * B$ is also a subset of $L^1(G)$, which is $\sigma(L^1(G), C^0)$ -compact by the above argument. If there exists some compact set K in G with $K > 0$, such that $W * B|_K$ contains the set of restrictions of some L^1 -ball to K , we may assume for simplicity that $B_1(1)|_K \subseteq W * B|_K$. It is clear that for any neighborhood U of $e \in G$ there exists $x_U \in G$ such that $|x_U U \cap K| > 0$. Letting U run through the system of symmetric neighborhoods contained in a given compact, symmetric neighborhood U_0 it follows from the compactness of $U_0 K$ that there exists a convergent subnet (x_ν) , with limit x_0 . Let (f_ν) be the corresponding family of normalized characteristic functions of the sets $x_\nu V \cap K$. Then it is evident that it is convergent to δ_{x_0} , which apparently cannot be represented by anything in $W * B \subseteq L^1(G)$.

Remark 3. Note that the tightness-assumption in a) is essential. Indeed, let (h_n) be any tight, bounded sequence in $L^1(G)$, vaguely convergent to the Dirac measure δ_0 , on any non-discrete, non-compact abelian locally compact group G . For any sequence (y_n) in G

without cluster point (i.e. tending to infinity) set $f_n := L_{y_n} h_n$ and $g_n := L_{-y_n} h_n$. Then both sequences tend to zero in the $\sigma(L^1, C^0)$ -topology. Taking them as sets (including their limit zero) W and B , they are thus both $\sigma(L^1, C^0)$ -compact. However, their products $f_n * g_n = h_n * h_n$ tend to δ_0 , which is not an element of $W * B \subseteq L^1(G)$ by the assumption.

Using a combination of the arguments for the proof of Theorem 2 and above allows to show:

Theorem 5. *Let W be a $\sigma(L^1, C^b)$ -compact (e.g. a weakly compact) subset of $L^1(G)$, G non-discrete. Then $\chi_K(W * M(G))$ is properly contained in $\chi_K L^1(G)$ for each compact set K with $|K| > 0$.*

Proof. We only have to show that we are actually in the situation of Theorem 4b. However, since any locally compact group is paracompact (cf. [15], p. 172) we may invoke the converse to Prohorov's theorem, saying that a $\sigma(M(X), C^b(X))$ relatively compact set over a paracompact space has to be tight (cf. [2], Theorem 2.6, see also [12], p. 124).

As preparation for another approach to local nonfactorization let us introduce the notion of *uniform integrability* in $L^p(G)$ (shortly u.i.). A subset $W \subseteq L^p(G)$ is said to have this property if for $\epsilon > 0$ there exists $C > 0$ such that $\int_{|f|>C} |f(x)|^p dx < \epsilon^p$ for all $f \in W$.

Lemma 6. *Let G be a unimodular group. Given $1 \leq p < \infty$ and a bounded, u.i. set $W \subseteq L^1(G)$ and $S \subseteq L^p(G)$, bounded. Then $W * S$ is u.i. in $L^p(G)$.*

Proof. Without loss of generality we may assume $S = B_p$. By assumption there exists $C > 0$ with $W \subseteq B_1(C)$ and for $\epsilon > 0$ there exists $C_\epsilon > 0$ such that $W \subseteq [B_1(C) \cap B_\infty(C_\epsilon)] + B_1(\epsilon)$. By the Riesz convexity theorem one has $B_1(C) \cap B_\infty(C_\epsilon) \subseteq B_1(C) \cap B_{p'}(C'_\epsilon)$ for some $C'_\epsilon > 0$. Since by the unimodularity of G one has $L^p(G) * L^{p'}(G) \subseteq C^0(G)$ (with $1/p + 1/p' = 1$, together with the corresponding norm inequality) it follows $W * S \subseteq [B_p(C) \cap B_\infty(C'_\epsilon)] + B_p(\epsilon)$, and the proof is complete.

