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A Physical Approach to the Construction of Cognition and to Cognitive Evolution

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Abstract:

It is shown that the method of operational definition of theoretical terms applied in physics may well support constructivist ideas in cognitive sciences when extended to observational terms. This leads to unexpected results for the notion of reality, induction and for the problem why mathematics is so successful in physics.

A theory of cognitive operators is proposed which are implemented somewhere in our brain and which transform certain states of our sensory apparatus into what we call perceptions in the same sense as measurement devices transform the interaction with the object into measurement results. Then, perceived regularities, as well as the laws of nature we would derive from them can be seen as invariants of the cognitive operators concerned and are by this human specific constructs rather than ontologically independent elements. (e.g., the law of energy conservation can be derived from the homogeneity of time and by this depends on our mental time metric generator). So, reality in so far it is represented by the laws of nature has no longer an independent ontological status. This is opposed to Campbell's 'natural selection epistemology'. From this it is shown that there holds an incompleteness theorem for physical laws similar to Gödels incompleteness theorem for mathematical axioms, i.e., there is no definitive or object 'theory of everything'.

This constructivist approaches to cognition will allow a coherent and consistent model of both cognitive and organic evolution. Whereas the classical view sees the two evolution rather dichotomously (for ex.: most scientists see cognitive evolution converging towards a definitive world picture, whereas organic evolution obviously has no specific focus (the *'pride of creation')*.

Keywords:

Cognitive operator theory; epistemological autoreproduction; human specific character of natural laws; incompleteness theorem of mathematics (Gödel) and physics; realism.

1. *Introduction*

The success story of human cognition is fascinating. We look around and analyse what we see, we make experiments to see and to analyse more, and all this we condense step by step to mathematically formulated sciences and laws of nature from which, in turn, we derive all the techniques which enable us to master nature to the amazing actual extent. But we have difficulties to explain why these methods are so successful. Davies (1990a) speaks of the algorithmic compressibility of the world saying that the world despite all its vast complexity can be described by relatively modest mathematical means, or, in other words, asking why induction is so successful. Similar is expressed by Wigner (1960) when speaking of the unreasonable effectiveness of mathematics in the natural sciences.

We will approach the problem of the obviously good relationship between physics and mathematics (or between observations and the interpretations we apply) by the assumption that both areas are based on phylogenetically acquired mental mechanisms which due to their possibly common root may bring about structural similarities. These similarities may enable mathematics to act as a model for what we 'see'.

When looking for such mechanisms we may refer to physics. In the first decades of the last century physicist had to accept that some of the well proven notions in classical physics can no longer be applied when subatomic dimensions or very high speeds are involved. In order to avoid future similar occurrences and to keep our notional reference frame in coherence with these or other novel experimental situations, physicists proposed to use as far as possible only those theoretical terms which can be defined operationally, i.e. by means of experimental devices rather than by means of protophysical notions such as those used in day to day life. This is realised by seeing observables as invariants of experimental measurement operators.

What we will propose here is to extend the demand for operational definitions of theoretical terms to observational as well as to mathematical terms. This is realised by considering both, the perceived regularities which we condense to the laws of nature and the mathematical structures which we recognize and then condense to axioms, as invariants of inborn cognitive and mental operators (theory of cognitive operators). A phylogenetic relationship (or homology) between the mental operators generating both the perceived and the mathematical regularities respectively may explain the high suitability of mathematical tools to extrapolate or complete observed data, i.e., it may explain the success of induction (par. 10). The extension of perceptional operators by means of experimental operators (i.e. by means of measurement devices or devices which assist perception such as microscopes or radio receivers) would lead to the completion of the classical world view if both the cognitive and the physical operators commute in the sense of operator algebra (quantitative extensions). Otherwise the physical operators will have invariants which no longer can be described in classical terms, and, therefore, would require the formation of novel, non-classical theories (qualitative extension), exceeding the classical world view. The mathematical analogon would be the algorithmic extension of elementary mathematical thinking exceeding the axiomatic basis previously established. These qualitative extensions, according to Gödel's incompleteness theorem, would require to modify the axiomatic basis (towards a 'non-classical' mathematics, so to speak). As a consequence there will be neither a definitive set of axioms in mathematics, nor will be there a definitive theory of everything in physics (Feynman 1965, Hawking 1979, Barrow, 1990). In other words, there is an incompleteness theorem which applies for both mathematical axioms and physical laws of nature.

If the laws of nature are defined as invariants of cognitive operators they are no longer objective but rather human specific (The law of energy conservation, e.g., which can be derived from the homogeneity of time, depends on the phylogenetically acquired time metric generator in our brain which defines what we perceive as homogenous in time). Let us define reality as a synonym for all what is independent from men and cannot be made, changed or influenced by men. According to usual thinking the only candidate to meet this requirement in full strictness are the laws of nature. But here the natural laws are human specific and this means that also reality can no longer claim to have an independent ontological status.

Cognitive operators are not the only ones which act upon the sensorial input. Acting operators such as limbs and other tools would as well modify our perceptions. They both are equivalent in so far they act upon what we see. That we nevertheless dichotomize action and perception is probably closely related to the mental formation of the notion of a real world: action, so our saying, refers to the individual's input to the world, whereas perception means the world's input to the individual. This we will discuss in more detail in par. 4 on 'theory of cognitive operators':

As a consequence, our cognitive tools and the theories they bring about can no longer be required to collect 'true' information from an objective outside world or to construct a 'true' world view. They rather have to predict correctly the effects of human actions and perceptions or, more generally, to improve acting competence. Then, however, our inborn epistemology can no longer be a 'natural selection epistemology' in the sense of Campbell (1973), and truth can no longer be used as an evaluation criterion for physical theories. So, the independent outside world may no longer serve as the great censor for our theories. But this is not necessary anyway because what we really need are theories which allow verifiable predictions - and this does not require information about the external world. If observables are defined as invariants of defining operators the effect of an acting operator will depend exclusively on how the defining and the acting operator are related to each other. In other words: what an action will bring about depends on how the object is defined, and not of the structure of the world. Of particular interest are those definitions where the observables concerned change linearly in time under the influence of the acting operator so that predictions can easily be realised. This is exactly analogue to the the Hamilton-Jacobi theory in classical mechanics from which we can learn how the physical reference frame of a given system has do be transformed in order to get variables with linear time dependency. It seems that cognitive evolution has chosen a similar way.

The functional equivalence between cognitive and acting operators may suggest that the most elementary observables have phylogenetically evolved as invariants of the most elementary acting operators. Indeed, locomotion, for example, brings about the category of spatial objects which, according to Piaget (1970), are mentally defined as invariants of motion. A consequence of this (which can be derived from the theory of classical mechanics) is the homogeneity of space and the conservation of motion or, what is se same, that force free bodies travel through space linearly in time. Similar applies to the other conservation laws in mechanics. Particularly the conservation of energy depends on the homogeneity of time. In other words: the world is simple and predictable if and only if the way we describe the world is closely related to the way we act upon the world. The reason why the world is predictable and seems to be simple is not that it has a simple structure and therefore will allow easy predictions.

The purpose of theories is not only to provide predictions for what will happen in nature or for what our acting will bring about. What we expect from scientific theories is to explain all known phenomena within the framework of one coherent world model. Such a model brought about by human brains has to explain everything from the big bang, the creation of our world, organic and then cognitive evolution and eventually up to the development of the scientific model itself. I.e., the cognitive phenotype has to reproduce itself in the same sense the organic phenotype has to do. A cognitive phenotype (or a world picture) which meets this requirement we will call consistent. Accordingly, further to the special cognitive phenotype we have acquired, there is an unlimited number of possible (and consistent) cognitive species similar to the many existing or possible organic phenotypes (i.e. species). Also biotic organisms are not required to be 'true' but rather to reproduce, i.e. (as biologists are used to say) to contribute to the survival of the genes concerned. As to the prediction competence of theories, it will be shown that the criteria of truth and consistency are equivalent. (Par. 10). (The term cognitive phenotype as used here means the totality of all what is cognitively relevant in an organism as opposed to the organic phenotype which refers to what is physically (or physiologically) relevant. Both phenotypes constitute together the general phenotype).

In the organic area we can modify our environment by means of inborn or technologically acquired tools (assimilation) with a view to modifying the selection pressure to be met by our adaptive efforts (Assimilation is a rather successful tool in all cases where we have no time to wait for evolution to solve our environmental problems by means of adaptation). Nothing equivalent seems to be in the cognitive area. The cognitive selection pressure is given by the laws of nature (‘a theory, if it wants to be ‘true’ (i.e., to be successful) has to reflect the laws of nature). According to classical thinking these laws are seen to be ontologically objective and unchanging in time. So, according to classical thinking, cognitive evolution, as opposed to organic evolution, would have no chance to modify its selection pressure. Within the theory of cognitive operators, however, as proposed here, laws of nature are subject to the requirements of technologically generated novel qualitative extensions and by this will modify the selection pressure for science. (Par.4)

From this it follows that neither organic evolution can be seen as a process of adaptation to an independent outside world nor is cognition a matter of depicting this world. Both evolutions are rather a process of self organising construction.

2. Elements of the classical world picture

Before we can hope to realise this program we have to show, that our existing notional reference frame, though it may well have evolved phylogenetically, nevertheless has nothing to do with adaptation to the structure of the world. Our inborn epistemology, therefore, - in contrast to conventional thinking - is not a natural selection epistemology in the sense of Campbell. For this we have to deal in more detail with the metatheoretical elements of classical physics.

2.1. Induction

The most enigmatic element of the human metatheory of physics is that unmade experiences could be derived and predicted from made experiences by means of induction. Thinking in terms of induction is the most elementary and the most frequently used strategy of organising our life. Be it in day–to–day life where we have to make our usual decisions on the basis of incomplete data or unconfirmed hypotheses, be it in science where we have to conceive theories on how to extrapolate empirical data, or be it in philosophy of science where we try to find a basis for teleology or determinism – inductive thinking dominates all we do, and it is probably the most successful of all the mental concepts men apply.

On the other hand, the obvious and uncontested success of induction is one of the greatest fascinations which philosophy of science was ever confronted with. Despite all philosophical efforts, we are more or less still in the same position as the one David Hume described 250 years ago: Universal laws can be justified only by induction which he took to be unjustifiable, although natural to us. A. F. Chalmers said (1982, p. 19) "Faced with the problem of induction and related problems, inductivists have run into one difficulty after another in their attempts to construe science as a set of statements that can be established as true or probably true in the light of given evidence. Each manoeuvre in their rearguard action has taken them further away from intuitive notions about that existing enterprise referred to as science. Their technical programme has led to interesting advances within probability theory, but it has not yielded new insights into the nature of science. Their programme has degenerated". Nearly the only progress achieved up to now is in clarifying and specifying the problem itself.

The key notion in this context is what Wigner (op. cit.) called "The unreasonable effectiveness of mathematics in the natural sciences" meaning that it is difficult to understand why so much of the complexity of the world can be described by so relatively simple mathematical formulae. Davies (1990a) has a similar idea in mind when following an idea of Solomonoff (1964) saying "All science is really an exercise in algorithmic compression. What one means by a successful scientific theory is a procedure for compressing a lot of empirical information about the world into a relatively compact algorithm, having a substantially smaller information content. The familiar practice of employing Occam's razor to decide between competing theories is then seen as an example of choosing the most algorithmically compact encoding of the data. Using this language, we may ask: Why is the universe algorithmically compressible? and why are the relevant algorithms so simple for us to discover?" Another version of the same question is (Davies 1990b) "... how can we know anything without knowing everything?", and, more generally: "Why is the universe knowable?"

As to the criticism of the notion of induction Popper (1982, p. 4) goes even one step further: Despite all practical success of inductive thinking, according to him natural science should dispense with induction at all as it cannot be justified. His argument is that a general principle of induction can be neither analytic nor synthetic. Were it analytic it could not contribute to the growth of knowledge and therefore would not be inductive at all. Were it synthetic it would have to be justified by another inductive principle of a higher order which would lead to an endless regression.

The problem, therefore, is as follows: on the one side we know that inductive thinking is very successful. On the other side we have no means to explain this. So, saying that the formation of inductive thinking must be the outcome of a selective process because it is so successful, has no explanatory value. Of the same quality is the argument that a recent species survived because it is fit if fitness is defined by nothing but by the resulting survival.

2.2. Reality

These and many other positions concerning induction have one thing in common: They arise from our intuitive conviction that there is something existing independently which we have to recognise without having any a priori idea what it could look like. In other words: all these positions arise from the claim to organise our life by means of the category of an independent reality to be described in terms of its structure. With Popper (1973), this is comprised in the term "growth of knowledge" to which induction has to contribute and which can be defined only in the context of some reality about which one might accumulate knowledge.

Davies is taking his stand even explicitly (1990a): "There exists a real external world which contains certain regularities. These regularities can be understood, at least in part, by a process of rational enquiry called scientific method. Science is not merely a game or charade. Its results capture, however imperfectly, some aspect of reality. Thus these regularities are *real* properties of the physical universe and not just human inventions or delusions. ... Unless one accepts that the regularities are in some sense objectively real, one might as well stop doing science."

The nearly generally agreed view that the problem of induction can and has to be solved only within the framework of an ontological reality is the most influential metaphysical element in all sciences.

