Localization properties of the weighted frames
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Abstract

The frames localized with respect to the Riesz basis and the frames self-localized are considered with semi-normalized weights put on atoms. The preservation of the localization properties of the frame and its dual frame under the applied weights is verified.

Frames $\mathcal{G} = \{g_i\}_{i \in I}$ generalize the idea of a basis in a Hilbert space $\mathcal{H}$ and consist of the indexed families such that the so-called frame operator $S f = \sum_{i \in I} \langle f, g_i \rangle g_i$ is invertible. Hence every element $f \in \mathcal{H}$ has an expansion of the form

$$f = S^{-1} f = \sum_{i \in I} \langle S^{-1} f, g_i \rangle g_i = \sum_{i \in I} f_i g_i,$$

where $f_i = \langle S^{-1} f, g_i \rangle$. The family $\{S^{-1} f_i\}_{i \in I}$ is again a frame and is called the canonical dual frame.

Localization properties of the weighted frames

Definition 1. A countable set $\mathcal{G} = \{g_i\}_{i \in I} \subseteq \mathcal{H}$ is self-localized if its Gramian matrix with entries $\langle g_i, g_j \rangle_{i,j \in I}$ has a prescribed off-diagonal decay in $|i - j|$, which in the case of polynomial decay gives the following condition for some $s > d$

$$|\langle g_i, g_j \rangle| \leq C |1 + |i - j||^{-s} \quad \forall i, j \in I.$$  

Theorem 1. If $\mathcal{G}$ is localized with respect to some Riesz basis, then $\mathcal{G}$ is self-localized.

References