Arguing once more as in the proof of Theorem 2 one obtains:

Corollary 7. *Let G be unimodular, non-discrete group. Let W be a uniformly integrable, bounded subset of $L^1(G)$. Then $W * L^p(G)$ is strictly contained in $L^p(G)$, even locally.*

Remark 4. Observe that a weakly compact subset $W \subseteq L^1(G)$ satisfies the assumptions of this result by the theorem of Dunford-Pettis (cf. [5], [11], §8.9).

Remark 5. The above arguments also suffice in order to show, for example, that for any given compact set K with positive measure, there does not exist $f \in L^1 + L^{p'}$ such that $f * L^p|_K \supseteq L^p|_K$.

3. Strong local non-factorization. In the case of an abelian group the following decisive improvement of Theorem 2 can be easily obtained (writing $\mathcal{F}L^1 = \{\hat{f} | f \in L^1(\hat{G})\}$ for the Fourier algebra on $G = (\hat{G})^\wedge$).

Theorem 8. *Let G be a nondiscrete, locally compact abelian group. Then for any nonvoid open set $Q \subseteq G$ and any equicontinuous set $W \subseteq L^1(G)$ there exists $h \in \mathcal{F}L^1$ (the Fourier algebra on G) such that for any $g \in W^*M(G)$ there exists a subset $V \subseteq Q$ of positive measure such that $h(x) \neq g(x)$ for all $x \in V$.*

Proof. Again we may assume without loss of generality that W is a bounded subset of L^1 . Given Q we choose another nonvoid, open set Q_1 with compact closure such that $Q_1 \subseteq \bar{Q}_1 \subseteq Q$. The regularity of $\mathcal{F}L^1$ (cf. [17], Chap. 6, § 1.3) implies that there exists $p \in \mathcal{F}L^1$ such that $p(\gamma) = 1$ for $\gamma \in \bar{Q}_1$ and $\text{supp } p \subseteq Q$.

Assuming now the converse of Theorem 8 to be true, i.e. that for any $h \in \mathcal{F}L^1$ there exists $g \in W^*M(G)$ such that $h(x) = g(x)$ a.e. on Q , it follows that we have for any h in the closed subspace $\mathcal{F}L^1_{\bar{Q}_1} := \{f, f \in \mathcal{F}L^1, \text{supp } f \subseteq \bar{Q}_1\}$ the identity $h(x) = h(x)p(x) = h(x)g(x)$ a.e. on G , and thus

$$\mathcal{F}L^1_{\bar{Q}_1} = \bigcup_{k=1}^{\infty} (p(W^*k B_M) \cap \mathcal{F}L^1_{\bar{Q}_1}).$$

Applying Baire's theorem it follows that there exists $k_0 > 0$ such that the unit ball of the Banach space $\mathcal{F}L^1_{\bar{Q}_1}$ is contained in the closure of $(p(W^*k_0 B_M) \cap \mathcal{F}L^1_{\bar{Q}_1})$ in $\mathcal{F}L^1_{\bar{Q}_1}$. Since $L^1_{\bar{Q}_1}$ is continuously embedded in L^1 and the L^1 -closure of $p(W^*k_0 B_M)$ is equicontinuous with respect to the L^1 -norm, it follows that the unit ball of $\mathcal{F}L^1_{\bar{Q}_1}$ is equicontinuous in the L^1 -norm, which is absurd, because the set $\{\chi f, \chi \in \hat{G}\}$ is not L^1 -equicontinuous for any nonzero $f \in \mathcal{F}L^1_{\bar{Q}_1}$ (cf. [9], Lemma 2.5).