This, however, becomes problematic when discussed within the so called **e**volutionary **e**pistemology (EE) (see Vollmer 1975). The EE was developed with a particular view to get a better understanding of human categories of perception and thinking, i.e. of our physical metatheory. The classical version (as I would like to call it) of EE declares these categories such as space, time, object, reality, causality etc. result from evolution in the same way as organic elements and features do. This, in classical parlance, would mean: in the same way as organic evolution is guided by adaptive forces, cognitive evolution is said to be the result of adaptation to the independent structures of an ontological reality. Campbell (1973) speaks in terms of a "natural–selection–epistemology". The general argument goes as follows: the theories we have designed to describe the structures of reality are surely incomplete or may have other strong deficiencies – reality itself, however, has been developed as a category of human thinking just because of the ontological character of outside reality. The fact that we think and act in terms of reality is taken as a proof that a sort of reality must exist. What is done here is to explain the formation of the category of reality by means of reference to its own content, i.e. to the existence of an ontological reality. Further to the fact that this would lead to circular inference, there is an even stronger objection: The existence of an ontological reality may, of course, have been a good reason for mental evolution to emulate it by a corresponding category of thinking. This argument, however, can not be reversed. It cannot be said that human mental phylogeny never would have brought about the category of reality if there were no such thing as an ontological reality, as long as other reasons can be found which are functionally conceivable and phylogenetically plausible though they do not refer to an ontological reality.

We will try here to show that such reasons can be given. The consequence, then, is that it is no longer an epistemological imperative to start from an independently predefined ontological reality which is said to determine in the long run both the strategies of mastering nature and the theories of analysing it – a reality which was also held responsible for all kinds of teleological thinking. Abandoning the notion of an ontological reality, however, will provoke some important questions: What does it mean to assert that the category of reality is a specificum of human thinking without any ontological quality of its own? What are the evaluation criteria for theories which are said to reflect the specificity of an independent reality? Can we renounce an ontological reality without bringing the well proven methods of empirical sciences into discredit? – leading questions which deterred most scientists from tangling with the ontological point of reality.

Most people when hearing that reality may not be really real would argue that ignoring the existence of tables, trees, traffic lights or what ever we find in our environment were sheerly unacceptable. Of course - but these are objects or facts which we can, at least in principle, alter or displace according to what we intend. Let us call this 'actuality'. By contrast, we will speak of 'reality' if something should be neither ignored nor can it be modified by what ever we may do. According to classical thinking this would apply in strict sense only for the laws of nature. So, disputing the ontological character of reality is reduced here to saying that there will be no definitive or objective laws of nature. It is evident that this view has no solipsistic consequences which people sometimes see when realism is disputed in general.

According to classical thinking objects we describe have independent properties which may change in time and according to interactions - except one property which we call identity and which is invariant under motion, i.e. identity is characterised by what we can follow with our eyes at its way through space and time. In quantum mechanics, however, there are cases where it is inappropriate to speak of independent properties. According to classical parlance properties of an object tell us what we can expect when dealing with the object concerned by means of measurement devices. From quantum mechanics we know, that measurement results cannot always be derived from objective properties. In a similar way the laws of nature are said to tell us what we can expect when dealing with nature in general. So, the laws of nature are the properties of nature, so to speak - and, similar to the properties of physical objects, they are not that objective as we expect them to be. They rather depend on our phylogenetically acquired cognitive phenotype which is neither definitive nor necessarily equal for all kind of beings. We mentioned already the example that the law of energy conservation depends on what kind of mental clock is implemented in our brain.

2.3. The separability of object and description

If we describe nature scientifically we usually do it by means of theories. As a language is first of all a descriptive tool we can consider also languages to be a theory (Diettrich, 1997). Theories and languages have in common that they aim at describing the world and the experiences made in the world in order to get helpful predictions or conclusions. The specificity of theories is based on the fact that they code certain laws of nature. The specificity of languages is based on the fact that they code our world-view by means of their syntax. (The space-time character, for example, of our world-view is coded syntactically by the forms of verbs and prepositions.) Also mathematics can be considered as theory in so far as it codes the constituting axioms. Coding something means incorporating it implicitly into the structure of the language or theory concerned so that no explicit reference is needed.

On the other hand theories and languages differ in so far as (scientific) theories can be checked empirically whether to be true or not. Even the change from the classical to the quantum mechanical metatheory was induced empirically. For languages, however, it is not usual to attribute to them truth-values. What is more: from a 'good' language it is expected that its syntax is free of any element derived from observations or experiences, i.e. from what has to be described by language itself. In other words, language is expected to be completely separable from its objects. Otherwise doubts will rise as to whether the verbal description of the world will be objective.

This is, more or less, the position of the analytical philosophy. It is said, that the solution of philosophical problems is mainly hampered by the lack of an 'objective' language, i.e. a language which is able to articulate human experiences but which itself is syntactically free of any experience or explicit ontological prerequisites. Otherwise, so the argument, we have to be prepared that such an ontologically 'polluted' language will prejudice the content of what is to be described and, therefore, can no longer be seen as an objective descriptional tool. The view gained ground that this can be avoided only if the rules of language are based exclusively on the 'objective' laws of logic and mathematics.

It was Hilbert who postulated the existence of a Platonistic 'objective' mathematics, i.e. a mathematics which mathematicians can discover but not invent. This led to the assumption that mathematics could be based on a complete and definitive set of axioms. We know, however, from Gödel that this is impossible. There is no closed system of axioms. We always have to be prepared to find cases where the axioms used have to be replaced by new and more appropriate ones. (see the review by E. Nagel, 1958). With this, mathematics and logic are epistemologically not concluded and never will be. I.e., mathematics is not Platonistic. These findings of Gödel have not only ruined definitively Hilbert's program for an objective mathematics but also the hope of analytical philosophers to realise their linguistic objectivity program by means of reference to logic and mathematics.

This does not exclude that a language, in so far it is based on our world-view, could well be objective - provided that the world-view can be traced back to objective laws of nature, i.e. to laws which, once they have been found, are no longer subject to future modifications in the sense mathematical axioms are. In the same way as everything we are interested in nature can be derived from the laws of nature, everything we are interested in mathematics can be derived from the axioms. This is the position of realism which refers to definitive (and therefore objective) laws of nature. It is equivalent to Platonism which refers to definitive (and therefore objective) axioms. As Platonism no longer holds, the only hope for languages to become objective is realism - and this exactly is what is comprised in the ordinary notion of language.

In day-to-day understanding, there is no reason to see language as a theory. It rather proceeds on the assumption that language is a universal and objective tool for the description of independently existing objects and processes, being able to convey any usual experience. Certainly, natural sciences sometimes require to extend ordinary language into mathematical areas, but this we do not see to conflict with the neutral character of language. Common sense understands that neither language nor mathematics would have any effect or influence on what they may describe. Mathematical methods, as we know, allow to extrapolate physical data and by this to predict new data, but this is not seen as an achievement of mathematics. We rather believe that it is the special physical structure of the world which would permit its inductive analysis. Experimental physical facilities and the results they produce represent another kind of language. They differ from ordinary written texts mainly by the fact that their decoding would require physical competence whereas the analysis of written communications needs language competence. On the other hand we know from physics that there are no absolutely interaction free relations between object and measurement devices, i.e. nature and the methods of its decoding cannot be completely separated from each other. So, strictly speaking, it should depend on the methods we apply what kind of statements we have to make on nature. Scientists try to avoid this difficulty by using only statements which they believe to be general enough not to depend any more on the experimental methods, i.e. on the "language" employed - or, in physical parlance: statements on nature should be invariant under the empirical methods applied. So, the knowledge of what we call the structure of nature is obtained through abstraction from the experimental techniques concerned - like the meaning of a message which could be defined as what is invariant under a change of language. According to naive understanding, language represents a generally unspecific capability independent from whether it is articulated in verbal or mathematical terms or in terms of experimental facilities. Nothing, we say, in the specificity of our life experiences is based upon the specificity of our descriptive tools. Language, within the limits of its competence, is seen to be objective and omnipotent. This is what expresses the naivety of the ordinary notion of language: to assume that content can be separated from representation.

From the functional point of view realism implies the idea that theories and the instances of their evaluation can strictly be separated from each other so that independent evaluation criteria can be found. This view is shared also by logicians when dealing with the notion of truth. In the same way as proximity to reality is seen as the criterion for the success of theories in natural sciences, truth is seen there as the criterion for the success of linguistic behaviour in its contribution to the overall behaviour. Accordingly the aim of natural sciences is seen to identify the (independent) structures of reality, and the aim of semantics is to identify universal conditions of truth.

A similar view on the universality of language (though not necessarily to what is existing, but with respect to what may be intended) is expressed by Searle (1971) in his "principle of expressibility" according to which everything that can be thought, can be said. This statement may be true because we base our thinking on the same syntax as our speaking. Thinking outside our syntax, therefore, is unthinkable. So, Searle's principle does not indicate so much the universality of language. It rather would denounce the weakness of our thinking being unable to comprise more than what can be said.

2.4. The conservation of identity

Another apriori (from classical physics too) is that identity is conserved in time. I.e. we do not consider cases where something will loose its identity and then will be 'reborn' at a later time. We rather would say that an object was invisible for a while, or that two equal (but not identical) objects have been involved . Identity cannot be interrupted without losing its character.

2.5 The 3-dimensional structure of visual perception

To see the world in 3 dimensions allows us to distinguish between the (visible) reduction of size due to physical compression and that one due to enlarged distance. But we cannot say that our space of visual perception is 3-dimensional because the world itself is 3-dimensional in character, and that apes which do not see the world in 3 dimensions were unable to jump from tree to tree, and, therefore, could not belong to our ancestors as Konrad Lorenz (1983) said. It is easy to show that appropriate and successful survival strategies could well be based on 2- or 4-dimensional perception spaces, independent from how many degrees of freedom are actually available.

With a 2-dimensional perception we would not know the phenomenon of perspective. Things are small or things are big, but they don not seem to be small because they are more distant and they would not seem to be big because they are nearer. Distance to the observer belongs to the third dimension which is excluded here. But objects, nevertheless, would shrunk in size if we use our legs to go backwards and they would enlarge their size if we go forward. So, with a 2-dimensional perception we would come to a world view according to which not only our hands and mechanical tools can modify objects but also our legs. With such a perception, an ape may well be able to jump from branch to branch. The only thing it has to learn is that it has to grasp the branch envisaged just when its size and position achieved certain typical values. If the perceived size of a branch will have doubled after three steps, the ape must know that it will arrive at it after another three steps and then has to grasp. If it has learnt to do so an external observer would find no difference between the moving strategies of such an ape and those based on a 3-dimensional perception. (It is evident that physical theories based on the inborn world view that objects could be 'deformed' not only by means of our hands but also by means of our walking or jumping legs would have no similarities with the theories we are used to use).

We can explain this by another example: Let us imagine locally fixed plants that have eyes and can see and which may have acquired a 2-dimensional perception. They would tell you that they have smaller and bigger companions. For us this would be due to different distances, but not for these plants. As soon, however, as they learn to communicate and would tell each other what they see, they would find out that what is small to one observer, might well be big to another one. After some perplexity they may construct a theory of relativity of size, saying that size is nothing absolute but depends on the relative position of observers - difficult to understand for someone who is used to live in a 2-dimensional perceptional space. Exactly the same happened to physicists when empirical evidence forced them to construct the theory of special relativity saying that time intervals are nothing absolute but depend on the relative motion of the observer - difficult to understand for someone who is used to live in a Newtonian world. (By the way, this analogon can be even extended: the (relativistic) limitation of all speeds in the 3-dimensional case ($v<c$) corresponds to the limitation of all lengths in the 2-dimensional case, as these can be defined only by means of the aperture $\alpha < 180°$).

The question whether modifications of visual perceptions should be interpreted geometrically or physically is well known from another case in physics: the orbits of planets could be considered as the effect of explicit gravitational forces (the physical solution) as well as geodetic lines within a 4-dimensional space (the geometrical solution as proposed by the theory of general relativity). As all this is just but a different interpretation of the same observations we cannot come to a decision on empirical grounds nor was adaptation or selection relevant when cognitive evolution of primates had to decide whether to see the visual world in 2 or 3 dimensions. In other words: perceptional spaces and systems of categories are purely descriptional systems which may tell us something on how we see the world but nothing on the world itself. So they cannot be the outcome of adaptation to the world. *From this it follows that our epistemology cannot be a natural (i.e. external) selection* *epistemology.*

But, is it not possible that our epistemology is an *internal* selection epistemology? i.e. that certain elements of our epistemology are easier to realise than others and therefore are selected by our cognitive evolution. The spatial shape of objects we describe in 3-demensional perception, for example, is something nobody has ever seen. All we see are 2-demensional projections on the retina of our eyes. The spatial character of perceived objects is a cognitive artefact which requires a lot of internal mathematical efforts to be realised. In 2-dimensional perception, however, things are as they appear. So, from the mathematical point of view, two dimensions are privileged with respect to three dimensions. On the other hand, 2-dimensional perception requires an explicit physical theory on the correlation between what our legs are doing and what wee see. In 3-dimensional perception this is solved implicitly. The correlation concerned is a matter of geometrical perspective comprised in the notion of 3-demensional spaces. The fact that our cognitive evolution ended in three dimensions rather than in two may not be due to mathematical economy (which, of course would favour 2 dimensions) but rather due to a very general principle: cognitive evolution tends to take away approved experiences from explicit description and to put it into implicit description, i.e. into the special character of the metatheory concerned. Then, the regularities we found are no longer a particularity of the 'world' but the 'logical' consequence of the metatheory we use. The most prominent example for this is Noethers principle showing that the conservation laws in physical mechanics (energy, momentum etc.) which govern everything in the world of mechanics can be derived from our space/time perception (conservation of energy results from the homogeneity of time, conservation of momentum results from the homogeneity of space, etc.). So, the laws of nature have an ontological character only if our cognitive variables such as space and time were ontological. They result from the way we see the world, i.e., they are human specific constructs. So, metatheories too are theories in the literal sens. They comprise the more elementary experiences in a way that everything what is articulated in terms of the metatheory implicitly satisfies the experiences concerned without further steps to be taken.

3. Adaptation vs. Assimilation in organic and cognitive evolution

That our natural epistemology cannot be an natural selection epistemology in the sense of Campbell does not dispense us from explaining why the evolution of our natural epistemology went just this way and not another one. Particularly it does not exclude the suggesting idea that organic and cognitive evolution must be linked to each other, or even more, that organic evolution has brought about cognitive evolution, i.e., that cognitive evolution may well be considered as the continuation of organic evolution by other means. From this it may suggest that a good theory of evolution is required to describe both organic and cognitive evolution in a strictly coherent way.

