Three-dimensional gravity and logarithmic conformal field theories

INSTITUTE FOR THEORETICAL PHYSICS

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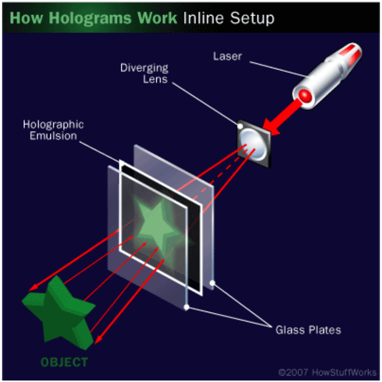
- CFTs degenerate to logarithmic conformal field theories (LCFT)
- LCFTs appear in condensed matter physics
  they describe systems with quenched disorder
  (spin glasses, quenched random magnets, percolation)

Outline

- Holography & AdS/CFT correspondence
- logarithmic CFTs
  - logarithmic pair
  - two-point correlators
- Three dimensional gravity
  - models of 3d gravity
  - evidence for gravity duals to LCFTs
- conclusion and outlook



- the number of dimensions is a matter of perspective
- physical systems can be described in different dimensions
- theory with less (two) dimensions is a theory without gravity
- theory with more (three) dimensions is a theory with gravity

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  - on the field theory side (CFT)
- calculations on the CFT side are easier
- the gravity side tells us what to expect,
  how to interpret the solutions

AdS/CFT – anti de Sitter spacetimes

- anti de Sitter spacetimes are maximally symmetric spacetimes
- in three dimensions

$$\bar{g}_{\mu\nu}^{\mathsf{AdS}}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = \ell^2 \left[-\cosh^2\rho\, \mathrm{d}\tau^2 + \sinh^2\rho\, \mathrm{d}\phi^2 + \mathrm{d}\rho^2 \right] \tag{1}$$

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- theories of gravity that we consider have AdS solutions
- we look for perturbations around AdS, i.e.

$$g_{\mu\nu} = \bar{g}_{\mu\nu}^{\text{AdS}} + \psi_{\mu\nu} \tag{2}$$

for some small ψ.

AdS/CFT – linearize around AdS

- consider higher curvature gravity with action

$$S_{\text{bulk}} = \int \mathrm{d}^3x \sqrt{-g}\,\{\,R - 2\Lambda + \mathcal{O}(R^2)\,\} \tag{3}$$

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- pure gravity (R) yields two solutions ψ^L and ψ^R

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- calculating correlators in this way is quite lengthy!

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- two-point correlators depend only on the conformal weights and a normalization constant

$$\left\langle \mathcal{O}^m(z, \bar{z})\, \mathcal{O}^m(0,0) \right\rangle = \frac{c_m}{2 z^{2h(m)} \bar{z}^{2\bar{h}(m)}} \tag{5}$$

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- special (holomorphic) function with $h = 2$ and $\bar{h} = 0$
- their two-point functions yield the central charges

$$\left\langle \mathcal{O}^L(z)\, \mathcal{O}^L(0) \right\rangle = \frac{c_L}{2z^4} \tag{6}$$

$$\left\langle \mathcal{O}^R(\bar{z})\, \mathcal{O}^R(0) \right\rangle = \frac{c_R}{2\bar{z}^4} \tag{7}$$

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- two-point functions are given by constants (central charges)

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- $\mathcal{O}^{\log}$ logarithmic partner to $\mathcal{O}$

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$$\langle \mathcal{O}(z)\, \mathcal{O}^{\log}(0,0) \rangle = \frac{b}{2z^{2h}}$$

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- obtain equations of motion

$$G^{(1)}_{\mu\nu}(\psi) = 0 \qquad (D^L D^R \psi)_{\mu\nu} = 0 \tag{12}$$

$$(D^{L/R})_\mu{}^\nu = \delta_\mu{}^\nu \pm \ell \varepsilon_\mu{}^{\alpha\nu} \nabla_\alpha \tag{13}$$

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- $c_{L/R}$ can not be zero
- for possible LCFT duals we need more metric modes so that the operators on the CFT side can degenerate
- $\rightarrow$ need higher derivative gravity theories

Topologically Massive Gravity (TMG)

$$S_{TMG} = \frac{1}{\kappa}\int \mathrm{d}^3x\sqrt{-g}\,\left\{R - \frac{2}{\ell^2} + \frac{1}{2\mu}\varepsilon^{\lambda\mu\nu}\Gamma^{\alpha}_{\lambda\sigma}\left[\partial_\mu\Gamma^{\sigma}_{\alpha\nu} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\alpha}\right]\right\} \tag{14}$$