Remark 6. In the above result it is even possible to replace the space $M(G)$ by the much larger space $T(G)$ of translation bounded measures on G (cf. [7]). In fact, since $T(G)$ coincides locally with $M(G)$ and $L^1 * T(G) \subseteq T(G)$ it follows again that $\mu \mapsto p(f * \mu)$ is a compact operator from $T(G)$ into $L^1(G)$. All other arguments remain unchanged. Since $T(G) \cong \bigcup_{1 \leq p \leq \infty} L^p(G)$ we have thus shown altogether:

Corollary 9. *Let G be a non-discrete, l.c.a. group. There does not exist a sequence $(f_n)_{n=1}^{\infty}$ in $L^1(G)$ such that*

$$\mathcal{F}L^1 \cap \mathcal{K}(G) \subseteq \bigcup_{n=1}^{\infty} \bigcup_{p \geq 1} f_n * L^p(G).$$

Remark 7. Theorem 8 may also be considered as a consequence of the non-compactness of the action of the dual group \hat{G} (under multiplication) on $\mathcal{F}L^1|_{Q_1}$ (cf. Olesen, [16]).

Remark 8. In contrast to the above remark it is possible to find on a non-compact (locally compact) abelian group G for any compact set $K \subseteq G$ some $f \in L^1_{loc}(G)$ such that $f * L^1|_K = L^1|_K$ (this will be explained in detail in a separate note).

In the case of a general non abelian group we do not have the characters $\chi \in \hat{G}$ at our disposition. However, we may use purely measure theoretic arguments in order to show that not all continuous functions can be factored over one out of a sequence of L^1 -factors, not even locally.

Theorem 10. *Let G be a non-discrete locally compact group, $f \in L^1(G)$ and $A \subseteq G$ a Borel set with positive measure. Then*

$$\{\chi_A f \mid f \in \mathcal{K}(G)\} \not\subseteq \chi_A(f * L^1).$$

*In particular, $\mathcal{K}(G)$ is not a subspace of $f * L^1(G)$, for any $f \in L^1(G)$.*

The main ingredient of our proof being of independent interest let us state it separately:

Lemma 11. *Let Y be a completely regular topological space (e.g. locally compact), and μ a positive Radon measure on Y (i.e. a Borel measure which is inner-regular for compact sets), satisfying $\mu(\{y\}) = 0$ for all $y \in Y$. Then one has: for any Borel set $A \subseteq Y$ with $0 < \mu(A) < \infty$ the set*

$$\{\chi_A h \mid h \in C^b(Y), \|h\|_\infty \leq 1\}$$

is not relatively compact in $(L^1(\mu), \|\cdot\|_1)$.

Proof. Given $\alpha := \mu(A) > 0$ we shall prove first that for $\varepsilon \in (0, \alpha)$ there exists a Rademacher-like bounded sequence $(r_n)_{n=1}^m$ in $C^b(Y)$ satisfying

$$\|\chi_A(r_n - r_m)\|_1 > (\alpha - \varepsilon)/2 \quad \text{for } n \neq m.$$

For this purpose let us construct inductively a tree of compact sets of A : $K_{(n,j)}$, $n \in \mathbb{N}$, $1 \leq j \leq 2^{n-1}$.

Starting with $A_{1,1} := A$ we choose a compact set $K_{1,1} \subseteq A_{1,1}$ with $\mu(K_{1,1}) > (\alpha - \varepsilon)$. Having found a tree $\{K_{n,j} \mid 1 \leq j \leq 2^{n-1}\}$ with