Unfortunately, this is in contrast to conventional thinking. As already underlined by Piaget (1967), we see organic evolution in terms of autonomous internal modifications (mutation, recombination etc.) to which the external world reacts by means of selection mechanisms, whereas in cognitive evolution we speak in terms of an autonomously existing and changing world to which intelligent beings react by means of formation of theories and learning. So, the attribution of actio and reactio is opposite in the theories we use to describe organic and cognitive evolution.

A further difference is seen between the way we see the development of empirical scientific theories and organic evolution: Science is said to be teleological in character, i.e., it will converge (though in sometimes rather roundabout ways) towards the hopeful end which physicists call the *'theory of everything'*, whereas organic evolution obviously has no specific focus towards which all species will converge, the *'pride of creation'* so to say.

3.1. Organic evolution

Let us start with organic evolution: it is wide-spread understanding that organic evolution has to meet the requirements of the environment by means of *adaptation*. Only appropriate adaptation will be honoured by natural selection. On the other hand adaptation in the wider sense may mean as well developing acting skills and tools by means of which the organism may modify just the environment according to the requirements of the organism itself. This is what we call *assimilation*. So, the discrepancy between the organism and the environment can be reduced from both ends, through adaptation (*internal solution*) and through assimilation (*external solution*). The world acts upon the organism by means of forcing it towards better adaptation, whereas the organism acts upon the world by means of assimilation.

One of the most popular methods of all animals for changing the environment is locomotion. Paramecia began early to use locomotion for escaping from adverse local conditions. That the relevant environment and its selection pressure is an artefact of the various species occupying it rather than an objective and external issue was seen already by Waddington (1959, p. 1636): "Animals are usually surrounded by a much wider range of environmental conditions than they are willing to inhabit. They live in a highly heterogeneous 'ambience', from which they themselves select the particular habitat in which their life will be passed. Thus the animal by its behaviour contributes in a most important way to determining the nature and intensity of selective pressures which will be exerted on it. Natural selection is very far from being an external force, as the conventional view might lead us to believe". Regarded from this aspect, life is a mode of world construction in the sense of Goodman (1984) rather than a process of exploring the world or of acquiring knowledge about the world as Lorenz (1983) said. In other words, evolution seems to aim at assimilation rather than at accommodation.

In deed, the higher organisms have evolved the more they refer to external solutions. Men, for example, when confronted with physical problems, count exclusively on external solutions. When ever we are confronted with a problem, we try to solve it by technical or social means. We do not wait for organic evolution to help us which would be totally inappropriate and would take by far too much time. That organic evolution will not converge towards a definitive and optimal species of universal competence has a simple reason: organisms (consciously or unconsciously) modify their environment and by this the boundary conditions to which evolution has to orient itself. So, there will be no completion of organic evolution.

3.2. Cognitive evolution

As to the evolution of cognitive capabilities, it is wide-spread understanding that it proceeds according to similar principles as organic evolution. Particularly the evolutionary epistemology (EE) (Campbell 1973, Lorenz 1966, Riedl 1980, Volmer 1980, Wuketits 1984) is based on the assumption that our cognitive instruments and the categories of our thinking have evolved in similar manners as the organic tools of life management such as metabolic and homoeostatic instruments or limbs as instruments for locomotion and other acting.

Within the EE, however, the question is open, how our cognitive instruments have evolved, and why just these and not others? We mentioned already Campbell′ s answer and his natural selection epistemology. According to him, our epistemology is a kind of encoded view of the world, in the same sense as the hoofs of horses are a kind of view of the steppe landscape at which they live, as said by Konrad Lorenz (1983). Most problems, however, can be solved in different manners which often are not related to each other. Horses and snakes, for example, though they may have developed in a similar physical environment, have entirely different organs of locomotion which have no structural element in common. So, the environment can neither tell us how an organism has to look like when to survive, nor what kind of cognitive apparatus it has to have. Within this line of thinking the world acts in two ways on cognition. One is by generating the sensorial input and by this provoking reactions, the other is by selecting the epistemologies concerned as described by Campbell. In contrast to the organic case there seems to be no analogue of assimilation, i.e. cognition seems to have no means of acting upon the environment The next paragraph will open the possibility to modify this view.

4. Theory of cognitive operator

To understand cognitive evolution from an organic point of view, we start from the CEE (Constructivist Evolutionary Epistemology, Diettrich, 1993) a constructivist extension of the classical evolutionary epistemology (EE). The particularity of the CEE is what we will call the theory of cognitive operators (TCO). This is based on a methodological element used mainly in physics, the so-called operational definition of physical terms. What does a operational definition mean? As is well known, classical physics failed to accommodate the phenomena of quantum mechanics and special relativity primarily because it got involved with a non verifiable syntax brought about by the use of terms that had not been checked as to whether they could be defined by means of physical processes.

In our day to day life this epistemological refinement is not necessary. We have a clear understanding of what the length or the weight of a body means, and we do not need confirmation from a tape measure or a scale for carrying on. The situation, however, is different with microscopic distances that are smaller than the atoms of the tape measure. Here, first of all, we have to decide what kind of experimental facility we will apply in order to define length or momentum. Physicists say that properties are defined as invariants of measurement devices. This even applies to the order in time of events which, under normal conditions, can easily be defined and detected. With very high speeds, however, the topology of events may depend on relative motions.

As this kind of experience requiring a realignment of our world picture may happen again and again, it suggests a generalization that can be summarized as follows: properties of whatever kind and of whatever subject have no ontological quality. Instead they are defined by the fact that they are the invariants of certain measurement operators. This contrasts with classical thinking in which properties are used for the objective characterisation of objects. One of the most important properties we usually attribute to properties, namely, independent existence, is based simply on the assumption of their independent ontological quality. In day to day life this is incontestable. The length of a body and its colour exist independently of each other and can be measured separately. This does not necessarily apply in subatomic regions, as we know. The position and momentum of microscopic particles cannot be measured independently of one another. Physicists learned from this that theoretical terms have to be defined operationally, i.e., they have to describe nature by means of theories in which only those terms are acceptable which can be defined by certain experimental facilities, rather than by means of categories which are defined by protophysical common sense. Let us look at the operator concept in more detail:

Acting (be it acting by our self or acting by other subjects or systems) means everything which modifies perceptions. Acting can be acting in the literal sense such as modifying a physical object and by this changing what we see of the object as well as acting by means of our locomotion limbs and by this changing the perceptions concerned due to perspective phenomena. The only direct access we have to objects are perceptions. Then we can use the notion of operators acting on perceptions and transforming them into other perceptions. We will apply here the formalism of quantum mechanics in the following way:

- φ, χ, .. are normalised vectors in Hilbert space and represent perceptions.

- O, P, ... are Hermitian operators in Hilbert space and represent operators acting on perceptions.

- λ_i are the (real) eigenvalues of Hermitian operators.

Let us consider all the perceptions φ_i (i 0 1,2,...) which are invariant under the action of an operator O:

$$(1) \qquad O\,\varphi_i = \lambda_i\,\varphi_i.$$

We will call this set of eigenvectors of O a representation of an object defined by means of the Operator O. Formula (1) can also be read in another Way: if the φ_i are defined by another Operator, say N, which commutes with O (i.e. ON = NO), then(1) represents a measurement process and the λ_i are the possible measurement results. This relationship is symmetric. Both O and N can be defining or measuring operators respectively. In the O-representation we can measure by means of N, and in the N-representation we can measure by means of O.

If N and O do not commute (NO ≠ ON), then N will transform a perception φ which was defined by means of O into a perception φ' which is no longer an eigenvector of O. In this case we will call N an acting operator rather than an measuring operator. The same applies for the opposite direction. *So, whether an operator can be used for action or measurement depends on whether it will commute with the operator defining the object in question.*

According to classical understanding, action and perception are two entirely different categories: action refers to the individual's input to the world, whereas perception means the world's input to the individual. With the theory of cognitive operators, however, the question whether an operator represents an action or an perception depends on how the object in question is defined. This sounds strange but is easy to illustrate. We mentioned already an example. In a 3-dimensional world furnished with objects which are defined as invariants of spatial transformation, locomotion is an measurement process (when walking around an object the various views perceived will inform us on the objects shape, i.e., we will 'measure' the shape). In a 2-dimensional world, however, locomotion is an acting operator because it modifies the shapes concerned (see 2.5). Another example: a hammer is an instrument designed primarily to alter certain objects. But a hammer, in its quality as an operator, also has invariants: objects and properties which would resist the hammer's strokes of a given strength. The hammer, then, can be used to measure mechanical properties such as the strength of materials. (Railroad workers do so when clicking with their hammers on the trains wheel in order to check their mechanical integrity). So, a hammer can be both an acting or measuring device depending on whether it will be applied to weak or strong materials. Another example: the well known fact that in microscopic dimensions position and momentum cannot be measured simultaneously, can be phrased as follows: the position operator will commute with the momentum operator in macroscopic systems, but not in microscopic (quantum mechanical) ones. I.e., the position operator is a measuring operator with macroscopic particles, but it may act at the momentum of subatomic particles.

Up to here we implied that the operators we spoke about were physical operators, i.e., measurement and acting operators in the literal sense or limbs for locomotion etc.

Further to these we will define *cognitive operators* which are implemented physiologically somewhere in our brain and which transform certain states of our sensory apparatus into what we call perceptions. The invariants of cognitive operators are the regularities we perceive. So, these phylogenetically acquired human specific cognitive operators are the definition operators for all the regularities we see. Insofar we condense observed regularities into laws of nature, these laws also are human specific rather than objective. For phylogenetic reasons all men have the same cognitive operators and, therefore, will see the same regularities and, therefore, will identify the same laws of nature. This is why they succumbed to the temptation to call their common experiences as the result of something real, i.e., non human specific.

This also holds for causal laws: Causal laws describe the effect of actions, be it human actions or interactions of physical systems.

$$(2) \qquad O\,\varphi = \varphi'.$$

If O is the acting operator concerned the causal law describes the transformation $\varphi \Rightarrow \varphi'$. O is the cause, the transformation is the effect. But φ' does not depend on O alone but also on φ and by this on the definition operator defining φ. In other words: there are no universal laws. Any law descibing the effect of an operator has to refer to the operator defining the system upon which the operator will act. For example: as we have seen, the law that locomotion will deform physical objects will hold in a 2-dimensional world but not in a 3-dimensional one. And the law of energy conservation depends on our mental time metric generator.

In realism the situation is entirely different. From the realism's point of view the objects we see and investigate are not the outcome of human specific cognitive definitions. They rather are what they are in an ontologically undebatable manner - and so are the laws describing their relationship.

This is the very difference between the constructivist approach discussed here and realism of what ever breed. It is not that we identified one or two natural laws as being mental constructs and that we conclude from this that all natural laws must be so. (This, of course, would be a dubious enterprise).

The argument goes the other way round: All the regularities we perceive are invariants of their defining cognitive operators and, therefore, are human specific. So, every thing we derive from them - particularly the natural laws - is human specific as well.

If we speak in terms of (e.g. visually) perceived regularities we mean for example that a system has a certain number of objects, or that these objects ar grouped togeher in symmetric forms, or. more generally, that what we see may be invariant under certain symmetry or other mathematical operations. In other words, mathematical operations are the defining operators for the structures we perceive. On the other hand we use mathematical structures in order to describe perceived regularities. So we have here the same situation as with the physical operators we just described. We use mathematical operations which define the structures we see and we use other operations to 'measure' or describe them. What we use for what is generally arbitrary but phylogeny has taken a certain predecision: We see and think in terms of plurality, i.e., we are used to say that objects can exist on more than one copy. The mathematical analogy is the basis for set theory. We think in terms of space which would enable us to arrange objects in lenear orders which then is the basis not only for geometry but also for the theory of functions. Mathematicians understand that visual structures are good metaphors for many things they deal with. But they do not accept these analogies as definitions. In order to escape perceptional analogies they developped the instrument of axiomatisation. Axiomatisation, they believed, will allow defining mathematical notions and objects without any reference to visual structures. But unfortunately also axioms have to be written in a language which as any language is infected by perceptional elements. So, the development of an objective language without any empirical refernece (the dream of analytical philosophy) is unfeasible. We rather have to accept the idea that perceived and mathematical structures have much in common as they are invariants of the same mental operators. This, I think, is the very reason why mathematics is so well suited for describing what we see, i. e. for what Wigner (op. cit.) called "The unreasonable effectiveness of mathematics in the natural sciences"

Similar applies for logics. Indeed, the elementary logical structures and procedures which we find and apply respectively in language are phylogenetically based human specifica like the perceptional structures upon which we will apply them in order to generate higher theories. Particularly the laws of logic cannot be explained as universalia in the sense of Leibniz which on grounds of their truthfulness would hold in "any possible world" (see 8.2)

Let us summarise: the TCO suggests to see percieved structures as well as mathematical and logical structures as invariants of phylogenetically acquired mental operators which due to their possibly common root may bring about structural similarities. These similarities may enable mathematics to act as a model for what we 'see'.

In this context the often discussed dichotomy of observational and theoretical terms is reduced to a rather secondary difference

Observational terms, comprising both the visually perceived regularities (patterns) and those we condense into theories and into what we call laws of nature, are considered to be invariants of phylogenetically evolved mental cognitive operators. Observational terms have developed phylogenetically in the unconscious parts of the human brain,

Theoretical terms are defined as invariants of operations represented by physical measurement devices. Theoretical terms are the outcome of conscious and rational efforts.

Nevertheless, observational terms remain privileged as the basic elements of any higher theories. We can modify theories according to observational data, but we cannot modify the genetically fixed mental operators and their invariants according to the requirements of special situations.