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- third order in $\partial g_{\mu\nu}$
- not parity invariant
- linearize around AdS ($g = \bar{g} + \psi$)
- obtain EOMs

$$(D^{\mu} D^{L} D^{R}\psi)_{\mu\nu} = 0 \tag{15}$$

$$(D^{\mu})_{\mu}{}^{\nu} = \delta_{\mu}{}^{\nu} + \frac{1}{\mu}\varepsilon_{\mu}{}^{\alpha\nu}\nabla_{\alpha} \tag{16}$$

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  - c_L vanishes

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- instead of ψ^m we get another solution

$$D^L D^L \psi^{\log} = -2 D^L \psi^L = 0 \tag{17}$$

$$\psi^{\log} = \lim_{m \rightarrow m_{\text{crit.}}} \frac{\mathrm{d}}{\mathrm{d}m} \psi^m(m) \tag{18}$$

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$$H \begin{pmatrix} \psi^{\log} \\ \psi^{L} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\log} \\ \psi^{L} \end{pmatrix} \tag{19}$$

$$J \begin{pmatrix} \psi^{\log} \\ \psi^{L} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\log} \\ \psi^{L} \end{pmatrix} \tag{20}$$

with $H = L_0 + \bar{L}_0$ and $J = L_0 - \bar{L}_0$.

TMG – LCFT

- at the critical point $\mu\ell = 1$
- Jordan cell appears
- two-point correlators match

$$\left\langle \mathcal{O}^L(z)\, \mathcal{O}^L(0) \right\rangle = 0 \tag{19}$$

$$\left\langle \mathcal{O}^L(z)\, \mathcal{O}^{\log}(0,0) \right\rangle = \frac{b}{2z^4} \tag{20}$$

$$\left\langle \mathcal{O}^{\log}(z,\bar{z})\, \mathcal{O}^{\log}(0,0) \right\rangle = -\frac{b \log(|z|^2)}{z^4} \tag{21}$$

with $b = -\frac{3\ell}{G}$.

TMG – LCFT

- at the critical point $\mu\ell = 1$
- Jordan cell appears
- two-point correlators match
- partition function matches

$$Z_{\mathrm{TMG}} = \prod_{n=2}^{\infty} \frac{1}{|1-q^n|^2} \prod_{m=2}^{\infty} \prod_{\bar{m}=0}^{\infty} \frac{1}{1-q^m \bar{q}^{\bar{m}}} \tag{19}$$

$$Z^0_{\mathrm{LCFT}} = \prod_{n=2}^{\infty} \frac{1}{|1-q^n|^2} \left(1 + \frac{q^2}{|1-q|^2}\right) \tag{20}$$

New Massive Gravity (NMG)

$$S_{NMG} = \frac{1}{\kappa} \int \mathrm{d}^3 x \sqrt{-g} \left\{ \sigma R - 2\lambda m^2 + \frac{1}{m^2} \left(R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2 \right) \right\} \quad (21)$$

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- fourth order in $\partial g_{\mu\nu}$
- linearization yields

$$(D^{+m} D^{-m} D^L D^R \psi)_{\mu\nu} = 0 \quad (22)$$

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$$(D^{m_1} D^{m_2} D^L D^R \psi)_{\mu\nu} = 0 \qquad (24)$$

$$\ell\, m_{1,2} = \frac{m^2 \ell^2}{2\mu\ell} \pm \sqrt{\frac{m^4 \ell^4}{4\mu^2 \ell^2} - \sigma m^2 \ell^2 + \frac{1}{2}} \qquad (25)$$

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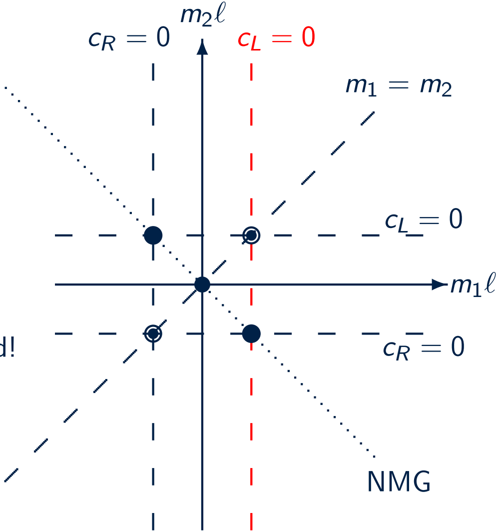
  - massive modes degenerate with each other but not with $\psi^{L/R}$

GMG – parameter space

- $D^{m_1} = D^L$:
  $T(z)$ has log-partner!
- $D^{m_1} = D^{m_2}$:
  $\mathcal{O}^M$ has log-partner!
- $D^{m_1} = D^{m_2} = D^L$:
  Rank three Jordan cell!
- $c_L = c_R = 0$: log-NMG
  Both $T(z)$ and $\bar{T}(\bar{z})$ logged!
- PMG! $c_L = c_R \neq 0$
  Enhanced gauge symmetry!