$$\mu(K_{n,j}) > (\alpha - \varepsilon) / 2^{n-1},$$

$$K_{(n,j)} \cap K_{(n,l)} = \emptyset \quad \text{for } j \neq l$$

and

$$K_{n-1,j} \supseteq K_{n,2j-1} \cup K_{n,2j} \quad \text{for } n \geq 2$$

we can find for $1 \leq j \leq 2^{N-1}$ a decomposition of $K_{N,j}$ into two disjoint Borel subsets, i.e. $K_{N,j} = A_{N+1,2j-1} \cup A_{N+1,2j}$ with $\mu(A_{N+1,2j-1}) = \mu(A_{N+1,2j}) = \mu(K_{N,j}) / 2 > (\alpha - \varepsilon) / 2^N$ (here it is used that μ is purely non-atomic, cf. [10], p. 63, Exercise). Using the inner-regularity of μ we may replace $A_{N+1,l}$ by a compact subset $K_{N+1,l}$ such that still $\mu(K_{N+1,l}) > (\alpha - \varepsilon) / 2^N$ for $1 \leq l \leq 2^N$. This completes the induction. For $n \in \mathbb{N}$ it is now clear (as a consequence of Tietze's theorem) that there exists $r_n \in C^b(Y)$ with $-1 \leq r_n(y) \leq 1$ for $y \in Y$ and $r_n(x) = (-1)^j$ on $K_{(n,j)}$. It is obvious that the required L^1 -estimate holds true.

Proof of Theorem 10. Since the Haar measure of a non-discrete group is of course non-atomic the result follows, combining Lemma 1 with Lemma 11.

References

- [1] J. T. BURNHAM and R. R. GOLDBERG, The convolution theorem of Dieudonné. Acta Sci. Math. **36**, 1-3 (1974).
- [2] J. B. CONWAY, The strict topology and compactness in the space of measures II. Trans. Amer. Math. Soc. **126**, 474-486 (1967).

- [3] J. DIESTEL and J. J. UHL, Vector measures. Math. Surveys **15**, Amer. Math. Soc., 1977.
- [4] J. DIEUDONNÉ, Sur le produit de convolution II. I. Math. Pure Appl. (3) **39**, 275–292 (1960).
- [5] N. DUNFORD and J. T. SCHWARTZ, Linear operators. New York 1958.
- [6] R. E. EDWARDS, Functional Analysis. Theory and Applications. New York 1975.
- [7] H. G. FEICHTINGER, Multipliers from $L^1(G)$ to a homogeneous Banach space. J. Math. Anal. Appl. **61**, 341–356 (1977).
- [8] H. G. FEICHTINGER, A compactness criterion for translation invariant Banach spaces of functions. Anal. Math. **8**, 165–172 (1982).
- [9] H. G. FEICHTINGER, Compactness in translation invariant Banach spaces of distributions and compact multipliers. J. Math. Anal. Appl. **102**, 289–327 (1984).
- [10] K. FLORET, Maß- und Integrationstheorie: Eine Einführung. Stuttgart 1981.
- [11] K. FLORET, Weakly compact sets. LNM **801**, Berlin-Heidelberg-New York.
- [12] D. H. FREMLIN, D. J. H. GARLING and R. G. HAYDON, Bounded measures on topological spaces. Proc. London Math. Soc. **25**, 115–136 (1972).
- [13] E. HEWITT and K. A. ROSS, Abstract Harmonic Analysis II. Grundle. Math. Wiss., Berlin-Heidelberg-New York 1970.
- [14] J. JANAS, Note on a theorem of Dieudonné. Acta Sci. Math. **47**, 461–464 (1984).
- [15] J. L. KELLEY, General Topology. Graduate Texts **27** (reprint of the 1955 ed., publ. by Van Nostrand), Berlin-Heidelberg-New York 1975.
- [16] D. OLESEN, On norm continuity and compactness of spectrum. Math. Scand. **35**, 223–236 (1974).
- [17] H. REITER, Classical Harmonic Analysis and Locally Compact Groups. Oxford Univ. Press 1968.
- [18] A. WIEHL, L'integration dans les groupes topologiques et ses applications. Act. Sci. et Ind., Nos. 869, 1145, Paris, 1940, 1951.

Eingegangen am 8. 10. 1985*)

Anschrift der Autoren:

Hans G. Feichtinger
 Institut für Mathematik
 Universität Wien,
 Strudlhofgasse 4
 A-1090 Wien

Walter Schachermayer
 Institut für Mathematik
 Johannes Kepler Universität
 Auhof
 A-4040 Linz

*) Eine Neufassung ging am 9. 6. 1986 und die Endfassung am 20. 11. 1986 ein.