Any set of operators acting on perceptions with an unambiguous separation of measuring from acting operators constitutes what we call a world picture. (We discussed the case how our world picture has to be modified if locomotion were taken as an acting operator). An equivalent possibility to define the term world picture is to say that it represents the total sum of our physical metatheories. One of the basic metatheories in classical hysics is that physical systems have independent and well defined properties (such as location and momentum) which can be measured independently. The quantum mechanical analogon is: the independency of the properties of a physical system is itself a property of the system depending on whether the Hermitian operators in Hilbert space representing the measuring device concerned commute or not.

Our classical world picture, to which we are subject in day to day life, is the result of phylogeny. The question will arise as to what prompted cognitive evolution to come to what decision. Let us try the following approach:

In case of a physical measurement (which is an action, of course) the result, in physical parlance, is the invariant of the measuring process. In other words, we use the invariants of a process to describe the effect of just that process, i.e., we describe the covariants of an operator by means of its invariants. This can be generalized into the cognitive area. The actions by means of which we explore the world can be considered as measurements (i.e., as perceptions in the broadest sense). Results of measurements (or, as one could say, the results of our experiences) then are views of the world and theories representing what we call the unchangeable and, therefore, the objective world (i.e., what is invariant under all our doing and acting). If we look however for the covariants of our action, i.e. what changes under the influence of our actions, we have to refer to what we said about the relationship between the covariants and invariants of measuring processes: the effect of action can be described only in terms of the invariants of action, i.e., in the terms of our world view.

If this is true, then the elementary categories of our perception must be the invariants of our most elementary action operators. But what are the most elementary action operators? They are not, as one might think, our hands and the tools guided by hands. They are, rather, our legs. By means of a few appropriate steps, we can change the environment of the room we are in into the environment of a blooming garden. Of course, we could achieve the same also by using our hands if we employ them do the necessary reconstruction work. But this is troublesome and time consuming. So, one of the most important human-specific operators is locomotion. Our world view, as a result, must be based on the invariants of this operator - and this is indeed the case. The most elementary descriptive category of our world view is the identity of extended objects and spatial structures defined as an invariant of locomotion. (This argument does not determine whether spatial structure is 2-, 3- or 4-dimensional).

This provokes the assumption that, from a phylogenetic point of view, the categories of description can be understood only through their capability to cope with the covariants of certain operators. From this it follows that evolution designed the cognitive phenotype in order to extend the action possibilities of the organic phenotype rather than to explore the world.

The program describing the covariants of operators by means of their invariants is well known and is often used in physics. Within the framework of Hamilton-Jacobi-formalism, the variables of a mechanical system can be chosen so that conservation laws (invariants) will apply for them. With this prerequisite cared for, the transformations describing the system's development in time can be found easily and explicitly.

On the other hand, the conservation laws themselves can be shown to be generated by the transformations considered. So the canonical total momentum (in this paper identified, in a more general way, as 'motion') brings about spatial translation, and the total energy (represented by the Hamiltonian) brings about translations in time. Something very similar applies for quantum mechanics. The system's development in time is generated by the Hamiltonian and the eigenvectors of the Hamiltonian constitute the reference frame by means of which this is described. This means that our elementary cognitive coordinates are what in the theory of Hamilton-Jacobi is called cyclical variables by means of which the motion of force free bodies can be described simply and linear in time. (The explicit solution of Newton’s equation of motion in classical mechanics is very difficult except in very simple cases. Hamilton and Jacobi have shown that in many cases a transformation of variables can be found so that the resulting variables will develop linearly in time and therefore will lead to simple calculations. The word cyclic has historical reasons and has nothing to do with what the word’s etymology may suggest).

Therefore, the existence of conservation laws and the closely related fact that in many cases the development in time of mechanical systems seem to be rather simple (i.e. algorithmically compressed as Davies would say) is not based on objective laws of nature and their always eulogized harmony and simplicity. It rather is related to the permanent co-evolution of acting and cognitive instruments which is necessary for predicting the consequences of acting, i.e. for action management. The perceived simplicity of the world is based on the phylogenetic decision to apply those cognitive variables for the description of the effects of elementary acting (such as locomotion) which are cyclic in the sense of the Hamilton-Jacobi theory. Complex is nature for us only if we deal with cases where additional forces with unusual properties depending on various variables will constitute a system with respect to which the 3-dimensional Cartesian coordinates of our phylogenetically acquired world view are no longer cyclic.

We can conclude: physical actions, and the cognitive operators we use to describe them, are brought about by the same organic operators (i.e., organic tools). Perceived patterns or regularities, and the instruments of mathematical thinking we use to describe them are brought about by the same cognitive operators. So mathematical patterns, perceived patterns and the results of our actions are literally homologous in so far as they have a common ontogenetic root, and this is the very reason why they can 'cooperate' so well with each another - as well as the various physiological mechanisms having brought about each other in the course of organic evolution.

5. The operational definition of space, time and causality

5.1. Space

The most crucial consequence of what has been said above, is that space, time and causality, which according to Kant are the necessary notions or categories respectively on which all external appearance is based, are not the only possible (and therefore necessary) categories. They are rather the phylogenetically evolved features of human perception and interpretation, defined operationally as invariants of certain actions and transformations. Let us look at this in more detail.

Following Piaget (1974), *the spatial metric* of our perceptional space (and therefore the topology comprised) is operationally defined by means of motion. The identity of extended subjects, therefore, is defined as an invariant of locomotion (Uexküll: "a body is what moves together as a unit"). This definition is probably the main reason for the major difference between what we call space and what we call time. Time is said to flow in an irreversible way; no one can retrieve any part of the past. We cannot move back and forth between two points in time. But we can do so quite well between two points in space. If we say we travel from point A to point B and than back again to A, we mean that the A where we started before arriving at B, and the A to which we arrived after leaving B, are not only equal but identical. To say so, however, is possible only if we can distinguish between "equal" and "identical" and if what we call identical is not influenced by our travel. This means that Identity is defined as the invariant of motion. And exactly this is the point. Only on grounds of such a definition can we call a change in spatial positions reversible, or more precisely: only on the basis of such a definition can we distinguish between the repeated return to the same A and travel along a sequence of equal As, i.e. between periodicity in time and space.

In a similar way, locomotion can change the visually perceived environment. We can transform the perception we call "forest" by means of walking, appropriately, into the perception we call "city". But this is not what we are accustomed to saying. More common is to speak in terms of an environment which, apriori, is multidimensional in character, i.e., comprising at the same time several structures which differ, first of all, in what we call their spatial positions. What we achieve, then, by means of our legs, is not a modification of the environment. We just "go" to places consisting of different structures and therefore experience different perceptions. What we call the multiplicity of the world, thus, is defined as invariant to changing our positions in that world. From the functional (and TCO) point of view, mentally generated spatial views belong to the most elementary theories we have at our disposal, by means of which we can forecast perceptions when walking - in the same way as the temporal structures stored in our memory inform us on what we can expect when repeating certain actions. So, both the formation of visual patterns and the formation of memory are first of all modi of extending life competence.

Spatiality and the spatial metric, as we have seen, are categories that are necessarily defined by the process of motion. On the other hand, motion cannot be explained without the notion of space in which motion takes place. From this it follows that motion itself will have brought about the mental category of spatial structures necessary to deal with motion. Exactly this is what we maintain: what an operator is doing can be explained only in terms of its invariants.

5.2. The arrow of time

Within the context of our day to day experiences we have a very clear understanding of what past and future is. Past is what embodies all the events we have experienced. Past is the source of all knowledge we have acquired. Future is the subject of our expectations. Future embodies the events which may happen and which we have to await in order to see if they really will happen. How can we express this by means of physical theories? Or, more precisely and according to the operationalisation concept: Are there devices or processes which can operationalise the terms past and future, i.e. the arrow of time?

Many efforts have been made in this direction. The result is short and disappointing (though not in the light of theTCO): In all cases where it is said that the arrow of time has been operationalised it can be shown that the direction of time was already comprised implicitly in the preconditions of the experiment. A typical example is the following: Shaking a box with black and white balls put in order according to their colour will always lead to disorder and never again to order. In physical terms: Entropy will increase in time and never decrease. Entropy, therefore seems to operationalise the arrow of time. But in this case the result will depend on what we do first, separating the balls or shaking them. Shaking before separating will lead to order. Shaking after separating will lead to disorder. So we already have to know what the terms before and after mean before we can do the experiment which is to tell us what before and after will mean. Another example: A hot physical body left in a cooler environment will always cool down. But this applies only if the collision processes between the atoms involved are endothermal, i.e. if the kinetic energy of the collision partners are higher before the collision than they are afterwards. If we have however exothermal processes which are characterised by the fact that the kinetic energy of the particles involved is higher after the collision, then the body will heat up rather than cool down. Here again we have to know what before and after means in order to define the collision process which will define the result of the experiment which is to define the arrow of time.

These are particular examples. I. Prigogine (1979, p. 220) has shown in a more general way that irreversible processes in thermodynamics cannot help us to operationalise the arrow of time: The existence of the so called Ljapunow–function – which is closely related to macroscopic entropy – is a prerequisite for the distinction between past and future also in microscopic systems. Unfortunately, the Ljapunow–function is ambiguous with respect to the arrow of time. It can be constructed in a way such that equilibrium will be achieved in the future as described in classical thermodynamics but it can also be constructed so that the equilibrium will be "achieved" in the past.

From all this one can make the hypothesis that in principle the arrow of time cannot be operationalised objectively, i.e. it cannot be derived from what we call nature. What past and future means, then, can be described only by means of a sort of mental operationalisation. The following definition, for example, may be suitable: From two perceived events A and B, A is said to be before B if we can remember A when B happens but not B when A happens. Of course, past is what we can remember but we cannot remember future. This "mentalisation" of past, present and future, I think, is very close to what Einstein (published 1972) may have had in mind when he wrote to his friend Bosso "that these categories are sheer illusions". But when referring to memory we have to be aware that, from the cognitive point of view, events themselves are already operators transferring the status before the event into the status after the event. So events, just as any operator, require the prior definition of the arrow of time. Without a definition of the arrow of time events and all we store in our memory in order to write history remain undefined.

By this, time turns out to be a mental modus which itself needs to have been brought about by operational means. In the same way as the spatial metric was generated by the process of motion (i.e., motion bringing about the category of space which is necessary for describing of motion), the category of time also must have been generated by operators (i.e., operators bringing about the category of time which is necessary for describing what operators are).

5.3. Causality

In order to constitute causality we must be able to identify patterns of events. If a number of events, say A, B, C, and D follow each other always at typical intervals independent of when the first one occurs (i.e., if the pattern is an invariant of translation in time), then we say that there must be a causal relationship between the events concerned. Otherwise the perceived regularity could not be explained. Causal relations, then, are defined as invariant patterns of time. This, however, requires more than just having a topology of events as provided by our memory. We also must be able to distinguish between shorter and longer intervals of time, i.e. we need a time metric defined by a mental metric-generator implemented physiologically somewhere in our brain. For example, that we say lightning is the cause of thunder but not the contrary, is based on the fact that the time between lightning and the next thunder is usually much shorter and varies less than the time between thunder and the following lightning. But the length of time intervals can be defined only by means of a time metric. If our time metric generator were of the kind that it would be accelerated after a flash of light and retarded after an acoustic event we might well come to the conclusion that thunder is the cause of lightning rather than the other way around. *The mental time metric-generator is therefore responsible for the causal order established and for the prognostic capability derived from it.*

The specificity of the metric generator has direct effects on the laws of conservation we record in physics (energy, momentum, etc.). Following Noether's theorem, these laws can be derived from the invariance properties of the equation of motion: invariance under a translation in time (i.e., physics is the same yesterday and today) implies the conservation of energy; invariance under translation in space (i.e., physics is the same in America and Europe) implies conservation of momentum; invariance under spatial rotations implies conservation of angular momentum. In other words: from the homogeneity of space follows the conservation of momentum and from the homogeneity of time follows the conservation of energy. What 'homogeneous' means, however, is exclusively a matter of the mental metric-generator concerned. This applies also to the other conservation laws which, therefore, are human specifics rather than objective properties of nature. As will be seen below (par. 8.1), the conservation laws constitute what one could call the cognitive reference-frame we use to describe actions and what those actions will bring about. Other conservation laws based on other cognitive operators would effect a different cognitive phenotype. We mentioned already that none of these cognitive phenotypes were privileged, i.e., methods and life strategies based on other operators may well be as consistent or efficient as ours are.

6. Induction and the compressibility of observational and theoretical terms

The TCO allows a somewhat unusual approach to the problem of induction: Perceptions (and observations) are related to each other according to what we call the regularities perceived. These regularities, as we have seen, are the outcome of special mental operators. A (scientific) theory on the relation between observations, therefore, can be "true" (i.e. it can extrapolate the data observed correctly) only if it would emulate the generating mechanisms. But how can we emulate these mechanisms if we do not have any access to the brain where they are implemented and if we have no means to analyse them otherwise? What we have is nothing but mathematical methods which – astonishing enough as Wigner said – would work very effectively in helping us to extrapolate observational data. Then, the conclusion is near at hand that there is a certain homology between the mechanisms generating mathematical, logical and other theoretical terms and those generating observational ones. This would explain, of course, why observational extrapolation (i.e. waiting for the observations expected or doing the experiments required) may lead to the same result as the mathematical extrapolation of observed data does. A helpful contribution for the solution of the problem of induction, therefore, were plausible hypotheses on a common metatheory of mathematics and observational terms.

The stated equivalence of observational and theoretical terms requires that we approach mathematics and logic under the same constructivist aspect as we do with the empirical world. There is already a certain tradition of constructivist approaches (see Lorenzen, 1975) having in mind mainly a better foundation of mathematics: Only if we knew how things came up could we understand why they are as they are. Unfortunately it is not enough to find a "generative mathematics" which generates all the mathematical rules or regularities we know as there is no guarantee that it would also generate those we may still find in the future. The only guarantee for generally succeeding is that we find a solution which emulates the actually implemented mental mechanisms. This generative mathematics, however, as well as Chomsky's generative grammar, is inaccessibly sited in the subconscious parts of cognition. All we know and all we have access to are their results. From them, unfortunately and as a matter of principle, we can not conclude the generating mechanisms. So we may well deduce grammatical rules from a generative grammar if we had it, but we cannot derive the generative grammar from the grammatical rules we know as intended by Chomsky.