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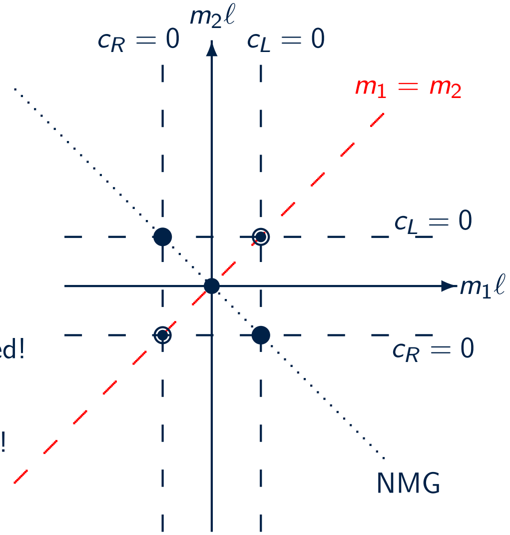

GMG – rank two LCFT dual

$$H\begin{pmatrix}\mathcal{O}^{\log}\\ \mathcal{O}^{L}\end{pmatrix} = \begin{pmatrix}2 & 1\\ 0 & 2\end{pmatrix}\begin{pmatrix}\mathcal{O}^{\log}\\ \mathcal{O}^{L}\end{pmatrix} \quad \text{and} \quad J\begin{pmatrix}\mathcal{O}^{\log}\\ \mathcal{O}^{L}\end{pmatrix} = \begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}\mathcal{O}^{\log}\\ \mathcal{O}^{L}\end{pmatrix}$$

$$\langle \mathcal{O}^{L}(z)\, \mathcal{O}^{L}(0)\rangle = 0$$

$$\langle \mathcal{O}^{L}(z)\, \mathcal{O}^{\log}(0,0)\rangle = \frac{b_L}{2z^4}$$

$$\langle \mathcal{O}^{\log}(z,\bar{z})\, \mathcal{O}^{\log}(0,0)\rangle = -\frac{b_L\ \log|z|^2}{z^4}$$

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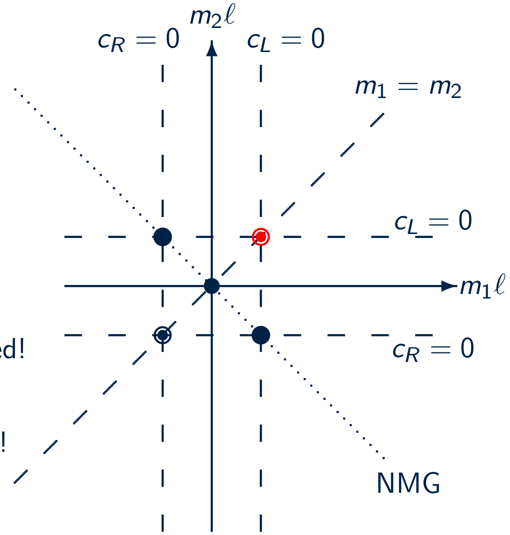

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and $diag(2,2,2)$ for J.

$$\left\langle \mathcal{O}^L(z)\, \mathcal{O}^L(0) \right\rangle = \left\langle \mathcal{O}^L(z)\, \mathcal{O}^{\log}(0,0) \right\rangle = 0$$

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LCFT duals to GMG

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  - in the log TMG and log NMG limits
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- each LCFT has another value for the new anomaly
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- is it possible to relate the new anomalies to each other – via degeneration of parameters?

$$c_L \longrightarrow b_L \longrightarrow a_L \qquad (26)$$

GMG – possible LCFT duals and their anomalies

We find for GMG

- a 'standard' rank two LCFT when $\mathcal{O}^{m_{1,2}} \longrightarrow \mathcal{O}^{L}$

  with new anomaly $b_L = \frac{6\ell\sigma}{G}\frac{1-m_{2,1}\ell}{1+2m_{2,1}\ell}$.

- a 'standard' rank three LCFT when $\mathcal{O}^{m_1} \longrightarrow \mathcal{O}^{m_2} \longrightarrow \mathcal{O}^{L}$

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- an 'exotic' rank two LCFT when $\mathcal{O}^{m_1} \longrightarrow \mathcal{O}^{m_2}$

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Thank you for your attention!