Universally reversible $JC^*$-triples and operator spaces

Richard M. Timoney

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Based on


A (concrete) $JC^*$-triple is a closed $E \subseteq A$ ($A$ $C^*$-algebra) such that

$$a, b, c \in E \Rightarrow \{a, b, c\} \overset{\text{def}}{=} \frac{1}{2}(ab^*c + cb^*a) \in E$$

**Examples:** $E = A$, $E = \ell_2^d$ row or column Hilbert space in $M_d(\mathbb{C})$. These are all TROs ($a, b, c \in E \Rightarrow [a, b, c] \overset{\text{def}}{=} ab^*c \in E$).

$E = \{x \in M_d(\mathbb{C}) : x^t = x\}$. More generally $E$ a $JC^*$-algebra ($a, b \in E \Rightarrow a^*, (ab + ba)/2 \in E$).

**Theorem** (W. Kaup 1984) $P: A \to A$ a linear projection of norm 1 \Rightarrow $E = P(A)$ is (isometric to) a $JB^*$-triple with triple product $\{a, b, c\}_P = P(\{a, b, c\})$.

Arazy-Friedman (1978): For $A = \mathcal{K}(H)$, described $P(A)$.
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**Theorem.** Given an abstract $JC^*$-triple $E$, there exists a universal (largest) TRO $T^*(E)$ generated by $E$.

More precisely, there exists an isometric embedding $E \xhookrightarrow{\alpha_E} T^*(E)$ onto a sub-$JC^*$-triple of a TRO $T^*(E)$ with the universal property

\[
\begin{array}{ccc}
T^*(E) & \xrightarrow{\pi} & T \\
\uparrow{\alpha_E} & \swarrow{\tilde{\pi}} & \searrow{\pi} \\
E & \xrightarrow{\pi} & T
\end{array}
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where $\pi: E \to T$ is any given triple morphism (i.e. $\pi\{a, b, c\} = \{\pi(a), \pi(b), \pi(c)\}$) with values in a TRO $T$, and $\tilde{\pi}: T^*(E) \to T$ is a TRO morphism (meaning $\tilde{\pi}[x, y, z] = [\tilde{\pi}(x), \tilde{\pi}(y), \tilde{\pi}(z)]$, $\tilde{\pi}$ unique given $\pi$.)
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\downarrow & \downarrow & \downarrow \\
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Consider $E \subset A$ a $JC^*$-triple also as an operator space.

**Corollary.** There exists a TRO ideal $\mathcal{I} \subset T^*(E)$ with $\mathcal{I} \cap \alpha_E(E) = \{0\}$ such that $E$ is completely isometric to $E_\mathcal{I}$, the operator space structure on $E$ determined by the isometric embedding $E \to T^*(E)/\mathcal{I}$ ($x \mapsto \alpha_E(x) + \mathcal{I}$)

$$
\begin{array}{c}
T^*(E) \\
\alpha_E \\
E \\
\pi \\
\end{array} \xleftarrow{\tilde{\pi}} \xrightarrow{\pi} \text{TRO}(E)
$$

Take $\mathcal{I} = \ker \tilde{\pi}$. (We call such $\mathcal{I}$ operator space ideals of $T^*(E)$).

We have a surjective map from such ideals $\mathcal{I}$ to $JC$-operator space structures on $E$.
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\[ \xymatrix{ & T^*(E) \ar[dl]_{\alpha_E} \ar[dr]^\tilde{\pi} & \\
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$T^*(E)$

$\alpha_E$

$\tilde{\pi}$

$\mathcal{I}$

$\pi$

$E \rightarrow TRO(E)$

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A (concrete) $JC^*$-triple $E \subseteq A$ is called reversible if

$$n \geq 2, a_1, \ldots, a_{2n+1} \in E \Rightarrow a_1 a_2^* a_3 \cdots a_{2n}^* a_{2n+1} + a_{2n+1} a_{2n}^* \cdots a_3 a_2^* a_1 \in E$$

$E$ is called universally reversible if $\pi(E)$ is reversible for all triple homomorphisms $\pi : E \to A$.

It is known that $E = A = aC$ algebra considered as a $JC^*$-triple is universally reversible, but $\ell_3^2$ is not.

$\ell_3^2$ has isometric representations as TROs (row or column in $M_3$) — clearly reversible — but also has a representation as annihilation operators in $B(\Lambda^2 \ell_3^2, \Lambda^1 \ell_3^2)$ which is not reversible.
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It is known that $E = A = a C^*$-algebra considered as a $JC^*$-triple is universally reversible, but $\ell^2_3$ is not.

$\ell^2_3$ has isometric representations as TROs (row or column in $M_3$) — clearly reversible — but also has a representation as annihilation operators in $B(\Lambda^2 \ell^2_3, \Lambda^1 \ell^2_3)$ which is not reversible.
Reversible subtriples

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$$n \geq 2, a_1, \ldots, a_{2n+1} \in E \Rightarrow a_1a_2^*a_3 \cdots a_{2n}^*a_{2n+1} + a_{2n+1}a_{2n}^* \cdots a_3a_2^*a_1 \in E$$

$E$ is called **universally reversible** if $\pi(E)$ is reversible for all triple homomorphisms $\pi : E \to A$.

It is known that $E = A = \mathbb{C}$ is a $C^*$-algebra considered as a $JC^*$-triple is universally reversible, but $\ell_3^2$ is not.

$\ell_3^2$ has isometric representations as TROs (row or column in $M_3$) — clearly reversible — but also has a representation as annihilation operators in $B(\Lambda^2 \ell_3^2, \Lambda^1 \ell_3^2)$ which is not reversible.
A (concrete) $JC^*$-triple $E \subseteq A$ is called reversible if
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It is known that $E = A = a_1C^*$ algebra considered as a $JC^*$-triple is universally reversible, but $\ell_3^2$ is not.

$\ell_3^2$ has isometric representations as TROs (row or column in $M_3$) — clearly reversible — but also has a representation as annihilation operators in $B(\Lambda^2 \ell_3^2, \Lambda^1 \ell_3^2)$ which is not reversible.
Suppose $E$ is a universally reversible abstract $JC^*$-triple [linear isometric class].

Then $\mathcal{I} \mapsto E_{\mathcal{I}}$ is a bijective correspondence between the operator space ideals of $T^*(E)$ and the $JC$-operator space structures of $E$ (operator space structures induced by linear isometries onto concrete $JC^*$-subtriples of some $A$).

If in addition $E$ has no ideals linearly isometric to a nonabelian TRO, then the only operator space ideal of $T^*(E)$ is $\mathcal{I} = \{0\}$. 

R. Timoney
Universally reversible $JC^*$-triples and operator spaces
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