To deal with the compressibility of mathematical terms means to pose the question: why can we describe the results of rather complex mathematical operations by relative simple expressions? How can we extrapolate ordered sequences of mathematical operations by explicit formulae, i.e., why does the principle of mathematical induction work? That this is a serious problem is known – at least in principle. Though mathematicians generally acknowledge that Peano by means of his five axioms has considerably contributed to understanding the world of natural numbers – particularly the fifth *"If the natural number 0 has some property P, and if further whenever n has P then so does n + 1, than all natural numbers have P"* is the basis of mathematical induction, one of the most important procedures in practical algebra. But Hofstaedter has rightly remarked that this does not provide a criterion to distinguish true from false statements on natural numbers. He asked (1979, p. 229): "..., how do we know that this mental model we have of some abstract entities called 'natural numbers' is actually a coherent construct? Perhaps our own thought processes, those informal processes which we have tried to capture in the formal rules of the system, are themselves inconsistent!" Well, at least in the constructivist context, they are not inconsistent as this term is not explained there. But the possibility remains that the formal rules we have established do not correctly or completely emulate the informal thought processes (i.e. what we called mental operators). The ongoing success of mathematical sciences, however, make it rather probable that mathematics is a fairly good theory of what the mental operators can bring about. It may even be a correct or true theory if the mental operators in the course of cognitive evolution contributed implicitly to their own conscious formalisation, i.e. to the development of mathematical and logical thinking. In other words: mathematics succeeds by means of compressing theoretical terms (e.g. by means of mathematical induction) because the mechanisms of generating theoretical terms and those compressing them are closely related to each other due to a special cognitive co–evolution having the effect that compressed and uncompressed terms behave alike and therefore are interchangeable.

The fact that a large number of empirical data can be described by a relatively simple mathematical formula, by a simple view or regularity or by just a few words (i.e. by a theory in general), we explained by their compressibility. On the other hand we can consider these formula etc. to generate the data in question in the sense that we can derive them from the generating theory. Within the framework of constructivism, however, there is nothing that is not generated, either by a physical or biological process, by a theory in the proper sense or by a mental operator generating what we perceive as regularities or laws. Compressibility, therefore, is not a special feature of some data or entities we have to investigate or to wonder about. It is rather the central characteristic of constructivism. The generating mechanisms (and only they) can tell us how we have to extrapolate given data or what we can conclude from certain observations, i.e. how we can apply mathematical or empirical induction. Without generating mechanisms neither extrapolation nor induction is anything but arbitrary and therefore useless and meaningless.

The difficulty of classical approaches towards the problem of induction follows from the idea that the operators generating the regularities of our perceptions are seen exclusively as non–mental external mechanisms. We say: regularities (such as symmetries) are in the outside, real nature and not in the way we see it. According to this it is generally understood that we have to extrapolate data from celestial mechanics according to the effect of gravitational forces as contained in Newton's laws. But we find it strange to understand why we usually succeed in extrapolating a number of sensual data perceived according to a regularity identified by means of nothing but the data given themselves – as if the regularity of the past data and of those to come were caused by the same reason. But exactly this is the case. There is of course a causal reason generating these regularities, but it is not an external one as gravitation is said to be. It is rather the internal mental operators generating the regularities in question. This is the very legitimation for empirical induction. As this applies for any kind of regularity, so also the laws of classical mechanics as described by Newton are nothing but the emulation of mental operators by means of what we call explicit external forces. What still has to be explained, however, is how to deal with the regularities we find in areas such as elementary particle physics etc. which hardly can be expected to be 'inborn' as the regularities of classical mechanics. This will be done in the section on physical extensions of cognitive operators (8.1).

7. Communication, meaning and the compressibility of semantics

Here it will be shown how the notion of communication and meaning will depend on the human cognitive phenotype. By this communication is a human specific construct which will exclude communication with beings of different cognitive phenotypes such as (probably) extraterrestrial aliens.

If all structures we perceive are only human specific artefacts which can only be defined as invariants of cognitive operators, then this must apply also for the perception (or interpretation respectively) of language structures, i.e. neither can a physical object have objective properties which can be used for an objective description, nor can verbal texts have an objective interpretation. Then the question will arise as to whether a text can carry an autonomous message, and if not, what the notion of communication will mean.

According to common understanding communication means that certain structures, for example texts, will be transferred from the sender to the recipient where they will actuate text specific reactions. The text will enable the recipient to draw conclusions insofar as he has understood (i.e. analysed) what we call the meaning of the text. Meaning, then, is something encoded in the text. For the recipient, therefore, meaning is an externally defined structure. A similar view is held by Hofstadter (1979) who believes in the general possibility to decipher context free messages. For him (p. 165) "... meaning is part of an object (or a text) to the extent that it acts upon intelligence in a predictable way." This would mean to concede meaning the status of an objective property in the sense of realism. Further to this, within the framework of the TCO, the notion of analysing a structure in order to identify the structure's inherent meaning is not explicable. Structures can be generated but not analysed. What we usually call an analysis refers to other structures which are generated by the same operation and which we, just because of this, perceive as "similar".

Both theories and their meaning (in their quality as invariants of cognitive operators) are mental artefacts. Their effect is that they connect data or statements with each other - and here they have a monopoly: There is no other possibility to connect data and statements but within the framework of a theory or a known meaning. From an observation alone one cannot derive a second one except by means of a theory which is able to do this. From the fact, for example, that one has seen up to now only white swans nothing can be derived on the existence of black ones, except there were a theory saying something on this matter. (Diettrich 1989, p. 78). Nor is it possible to derive from an isolated statement a second one without having knowledge of the context of meaning. If we nevertheless try sometimes to derive statements from each other then only on the basis of tacit assumptions on the context..

Under these circumstances, to perceive a text or any other structure can only mean to reproduce it through the recipient's own generative means. If these means are insufficient, they will have to be modified accordingly by the recipient himself. This is what we call learning, and the text which has effected this is termed a piece of information. Information is something the recipient did not know before, i.e. what he could not reproduce by own means. To understand a text shall mean that the recipient is not only able to reproduce the text but also to draw the same (or similar) conclusions from it or infer the same texts as the sender. But what does it mean to make inferences and particularly inductive inferences within the context of constructivism? In common thinking all things which can be derived from each other by extrapolation or by inductive inference, just by this, represent certain relations. Under constructivist aspects, however, relations of what ever kind can be defined only through common generative mechanisms (operator, theory etc.).

We can now say: a recipient will understand a text in the sense intended by the sender if he not only reproduces the text but if he reproduces it by the same (or similar) mechanisms as used by the sender. Only under these circumstances the recipient has, further to the text in question, also all the other texts at his disposal to which the sender could refer, i.e. they can both draw the same 'conclusions'. Strictly speaking this does not require that the generating mechanisms are structurally equal as long as they produce the same. But as they do so, more or less, with all men it can be assumed that this is due to their phylogenetically acquired common metatheory. In this case they would not only be functional but also structural homologa.

Let us summarise: to say that the recipient has understood the meaning of a text means that the recipient has interpreted the text within the same theory which the sender has used to generate the text, or, in other words that the recipient has decoded the text in the same way as it was encoded by the sender.

A prerequisite for communication is that those concerned have the same (or at least a similar) cognitive phenotype. Only then they would think in terms of the same categories and would deal with the same things and phenomena about which they can inform and talk to each other through pointing to the object in question or by means of interpreters. In other words: the perceived worlds of those who want to communicate with each other must be largely isomorphic. This is the case if Chinese and European speak with each other - however different their languages might be in detail, as long as they speak the same metalanguage. This means: they all think and speak in terms of space and time and of subjects living in space and moving in time. They speak of time that proceeds always in the same direction. The objects of time they call events; and if events come always in the same order they call some of them causes and others effects. In this respect all human languages are alike and, therefore learnable by everyone. The languages of our world can differ considerably in words and grammar and in the metaphors they use to enlarge the language competence - but all this is negligible as compared to the differences which another cognitive phenotype may bring about. If, for example, one would deal with objects having an identity defined as invariant of motion within a (3+1)dimensional space time continuum (as we do), whereas the other one describes objects as being the eigenvector of certain operators in Hilbert-space (as it may happen with certain extraterrestrial aliens), i.e., if the communication partners use different defining operators, nothing could be compared. In this case the perceived worlds are not only furnished with different objects but also are syntactically structured in different ways.

Some years ago a NASA rocket launched a copper plate into the space with some elementary information engraved about men and the terrestrial environment. This was based on the assumption that the same laws of physics would apply everywhere in the universe and that extraterrestrials, however else they might be structured, had to adapt to these laws and, therefore must have brought about equivalent cognitive structures in the course of their evolution. As the laws of nature, as we have seen, characterise our cognitive and empirical phenotype rather than the world we live in, men will identify the same laws of nature where ever he will be in the universe. From the human point of view the laws of nature are indeed universal as men, so to speak, carry their own laws with them where ever they go. The same would apply for extraterrestrials. They as well would identify laws of nature which from their point of view would be as universal as ours but which must not equal them. Even when visiting the earth, there is no reason for them to modify their world view and the laws comprised towards our laws of nature. By this, the prerequisite for the success of the NASA experiment is not fulfilled because extraterrestrials, if any, cannot adapt themselves to laws which are not their own.

This does not mean that we could not come to a kind of working arrangements with extraterrestrials when meeting them. After a period of cohabitation we might have learnt how they will behave in what situation. This might lead to a modus vivendi. But we cannot understand them, i.e. we cannot extrapolate their behaviour into new and unknown situations. Understanding is possible only on the ground of similarities. These must not be necessarily cognitive ones. If such beings were physically closed and of more or less fragile structure and would not be fixed in the ground like plants but rather could walk or fly around, they have to avoid collisions with other objects like we have. Then we could understand their habits of moving at least to this particular extent.

The question of the compressibility of the world (i.e. why observational data can be successfully extrapolated and, therefore, why induction works) can be transferred into the linguistic area. We can speak of the compressibility of language and we can ask why we can extrapolate texts semantically, i.e. why we can draw correct conclusions from a text. The problem of induction, then (how can we successfully generalise physical data transmitted from nature?), corresponds to the problem of communication (how can we successfully generalise verbal data transmitted from other persons?).

We see here the parallel between sensual and linguistic perception. Both result from mental operators acting upon sensual or linguistic stimuli respectively. The invariants of either operators present themselves as structures. In the sensual case we perceive this structure as regularities which would allow us to complete observations, or, as we would say in most cases, to extrapolate perceived data. In the linguistic case we perceive the structure produced as meaning which would allow us to draw the "correct" conclusion from the text given or, as one could say, to extrapolate the text semantically. Regularities and meaning or extrapolation and logical inference respectively are analogues categories in the sensual and linguistic area.

8. Extensions

Extension here means extending the inborn tools of perception or thinking by means of artificial scientific or technical instruments or methods. Particularly we will ask if extensions may affect our world picture and by this may contribute to the construction of our epistemology.

8.1 Physical extensions of perceptions and the notion of reality

Typical of most empirical sciences is the use of instruments and measurement devices (measurement operators) by means of which we extend the range of natural perception in ways similar to those we use to extend our inborn physical capabilities by means of tools and engines. We know that there are measurement devices, for example in quantum mechanics, bringing about results which do not fit into our classical world picture. The question will arise as to when a measurement can be described within the classical world picture and when it will require novel, non-classical approaches.

8.1.1 Quantitative extensions

We will speak of *quantitative extensions* if the inborn perception operators (i.e. the phylogeneticly acquired definition operators) and the measurement operators commute in the sense of operator algebra. In this case both operators will have a spectrum of invariants (i.e. eigenvectors) which can represent each other. This means that the results of the measurement operations can be presented in the terms of invariants of the inborn cognitive operators, i.e., in terms of our classical world view.

8.1.2 Qualitative extensions

We will speak of *qualitative extensions* if the inborn perception operators (ore the later in the course of scientific development constructed defining operators) and the measurement operators do not commute. Then the results can no longer be presented in a classical manner and would require new, non-classical theories. As the set of possible measurement devices is, in principle, unlimited, it can never be excluded that qualitative extensions of previously established operators will bring about modifications of the previously established world view and of the theories associated with it. So there will never be a definitive world view and there will never be a definitive 'theory of everything'. No objective laws of nature will ever be formulated. Those laws that we have, we have 'constructed' in a human-specific way in the course of human evolution. They never will converge towards a definitive set of laws except within the context of a limited set of operators, i.e., if we desist from further experimental research exceeding these limitations. What we actually do when we do science is to construct a world that we then believe we analyse by doing science. In other words: analytical in the sense of deepening our knowledge is characteristic of science only within quantitative extensions.

The notion of a theory of everything is equivalent to the notion of reality. Reality, so our understanding, is independent of what ever we do or can do. So it must be characterized by objective laws of nature. We have seen that there are no objective laws of nature. But further to this, the notion of reality cannot even be defined operationally (and this is what we require of all meaningful scientific notions). To require of reality structures that they are independent of all human action, i.e., structures that are invariant under all possible operators, would deprive reality of just the specificity necessary for being a non-trivial notation. The operator which is to define reality must be resistant against anything men can do, i.e. it must commute with all other operators. Unfortunately, only the trivial unity operator meets this requirement. A non-trivial reality can result, thus, only through being invariant under particular operations such as all the operations applied in the past up to now (rather than all possible operations). In this case reality would reflect all the perceptions and experiences men have ever had and made. This is exactly what we have in mind when we speak in ontological terms about a reality which - according toour current knowledge - has this or that structure. Reality, so to speak represents the sum total of our actual knowledge. Therefore let us call it actual reality. That reality can be defined only as actual reality, i.e., with reference to what we experienced in the past does not mean that there is a well defined remainder which we come to know somewhere in the future. What we will experience in the future which we can use to bring reality up to date depends on what qualitative extensions we may bring about - and this is an entirely open set. There is even not a guarantee that our knowledge based competence will increase on and on. If we are forced to emigrate into an new and unknown biotope it may well happen that our acquired competence is useless and that new tools und means have to be build up.

Sometimes it is argued that the absence of objective laws of nature would open the door for sheer arbitrariness. This is not the case. That a different time metric generator in our brain may replace the variables for which we identified conservation laws (energy, momentum, angular momentum, etc.) by another set of conservation variables, does not mean that we would have less problems (such as with energy provision). We just would have different ones. The laws we actually found and the categories we actually apply constitute, so to speak, the categorical reference frame we use to describe and to master our life problems. The fact that the evolution of our cognitive phenotype might well have brought about another cognitive phenotype with another cognitive reference frame (i.e. the fact that our reference frame and the laws of nature concerned are not objective) does not allow the conclusion that cognitive reference frames are per se irrelevant and therefore could be ignored. Similar applies to our organic phenotype. That we have just two legs rather than one or three is not due to an objective law of nature. It rather is a specificum of human evolution. But this does not mean that we could ignore the number of our legs when to develop walking techniques.

From this it becomes evident that what we call reality cannot be brought about by adaptation to an independently extant or ontological reality. Under these circumstances we may well ask why, then, did cognitive evolution bring about the category of reality? A possible answer to this question is that we have to immunize our perceptions against doubts and distrust, particularly in situations where quick reactions are required. This is exactly what the notion of reality does. Within our day-to-day realism we consider our perceptions as representations of what is real rather than as the outcome of deliberate cognitive interpretation. In this way, time consuming (and, therefore, possibly dangerous) considerations as to whether these interpretations could be improved on do not arise. Careful reflections on how to interpret the results of physical measurements are no doubt useful. But in view of a freely walking lion to fall into rumination on to what extent our conclusions from the reality position are relevant - this might be risky. So, reality in its quality as the sum total of all we derived from past experiences has to be taken serious and objective, and to do so whenever it is required is a very meaningful result of our cognitive evolution. Thinking in terms of reality can be regarded as a kind of "cognitive burden" incorporated during the course of cognitive evolution. I.e.: in whatever direction our cognitive evolution may proceed, reality remains an irreversible category - similar to the developmental constraints in organic evolution called "genetic burden".

When the results of new experiments contradict what has been found before we have to decide which result we trust more and, in the case given, we have to change the theory concerned. When, however, the new results can not even interpreted within the notional framework used before, i.e. when the new experimental operator and the conventional ones do not commute, then we have to change the metatheory concerned.

In parlance of realism it is said that only empirical evidence can force us to change our world picture. Sometimes, however, a metatheoretical change can be caused by purely theoretical consideration, and is by this a human ‘construct’. Even more, the evolution of theories and knowledge is generally not predictable since new results would not determine their theoretical interpretation, nor would open theoretical questions determine the experimental measures to answer them. In other words, empirical evidence does not say what theoretical interpretation it will bring about. On the other hand, new developmental lines have been created very often in the history of physical theories without any empirical ‘cause’. Fresnel's interpretation of refraction phenomena of light by means of a wave theory (1816) led to the idea of the world ether and to the Michelson experiment, from there to the theory of general relativity, to the mass-energy equivalence and from there eventually directly to modern elementary particle physics. Fresnel's decision, however, was not a logical must. Quantum mechanics has shown that neither the corpuscular nor the wave aspect of light have an ontological quality. They are rather purely theoretical concepts. Refraction phenomena do not require a wave theory. They rather can be derived directly from the quantum mechanical uncertainty principle: an atom beam of given momentum passing a slit diaphragm will not follow the geometrical path as this would mean that both momentum and future location are precisely defined - contradicting the uncertainty principle. The beam rather is refracted exactly to an amount predicted by both wave theory and quantum mechanics. This is an idea which in principle could have been derived already from the work of W. R. Hamilton (1805-65) who embedded classical mechanics formally into a kind of wave mechanics. Nobody could say where we would be today if Fresnel and his time would not have embarked on the wave theory. May be we would have neither particle physics nor nuclear energy.

8.2 Algorithmic extensions of mathematical thinking

As already mentioned, the theory of cognitive operators requires that not only the regularities we find in sensory perceptions have to be seen as invariants of certain mental operators, but also the regularities we find in logical and mathematical thinking. Particularly the laws of logic cannot be explained as universalia in the sense of Leibniz which on grounds of their truthfulness would hold in "any possible world". This view is implicitly held, for example, by Vittorio Hösle (1988) when he writes "the statement S 'there is no synthetic a priori' is obviously itself an a priori statement. So S contradicts itself and its negation, therefore, must be true". There are, of course, categories which, for phylogenetic reasons, are used by all men. Logic as a scientific discipline deals with the structures which can be constructed on this phylogenetically established basis which we later on would furnish with empirical and other theories. Konrad Lorenz (1983) speaks of our 'forms of intuition' (Anschaungsformen) which cannot be derived from any individual experience and, therefore, are *ontogenetic a prioris*, but which, however, are the outcome of evolution and so are *phylogenetic a posterioris*. What we call synthetic a priori reflects nothing but the inborn human specific ways of thinking which outside this framework cannot even be articulated. What is more, no statement at all can be articulated beforehand and outside the framework of human categories if we want to understand it. So it is impossible to find statements which could be accepted by any sufficiently complex intelligence, irrespective of its phylogenetic background and which, therefore, could be called universal. Even the question if a certain statement expressed by an intelligence A would mean the same as what another intelligence B has formulated, can be replied only if the categories of thinking of A and B can mapped on each other which is possible only on the ground of a transformation which necessarily is human specific as well. In other words: the notion of universal synthetic a prioris cannot be logically explicated. Statements dealing with the existence of universal synthetic a prioris, as advocated by Hösle, are neither false nor true. They are empty. This is well in accordance with the views of Kant, insofar as there are forms of intuition prior to any experience - but only prior to any individual experience, not prior to any phylogenetic experience. The phylogenetically accumulated experience, as represented in our picture of the world, and the categories of our thinking and perceiving are the result of an permanent co-evolution. The idea that what is a priori for the individual is a posteriori for the species was articulated already before Lorenz (1983) by Spencer (1872) and Haeckel (1902). A summary is given by Oeser (1984).

If there is really a relationship between mathematics and perception as postulated here, then the phenomenon of qualitative extensions must occur also in mathematics (Diettrich 1994b). This sounds strange, but there is some plausibility behind this idea. Similar to the operators generating sensual perception which can be extended by physical facilities, the mental operators generating our elementary mathematical conceptions can also be extended through higher and more complex mathematical calculi. This is what mathematics does as science. Insofar as the higher mathematics used is based on appropriate axioms, i.e.

(in the parlance of the TCO) on axioms which emulate correctly the cognitive operators concerned, there is no reason from the classical point of view to believe that this will lead to "non-classical" statements, i.e. to statements which can no longer be formulated within the syntax constituted by the axioms concerned. This view substantiated the confidence in Hilbert's program of the complete axiomatisation of mathematics - or, in the terms used here, the confidence that mathematics can extend itself only quantitatively.

From Gödel, however, we know (see E. Nagel, op. cit.) that there are mathematical procedures which, though entirely constructed by means of well proven classical methods, will lead to statements representing a truthfulness which can no longer be derived from the axioms concerned. Mathematics has turned out to be as incomplete as classical physics. In either case, just the application of well-tried and sound methods and procedures can lead to results which cannot be extracted from the foundations of these methods and procedures. We must therefore conclude that we cannot be sure that there will be no surprises of a similar kind in the future. Indeed: just as experimental operators, though constructed entirely according to the rules of classical physics, may lead to results which cannot be described in classical terms, there are also mathematical calculi which, as shown by Gödel, though based entirely on well tested axioms, can lead to statements which cannot be proven within the context of these axioms - and this can happen again and again. So we have qualitative extensions in physics as well as in mathematics and we can define accordingly:

We will speak of *quantitative extensions* if the truth value of the terms achieved can be derived from the axioms used.

We will speak of *qualitative extensions* if the truth value of the terms achieved cannot be derived from the axioms concerned though the calculi used are completely based on these axioms. In this case the axioms themselves have to be extended in order to make the truth value in question derivable.

Qualitative extensions, be it in physics or mathematics, are purely emergent phenomena which cannot be predicted as they, by definition, cannot be derived from previous knowledge. The blueprints of quantum mechanical devices are entirely classical in character and nothing provoked the idea that the results they may bring about could no longer be interpreted within classical theories. The same applies to mathematics. There is no general criterion telling us if a given calculus will exceed its own axiomatic basis.

With this, the existence of non-classical theories in physics and the incompleteness theorem of Gödel are homologous cognitive phenomena. Neither is there a definitive set of physical theories (i.e. a theory of everything) explaining and describing all (also future) physical problems nor is there a definitive set of mathematical axioms determining the truth value of all possible mathematical statements.

As to qualitative extensions, the only difference between the physical and the mathematical situation is that we already have in physics two non-classical theories (quantum mechanics and special relativity) and that we can say precisely under what conditions we have to apply them, namely (simply spoken) in subatomic areas and with very high speeds. In mathematics we only know from Gödel's theory that there must be non-classical phenomena, but we do not know what they are and, more particularly, we cannot say which operations would bring us out of the classical domain. Is it the notion of cardinal or ordinal numbers, or the notion of set or of infinity, or is it the combined application of these notions which constitute the cause of non-classical mathematical phenomena? Will logic turn out to be as incomplete as physics or mathematics? And what will happen if we deal with more and more powerful computers? Up to now we do not know. But when we will, we will have modern, non-classical mathematics as well as physics.

The astonishment of mathematicians with respect to Gödel's proof continues, unbroken. Literature is full of respective manifestations. Among others the explanation was proposed that the brain's action cannot be entirely algorithmic (Lucas 1961, Penrose 1989). Further to the fact that it is not quite clear what in a neural network such as the brain could be non-algorithmic, this kind of reasoning is not necessary at all. What follows from Gödel's proof is only that what certain mathematical calculi can bring about is not necessarily the same as what a combination of them could generate. Similar applies to physics: apparatus, though constructed entirely according to the laws of classical physics, would not necessarily reproduce the laws of classical physics as seen in quantum mechanics (see scattering experiments with atomic rays). But no physicist would draw from this the conclusion that something in our natural sciences could not be natural.

In contrast to physicists who suggested as an explanation for their respective experiences that they had happened to come into domains of nature where other and unpredictable laws would rule, mathematicians hesitated to admit the idea that mathematical research is empirical in the sense that it would lead to really new discoveries which by no way could have been expected, even not a posteriori. If mathematics had its own specificity at all as included in the notion of Plato's reality, then, according to general mathematical understanding, this must be something which is included in the very rudiments and which, from there, would determine all possible consequences. In other words: if there is such a thing like Plato's reality it must reveal itself by the fact that a consistent mathematics can be based only on particular, well defined axioms (the analogy to the laws of physical reality, so to speak). Once they have been found - as per Hilbert's conviction - they would settle once and for ever the "phenotype" of all future mathematics. Mathematics, then, would be nothing but a kind of craft filling up the possibilities opened by the axioms identified - similar to physics which, according to prevailing understanding could do nothing but look for the applications of the "theory of everything" once it has been found.

In the beginning it was hoped, that extending or modifying the axioms in view of the unprovable statements concerned could solve the problem. Unfortunately the new axioms would be in no better situation, as for any set of axioms unprovable statements may be found. This applies also for physics. Of course, we can modify theories according to 'unprovable' phenomena, i.e. new phenomena which cannot be formulated within the existing theories or metatheories respectively, and we did so when establishing quantum mechanics - but this will provide no guarantee that similar things will not happen again and again. So, neither in physics nor in mathematics can a 'tool for everything' be found by means of which all problems concerned, present and future, can be solved in a purely technical or formalistic manner.

The relationship between physics and mathematics as suggested by the TCO constitutes a certain heuristic balance. Experimental physics is no longer privileged in providing information from the outside world, whereas mathematics has to set it in theories. Instead, hopes are reasonable that a possibly successful study of non-classical mathematical phenomena could be a key to better understanding non-classical phenomena in physics too - and vice versa. In a way, physics and mathematics can see each other as very general theories. So, mathematics could outgrow the role of an auxiliary science, which it has hold from the outset of empirical science, into the role of an heuristic partner of equal rights. Strictly speaking, this has already happened. Of course, that we consider the world to be algorithmically compressible reflects nothing but the suitability of mathematics for prognostic purposes in physics. This is what physicists call "the unreasonable effectiveness of mathematics in the natural sciences" which, in the light of the theory of cognitive operators, might well be reasonable.

8.3. Semantic extension of language

We discussed the equivalence of language and the theories in physics and mathematics. This raises the question as to whether there is an analogue of quantitative and qualitative extensions also in language. Are there qualitative developmental tools, i.e. tools which can extend in essence the set of what can be articulated meaningfully? Schneider

(1992) distinguishes here between calculus and fantasy. Calculus refers to what can be syntactically or axiomatically formalised in language as it may be brought about by a generative grammar and which may determine the quantitative extension of language. Fantasy denotes the source of all what cannot be syntactically reduced, i.e. it denotes the qualitative extensions of linguistic competence. However, the question remains open what principles fantasy applies when extending our competence. As to the qualitative cognitive extension in physics and mathematics, the qualitative added value does not result from a continuous development but rather from the combination of existing but incompatible elements (such as particle and wave picture) towards a new functional unit (it is, so to speak, the old phrase of the whole which is more than the sum of its parts).

This suggests that also in the linguistic area qualitative extension of competence will be brought about by the combination of different and previously not compatible functional elements, i.e. by what one could call metaphorisation. We discussed already the metaphorisation of the notion of space, i.e. the transfer of notions from the context of space into notional areas which have no spatial connotation. An example is the transfer of the notion of human acting into the outside world where it is used as ‘force’. Physical forces act upon objects in a similar way as men do by means of their hands. Whenever objects change their behaviour we say that this is due to the effect of forces. Even if bodies do not change their speed we assign this to the force of inertia - as if a personalised nature ‘acts’ upon its elements. Another example is the metaphorisation of the human body and its function (well known in linguistic quarters. See ‘frontal’, in ‘view of...’, ‘to keep balance’, ‘a behaviour which looks like...’ etc.). The quantitative and qualitative extension in the sense discussed here corresponds to what some authors call syntactical and semantical metaphors respectively.

All qualitative extensions have in common that their consequences cannot be predicted. Novel and incompatible measurement results leave undecided in what kind of novel theories or world views they will be incorporated. Also the metaphorical connection of semantically incompatible elements leave open into what semantic context they will be integrated. It seems plausible that the development of language from the very beginning to its actual richness can be considered as a process of successive metaphorisation.

Quantitative linguistic extensions, by contrast, can be seen as adaptation to special requirements such as the differentiation of technical terminology or the many words for the whiteness of snow according to different meteorological conditions. What characterises qualitative extensions is that they do not result from existing requirements. They rather open the door for novel possibilities which can be realised only in semantic applications still to be identified. By this, qualitative, i.e. metaphorical extensions determine the boundary conditions for further quantitative extensions.

9. Metamathematics

Here we will deal with the question if the metatheory of mathematics is human specific and hown it could be brought into line with the TCO.

As we already mentioned, also mathematics (and logics) has to be seen as a language we use to describe certain specificities of our perception. In this respect mathematics can be as little neutral as ordinary language. Just as with language, mathematics derives its specificity from the cognitive operators which operationalise mathematical terms. So, mathematics can express only special statements. As the constituting operators are inborn and more or less equal for all men, it seems evident for us that their invariants are universal entities. Here, even more than with the categories of our perceptions, it is difficult to understand that the elementary notions of mathematical and formal thinking are purely human specifica. It is rather a very intuitive view that there is something such as a notional reality, sometimes called Platonic reality.

If we start from the suggesting idea that operators constituting the structures of mathematics and those of sensory perceptions (for phylogenetic reasons) are related to each other, then the mathematical structures and the sensuously perceived structures themselves must show similarities. This would explain why mathematics does so well in describing the regularities we perceive, i.e. why the world is algorithmically compressible: the physical world - which is the world of our perceptions - is itself, on the ground of its mental genesis, algorithmically structured. Perceived regularities and mathematical structures are phylogenetic homologa. This is the reason why the formulation of (physical) theories in terms of the mathematics we are acquainted with is an essential prerequisite for their capability to emulate the genesis of perception and, therefore, for their truthfulness. From the classical point of view (i.e. within the theory of reality) the algorithmic compressibility of the world or, what is the same, the success of induction cannot be explained.

But what, then, are the specifica which mathematics and the world of our perceptions have in common so that the two areas can consider each other as their successful theories? This is difficult to say as we have to abstract just from these specifica, what is possible only if they themselves do not belong to the most primitive elements of our thinking. The following might be a clue: to the very beginnings of our inborn ways of thinking belongs the fact hat we use the same kind of cut by means of which we separate ourselves from the outside world, we use to separate the outside world itself into single subjects to each of which we attribute an independent identity. This approach is not compulsory. Quantum mechanics shows how the entire (physical) universe can be seen as a unity which can be described by a single wave function. Each division of the universe into subsystems is a matter of the categories applied and therefore is arbitrary as phylogenetically acquired categories are not determined either. Our inborn category of identity allows us to separate systems into discernible entities. It is therefore constitutive for the notion of plural (and, therefore, for the notion of set) as well as for the notion of cardinal numbers.

A second clue is the following: It can be shown that the relatively elementary categories of moving and counting are homologue in physics and mathematics:

Let us, in analogy to Peano, generate what we call integer (ordinal) numbers denominating a certain position within a series of equal elements, roughly speaking by defining that each integer number n has just one successor n' and just one predecessor $'n$. Adding a given integer number p to n means that we have to perform the successor p times. Then, p is a cardinal number as it indicates the amount of processes to be applied rather than a topological position. Let us call $+p$ (or $-p$) the operator performing p times the successor (or predecessor) of n and by this generating the ordinal number m. Each ordinal number, then, can be understood to be generated from another one by means of a counting operator. Even when performing the direct successor as to be done in ordinary counting we have to indicate that we do this just once (rather than several or no times). Also here, "one" is first of all a cardinal number. (counting operators, so to say, tell us how far to proceed at the scale of numbers.) But where do we have to start when this is to be a procedure defining ordinal numbers? There must be a first ordinal number to which counting operators can be applied. Peano bypassed this problem by stating axiomatically that 0 is an ordinal number. From the constructivist point of view, this is hardly a solution. What we need are plausible assumptions on a mental generator which could help us to avoid axiomatic settings of ordinal numbers.

Concrete counting first of all means identifying the subjects to be counted. Moving, as we have seen, was a way of defining the identity of subjects. But how can we define or identify countable subjects if there is no movement? Here we need other operators generating countable subjects in the form of their invariants. In general, what we will declare to be a subject will depend on the operator applied. Many of the existing operators have been established phylogenetically in such a way as to have the same invariants as the moving operator has, i.e. once something is identified as an invariant of moving, it will also be identified when at rest. Generally, however, and particularly in new cognitive territories, what we may identify as a subject it is entirely open. It will be a matter of what kind of cognitive operators we apply.

This can be explained as follows: Let us start with what a certain operator has generated. This we have to consider as the domain of definition D for any further consideration or acting. D has no structures except those generated by an operator, say Z, applied to D. The result may be U_a. Z applied to U_a may lead to U_b and so on. So we will get a series U_i with i = a, b, c, ... ; Let us call Z a counting operator. It would generate the subjects to be counted as well as the order in which they will appear: Each U_i has just one successor $U_{i'}$ with $U_{i'} = Z(U_i)$. U_a is the successor of D. If there is a *j* with $U_{j'} = U_j$ i.e. if there is an element which is Eigenelement of Z and which, therefore, is its own successor, the series shall be called finite, otherwise infinite. What the U_i are (i.e. what we count) and particularly whether their series is finite or not, depends on D and Z. If D itself is eigenelement of Z, i.e. if the application of Z does not bring about anything but D itself, we say D is empty with respect to Z. So is the last Element $U_{j'}$ (if there is one): When Z has all "counted away", i.e. when Z generates nothing more, the remainder is empty with respect to Z. Let us call Z^n the repeated application of Z which will lead to U_n. If U_n is a last element, then Z^n is a projection operator, i.e. $(Z^n)^2 = Z^n$. In quantum mechanics projection operators are used to define properties of a system. By analogy, we can say here that Z_n defines a property of D with respect to Z called the cardinality of D.

When counting, we are not obliged to set D = U_0 . We can also start with D = U_m, i.e. we can start counting from *m* on. Nor is D a kind of natural starting point. As mentioned above, D itself is the outcome of an operator applied to something generated before by another operator and so on up to the hardware roots of cognition and eventually to the beginnings of life at all. The ordinal number 0, therefore, is not generally distinguished. It just indicates the level chosen as domain of definition for the counting operator in question.

Let us summarize: The beginning of numerical thinking can not be a given set of ordinal numbers M, axiomatically characterised by Peano. It is particularly impossible to define cardinal numbers by means of a metric to be set up in M. This would require the definition of a pair (*n*, *m*) of ordinal numbers, i.e., the number two (a pair) has already to be known as a cardinal number. We rather have to start with mentally defined counting operators which generate ordinal numbers. Let Z_m be the counting operator which, when applied to 0, will generate m. So Z_d will generate *d*. Z_d applied to *m* may generate *n*. So we generated three numbers, *m*, *d* and *n*, where *d*, as easily can be seen, meets all the requirements of a definition of the distance between *m* and *n*. Thus, it is counting which generates both the numbers and the metric in the set of ordinal numbers.

This seems to be the same procedure the mental apparatus uses to generate the category of the spatial metric. According to what Piaget (1970, p. 58) found with children, it is not the category of space which allows us to define motion as mapping a line in space to the scale of time. It is rather motion which generates the category of spatial structure. The most primitive intuition, as Piaget called it, (next to the notion of time) is not space but motion. Just as it is impossible to be able to come from one number to another without a counting (or equivalent) operator, we cannot distinguish points in space except by attributing them to a path of motion. Counting and moving are analogue terms within the genesis of homologous algebraic and geometrical structures. It is this homology which allows us to extrapolate the observations of motional phenomena in an empirically verifiable manner. The continuity of any physical motion for example is a cognitive phenomenon, i.e., it is part of our metatheory of physics, and not the consequence of an independent law of nature. Formulating discontinuous motions would require a spatial metric which, on the other hand, is only defined by means of the category of motion itself. Discontinuous motions, therefore, can not be realised within the human cognitive apparatus, i.e. within our metatheory of physics. By this, the degrees of freedom of actual motions are drastically reduced. The same applies for the compactness of numbers we use to establish metric spaces and (regular) analytical functions in metric spaces. Discontinuity of a set of numbers is defined only within the context of a previously defined metric. So, numbers generated by a metric defining (counting) operator are per se compact. Analytical functions in metric spaces are, therefore, born candidates to describe the phenomena of mechanics. This altogether strengthens the assumption that what Davies called the algorithmical compressibility of the world is essentially based upon functional homologies between the mental roots of perceptual and mathematical procedures.

The close relationship between spatial perception and mathematics can be seen also from another example: spatial coding of mathematical notions also from areas outside geometry is probably the very beginning of mathematical heuristics. This means the visualization of sets as a closed figures with points inside representing the set's elements as well as seeing ordered sets as spatial chains. Similar applies to the basic notions in topology such as 'exterior', 'boundary' and 'interior' points, 'isolated' points etc. Even the notion of cardinality of sets comprises a certain geometrical coding. The cardinality of sets cannot be defined operationally as the process of counting or mapping in pairs requires that the elements concerned differ at least in one property defining their identity (for example in their position with respect to the counting device). You cannot 'pick out' an element which has not a well defined geometrical position. Similar applies to the notion of plurality. That something exists in several but equal copies is plausible only if these copies differ in their spatial position.

10. Epistemological autoreproduction

Here we will deal with the question how to find possible criteria for a successful epistemology when evaluation by an independent outside reality is no longer feasible.

The difficulty we have in accepting the notional character of our experiences as human-specific constructs differs with space and with time. As to the notion of space, undoubted (except perhaps by naVve realists) is that the spatial patterns we perceive are not objective in the sense of their being considered views of real structures, i.e., the world is not necessarily as what it appears to be. Here, with space, we quite readily attribute to our world view a reduced objectivity. Not so with time: The recorded time topology of events we consider to be real. The order of events is as we have perceived it, actual. The past is as it was and even God cannot change it a posteriori, we are used to say. Weizsäcker (1985) called this the 'facticity of the past'. Actually, however, events can only be defined as the results of cognitive or scientific interpretations, just as visual patterns can only be defined as invariants of cognitive operators. Events, as such, have as less clearly defined outlines than visual patterns have. A modification of the interpretations of events used (for example, in the presence of a novel theory) may well effect the past. An experiment may have been made in the beginning of this century documenting unambiguously a speed faster than that of light. After the appearance of the theory of relativity, the protocol of the same experiment would have had to be rewritten in using the relativistic formula which would lead to a speed less than that of light. A similar revision would have to be made if evolution would have changed our cognitive operators. But because this has not happened during historical times, the illusion arose of both the facticity of the past and the objectivity of the laws of nature.

The allegation that the historicity of the world is a human specific artefact is the more problematic as it is based (through CEE) just on what is known about biological evolution, and this deals explicitly with the historical order of phylogenetic events. Said another way: on the one hand our world view is the construct of our cognitive and experimental apparatus, on the other hand, just this world view is what physics and biology refer to, particularly when describing the development of the human brain and the operators established there. What, then, is hen, and what is egg? Is it the real world we live in and which developed in the course of biotic evolution up to and including the brain's functions, or is it just these brain functions which bring about the view of a real world as a tool for both articulating and solving our problems? Formulated differently: are perceptions brought about by nature, or is nature a category brought about by our cognitive apparatus? This dichotomy is the reason for the frequent accusations which say that the EE is circular in so far as not only the categories of space, time and causality are interpreted in phylogenetic terms but also the notion of reality and nature -- the latter comprising phylogeny itself. So, phylogeny is interpreted by phylogeny, which is circular.

Actually, however, no real dichotomy exists as long as there is certainty that perceptions and nature condition one another through generating one another. This certainty is provided by the fact that our cognitive phenotype constructs a world picture which permits an understanding of the genesis of just this cognitive phenotype by means of evolution within the framework of just this world picture. In other words: a world picture brought about by human brains has to explain everything from the big bang, the creation of our world, organic and then cognitive evolution and eventually the development of the world picture itself. I.e., the cognitive phenotype has to reproduce itself in the same sense the organic phenotype has to do. A cognitive phenotype (or a world picture) which meets this requirement we will call consistent. Accordingly, further to the special cognitive phenotype we have acquired, there is an unlimited number of possible (and consistent) cognitive species similar to the many existing or possible organic phenotypes (i.e. species). Also biotic organisms are not required to be 'true' but rather to reproduce, i.e. (as biologists are used to say) to contribute to the survival of the genes concerned. Thus, not only organic ontogenesis but also cognitive evolution has to be understood as circular, autoreproductive process in the subsequent sense:

In the biotic area the following holds: the epigenetic system of an organism is what determines how the genome's structure is to be interpreted and expressed into the phenotype. Identical reproduction is possible, however, only if the epigenetic system brings about a phenotype comprising the epigenetic system itself.

In the cognitive area the following holds: the cognitive apparatus (and all the science based on it) is what decides how the sensory input is to be interpreted and which world view will be conveyed. The knowledge acquired in this manner is consistent and reproducible, however, only if the cognitive/scientific apparatus generates a world view that includes the cognitive/scientific apparatus itself.

A genome on its own cannot determine the phenotype in the sense of providing a 'blueprint' - it rather represents one of several levels in the process of autoreproduction - nor can the sensory input dictate its own interpretation, and, by this, the reactions it will effect. This limitation does not contradict the fact that, within the context of a given organic or cognitive phenotype having a given interpretative machinery, a genetic mutation as well as a new perception may lead to reproducible modifications of our physical constitution or of our theories. This means that, as long as the epigenetic system remains unmodified, a given genetic mutation will always produce the same phenotypic change; and as long as our cognitive apparatus and our scientific theories also remain unmodified, a given sensorial input will always lead to the same reading. What we have to avoid, however, is concluding that what mutations and perceptions initiate is also what they determine. Determinism is possible only within a given scheme of interpretations, i.e., outside qualitative extensions changing the interpretation concerned. The same limitations hold for adaptation. Adaptation makes sense only as long as there are no qualitative extensions as these will modify the requirements to be met, i.e. the selective pressure. The world seen as the sum total of the boundary conditions of our acting is subject to a permanent actualization, as acting aims at changing just these conditions in order to make further and more ample acting feasible. This begins with the organic phenotype which defines the constraints for evolutionary 'acting', which in turn changes the constraints for further evolution (evolution meaning the evolution of its own boundary conditions). And it ends with the cognitive phenotype that defines, through our world view, which kind of scientific acting is possible due to which the world view itself may be affected -- a paradigmatic shift in the sense of Kuhn, so to speak. The world as object of adaptation can be defined only for the time between two 'paradigmatic changes', i.e., between two qualitative extensions.

Circularity, a devastating objection for any theory within the context of classical realism, becomes (in the sense explained here) a necessary prerequisite for any complete constructivist approach. A world view brought about by a cognitive phenotype is consistent if and only if the world concerned enables the genesis of the cognitive phenotype. The role of circularity constitutes the key difference between realism (of whatever kind) and constructivism as presented here. Realism requires of life mastering methods consistency with an independent outside world. A constructivist interpretation of the world as proposed here, however, needs only to reconstruct itself. The various epistemologies mentioned here can be explained more clearly when they are classified according to how they meet their functional requirements:

(a) *Structural approaches*. The most elementary position taken is that cognitive constructs (perceptions) have to delineate correctly the structures of the environment, since the strategies devised to meet the requirements of the environment are believed to be derivable from those structures. This is the basis for most kinds of *realism*. Physical knowledge is reliable (i.e., it allows verifiable predictions) if and only if it is 'true', i.e. if it is derived from perceptions and their 'true' theoretical interpretations. Both perceptions and true theories are seen to depend on the structure of an external world. Knowledge when being true is irreversible, additive and converges towards a complete and definitive set of laws of nature. The progress of knowledge is based on inductive inference. The success of induction cannot be explained in the structural picture. If epistemology is seen as a matter of cognitive evolution it is understood that acquired mental categories (reality, space, time,...) depict ontological categories.

(b) *Functional approaches*. Cognitive constructs have to contribute to meeting the requirements of the environment, but not necessarily by means of delineating environmental structures but rather functionally. The notion of 'truth' (in the structural picture) corresponds to the notion of 'viability'(in the functional picture) (see E. von Glasersfeld, 1995, p. 22). Physical knowledge is reliable (i.e., it allows verifiable predictions) if it is derived from perceptions (or phenomena) which depend on an external world and their interpretation by means of theories which no longer must be true but viable. The progress of knowledge is based on inductive inference. (What succeeded in the past will also succeed in the future). The success of induction cannot be explained in the functional picture (A.F.Chalmers 1982). If epistemology is seen as a matter of cognitive evolution it is understood as 'natural selection epistemology'(D.T. Campbell, 1973).

(c) *Constructivist approaches in the sense of CEE*. Physical knowledge is reliable (i.e., it allows verifiable predictions) if it is derived from perceptions and their appropriate interpretation, but neither perceptions nor their (viable) interpretations need the evaluation by an external world. The most elementary prediction, i.e. prediction by means of linear extrapolation, is possible only if the development in question is linear in time. For this it is sufficient and necessary that the phylogenetically acquired observational terms are cyclic variables (in the sens of Hamilton-Jacobi) with respect to the elementary human action operators. This resulted in the metatheory of classical physics, i.e. in the mental notion of time, space, spatial identity, locomotion, momentum etc. and to the conservation laws of classical mechanics. More sophisticated actions (particularly qualitative extensions) require 'non-classical views', i.e., a redefinition of our notional reference frame, i.e., a redefinition of what we consider to be an observation or a phenomenon with a direct effect on what we call the laws of nature. The structure of our perceptional world therefore will depend on what men can do by natural or technical means and it will change according to possible qualitative extensions brought about by novel experimental development. Knowledge is irreversible, additive and convergent only within quantitative extensions. Outside it will depend on what non-classical metatheory we will use to respond to the qualitative extension concerned. The

progress of knowledge within quantitative extensions is based on inductive inference. Induction succeeds because and as long as we describe the ‘world’ in terms of cyclical variables. An epistemology comprising the notion of time and development in time is consistent if it can explain its own genesis.

11. Conclusion

The relationship between physics and mathematics as suggested by the theory of cognitive operators (TCO) (i.e. by what is embedded in the constructivist evolutionary epistemology, CEE) constitutes a certain heuristic balance. Experimental physics is no longer privileged in providing information from the outside world, whereas mathematics has to set it in theories. Instead, hopes are reasonable that a possibly successful study of non-classical mathematical phenomena could be a key to better understanding non-classical phenomena in physics too - and vice versa. Mathematics, then, would not only help us to extrapolate successfully physical data; it also could contribute to the conception of novel physical theories as was already the case with Dirac: he found in his theory for purely mathematical reasons that elementary particle could have negative energies. Instead of rejecting this unbelievable result he postulated the existence of antiparticles having negative energies which after a while really have been detected. In a way, physics and mathematics can see each other as very general theories. So, mathematics could outgrow the role of an auxiliary science, which it has hold from the outset of empirical science, into the role of an heuristic partner of equal rights. Strictly speaking, this has already happened. Of course, that we consider the world to be algorithmicly compressible reflects nothing but the suitability of mathematics for prognostic purposes in physics. This is what physicists call "the unreasonable effectiveness of mathematics in the natural sciences" which, in the light of CEE, might well be reasonable.

The notion of truth provides a measure for how much a theory, a statement or a proposition meet the requirements of an objective (physical or mathematical) reality. Reality, however, can only be defined as actual reality within the context of a given metatheory. Actual reality can be considered as synonym for all experiences made by men and all knowledge acquired up to now. The problem is, that future knowledge will be an additive completion of past knowledge only if no qualitative extensions are involved. As soon as qualitative extensions will occur, past knowledge itself has to be rewritten in the light of a new metatheory. This is the very reason why knowledge never will converge towards a theory of everything. In so far the notion of reality is a constituent for the notion of truth, and as the notion of rality has to be replaced by the notion of actual reality, it follows that truth can no longer be used as an evaluation criterion for theories, except with respect to a given metatheory.

Within the CEE, truth as an evaluation criterion for theories has been replaced by the requirement that theories have to contribute to an reproducing cognitive phenotype, i.e. a phenotype which is able to explain its own genesis - in the same sense as biotic organs have to contribute to the reproduction skills of their organisms. The fact that there is no absolute truth which would apply in all metatheories corresponds to the fact that there are no unspecific organs which could do their duty in all kinds of organisms. From the functional point of view the requirement of reproduction does as well as the requirement of truth does.

The most promising effort of the constructivist approach proposed here aims at a coherent description of organic, cognitive and scientific evolution. Let us summarise the results in a synoptic view of the various notions in organic and cognitive evolution:

Organic Evolution:	**Cognitive evolution:**
Ortganic structures are invariants of the reproducing operator (epigenetic expression of genetic structures). A genetic mutation	The structures of perceptions are defined as invariants of physical operators (represented by physical or organic devices) or of

may bring about (I) a certain phenotypic modification in a reversible and unambiguous manner (defining mutations) or (II) it may interact with the epigentic system and by this may induce a sequence of non identical reproductions. Whether I or II will occur (i.e., whether there will be a linear or a non-linear genetic effect) depends on whether the genetic and the epigenetic operators will commute or not.	cognitive operators (implemented in our brain). Whether the other operators we apply are measuring or acting operators depends on whether they will commute with the defining operators or not.
It is wide-spread understanding that organic evolution has to meet the requirements of the environment (Adaptation).	It is wide-spread understanding that cognitive evolution aims at identifying laws of nature to which we condense the perceived regularities in order to allow predictions (*Adaptation*).
Accordingly, only appropriate adaptation will be honoured by natural selection.	Accordingly, only when the laws of nature concerned are sufficiently known, predicting (and, therefore, meaningful planning) is possible.
On the other hand, evolution brings about acting skills and tools in order to modify the (external) environment in order to meet better the requirements of the organism. Novel actions are called qualitative if their effect can no longer be mastered by established means but require novel tools and strategies (e.g.: emigration from aquatic to terrestrial environment requires new limb structures).	On the other hand, scientific evolution may extend the inborn cognitive operators by means of measurement operators in order to improve our knowledge of the laws constituting the (external) nature. These extensions will be called qualitative if they require novel, non-classical laws defining a different nature (*Assimilation*).
So, the discrepancy between the organism and the environment can be reduced from both ends, by Adaptation, (internal solution) and by Assimilation (external solution).	So, deficiencies in predicting can be reduced by improving our knowledge within the framework of established natural laws, i.e. by *Adaptation* (*internal solution*) as well as by modifying the laws representing the world by means of non-classical modifications of the world view. *Assimilation* (*external solution*).
Actions can be realised only when based on the action operators given or their combinations.	*Predictions can be realised explicitly only when expressible in terms of variables which are cyclic (and therefore linear in time) with respect to the operators ruling the system.*
The higher organisms have evolved, the more they refer to assimilation. (Man, when confronted with physical problems, counts exclusively on external solutions, rather than to wait that evolution will provide him with better *adaptation*).	The higher organisms have evolved, the more they refer to cognitive *assimilation*. Men, when confronted with qualitative physical extensions, would respond by non-classical modifications of the world view rather than by expensive *adaptation* of conventional theories such as D. Bohm's 'hidden variables' designed to keep quantum

	mechanics classical). Primates constructed a 3-diemsional world for the management of locomotion processes rather than a 2- or even 1-dimensional world as invented by kinesthetic animals
As evolving species modify their environment the selection power will not be static. Evolution, therefore, will not converge towards a definitivum or optimum - (No definitive species of universal competence) - the pride of creation so to say. (No completion of organic evolution)	As qualitative extensions of inborn or otherwise established cognitive operators can never be excluded, there will be no definitive set of *theories (No theory of everything*). From applying this concept to the inborn mental operators generating mathematical thinking and their algorithmic extensions it follows that there will be *no definitive set of axioms* as shown by Gödel. (*No completion of cognitive evolution*).
There is no objective acting strategy: whether walking, running, crawling, swimming or flying is best for locomotion depends on the organic phenotype.	There is no objective theory. What a theory describes and how it does depends on the inborn (or scientifically extended) notional reference frame, i.e. on the cognitive phenotype.
An organic phenotype (and the action tools comprised) cannot be said to be"true" or "false". When to survive it must reproduce.	A cognitive phenotype (and the theories comprised) cannot be said to be "*true*" or "false". It rather has to *reproduce*, i.e. it must be able to explain its own genesis.
Organisms from different species cannot communicate genetically i.e. interbreed.	Different cognitive phenotypes cannot communicate *semantically* (e.g. verbally)
Genetic mutations, once they passed all internal (epigenetic) and external selection mechanisms (of the physical and the social environment) can be transferred directly to the next generation.	*Scientific ideas*, once they passed all selections by existing knowledge, empirical evaluation and by the scientific community, *can be transferred directly to the next generation. There is no reason to say that organic evolution is Darwinian whereas cultural/scientific evolution is Lamarckian in character*.

Let us conclude with the question as to what all this may be good for. As this is mainly a metatheoretical treatise empirically verifiable facts are not involved. But scientific goals are often based on certain metatheoretical assumption. The many efforts, e.g., undertaken to find an operational definition for the arrow of time are based on the conjecture that the arrow of time has an ontological quality. If the arguments used here were accepted, the research in the field has to be redefined.

The price, however, we have to pay for these and other findings is high. We have to accept

- that there is no 'natural selection epistemology' (in the sense of Campbell);
- that the laws of nature are phylogenetically acquired human-specific artifacts;

- and, therefore, that there will be no reality in the sense of objective laws of nature
- that there will be no complete set of physical theories ('theory of everything') and
- that there will be no meaningful context-free communication (as necessary, for example, when contacting extraterrestrial beings).

On the other side, some explanations and possibilities are offered that could hardly have been expected from outside the CEE:

- the incompleteness of physical theories and the incompleteness of mathematical axioms discerned by Kurt Gödel have the same cognitive roots;
- the algorithmic compressibility of the world (which is equivalent to the success of induction) is due to the homology of (inborn) cognitive mechanisms and the (inborn) mechanisms of mathematical thinking.
- the phylogenetically based homology of mathematical and perceived structures makes mathematical and physical research heuristically relevant to each other.

Of particular interest is the link between organic and cognitive tools: if cognitive tools are to describe the covariants of an operator (i.e., what the operator effects), they have to be designed in terms of the operator's invariants (a principle which has been re-invented by physicists within Hamilton-Jacobi and quantum mechanical formalism). Cognitive evolution (including the epistemology here applied), therefore, is brought about by organic evolution (and by the evolution of experimental tools) rather than by trial and error and selection. And, vice versa, organic (also experimental) evolution is guided by the possibilities provided by cognitive tools.

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