Chapter 7

Production, Capacity and Material Planning
Production, Capacity and Material Planning

- **Production plan**
  - quantities of final product, subassemblies, parts needed at distinct points in time

- **To generate the Production plan we need:**
  - end-product demand forecasts
  - Master production schedule

- **Master production schedule (MPS)**
  - delivery plan for the manufacturing organization
  - exact amounts and delivery timings for each end product
  - accounts for manufacturing constraints and final goods inventory
Production, Capacity and Material Planning

Based on the MPS:

- rough-cut capacity planning

- Material requirements planning
  - determines material requirements and timings for each phase of production
  - detailed capacity planning
Production, Capacity and Material Planning

- End-Item Demand Estimate
- Master Production Schedule (MPS)
- Rough-Cut Capacity Planning
- Detailed Capacity Planning
- Material Requirements Planning (MRP)
- Material Plan
- Purchasing Plan
- Shop Orders
- Shop Floor Control

Updates
Master Production Scheduling

- **Aggregate plan**
- **demand estimates for individual end-items**
- **demand estimates vs. MPS**
  - inventory
  - capacity constraints
  - availability of material
  - production lead time
  - ...
- **Market environments**
  - make-to-stock (MTS)
  - make-to-order (MTO)
  - assemble-to-order (ATO)
Master Production Scheduling

**MTS**
- produces in batches
- minimizes customer delivery times at the expense of holding finished-goods inventory
- MPS is performed at the end-item level
- production starts before demand is known precisely
- small number of end-items, large number of raw-material items

**MTO**
- no finished-goods inventory
- customer orders are backlogged
- MPS is order driven, consists of firm delivery dates
Master Production Scheduling

- **ATO**
  - large number of end-items are assembled from a relatively small set of standard subassemblies, or modules
  - automobile industry
  - MPS governs production of modules (forecast driven)
  - Final Assembly Schedule (FAS) at the end-item level (order driven)
  - 2 lead times, for consumer orders only FAS lead time relevant
Master Production Scheduling

- MPS - SIBUL manufactures phones
  - three desktop models A, B, C
  - one wall telephone D
  - MPS is equal to the demand forecast for each model

<table>
<thead>
<tr>
<th>WEEKLY MPS (= FORECAST)</th>
<th>Jan</th>
<th>Feb</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Week</td>
<td>Week</td>
</tr>
<tr>
<td>Product</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Model B</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Model C</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>Model D</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>weekly total</td>
<td>3100</td>
<td>3000</td>
</tr>
<tr>
<td>monthly total</td>
<td>12200</td>
<td></td>
</tr>
</tbody>
</table>
Master Production Scheduling

MPS Planning - Example

- MPS plan for model A of the previous example:
- Make-to-stock environment
- No safety-stock for end-items

\[ I_t = I_{t-1} + Q_t - \max\{F_t, O_t\} \]
\[ I_t = \text{end-item inventory at the end of week } t \]
\[ Q_t = \text{manufactured quantity to be completed in week } t \]
\[ F_t = \text{forecast for week } t \]
\[ O_t = \text{customer orders to be delivered in week } t \]

<table>
<thead>
<tr>
<th>INITIAL DATA Model A</th>
<th>Jan</th>
<th>Feb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Inventory =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>orders Ot</td>
<td>1200</td>
<td>800</td>
</tr>
</tbody>
</table>
Master Production Scheduling

- **Batch production:** batch size = 2500
  - \( I_t = \max\{0, I_{t-1}\} - \max\{F_t, O_t\} \)
  - \( Q_t = \begin{cases} 
  0, & \text{if } I_t > 0 \\
  2500, & \text{otherwise}
  \end{cases} \)

- \( I_1 = \max\{0, 1600\} - \max\{1000, 1200\} = 400 > 0 \)
- \( I_2 = \max\{0, 400\} - \max\{1000, 800\} = -600 < 0 \Rightarrow Q_2 = 2500 \)
- \( I_2 = 2500 + 400 - \max\{1000, 800\} = 1900, \text{ etc.} \)

<table>
<thead>
<tr>
<th>MPS</th>
<th>Jan</th>
<th></th>
<th>Feb</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Inventory = 1600</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>forecast Ft</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>orders Ot</td>
<td>1200</td>
<td>800</td>
<td>300</td>
<td>200</td>
</tr>
<tr>
<td>Inventory It</td>
<td>1600</td>
<td>400</td>
<td>1900</td>
<td>900</td>
</tr>
<tr>
<td>MPS Qt</td>
<td>2500</td>
<td></td>
<td>2500</td>
<td></td>
</tr>
<tr>
<td>ATP</td>
<td>400</td>
<td>1400</td>
<td>2200</td>
<td>2500</td>
</tr>
</tbody>
</table>
Master Production Scheduling

- Available to Promise (ATP)

  \[ \text{ATP}_1 = 1600 + 0 - 1200 = 400 \]
  \[ \text{ATP}_2 = 2500 - (800 + 300) = 1400, \text{ etc.} \]

- Whenever a new order comes in, ATP must be updated

- Lot-for-Lot production

<table>
<thead>
<tr>
<th>MPS</th>
<th>Jan</th>
<th>Feb</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Week</td>
<td>Week</td>
</tr>
<tr>
<td>Current Inventory = 1600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>orders Ot</td>
<td>1200</td>
<td>800</td>
</tr>
<tr>
<td>Inventory It</td>
<td>1600</td>
<td>400</td>
</tr>
<tr>
<td>MPS Qt</td>
<td>0</td>
<td>600</td>
</tr>
<tr>
<td>ATP</td>
<td>400</td>
<td>0</td>
</tr>
</tbody>
</table>
Master Production Scheduling

- MPS Modeling
  - differs between MTS-ATO and MTO
  - find final assembly lot sizes
  - additional complexity because of joint capacity constraints
  - cannot be solved for each product independently
Master Production Scheduling

**Make-To-Stock-Modeling**

\[ Q_{it} = \text{production quantity of product } i \text{ in period } t \]
\[ I_{it} = \text{Inventory of product } i \text{ at end of period } t \]
\[ D_{it} = \text{demand (requirements) for product } i \text{ in time period } t \]
\[ a_i = \text{production hours per unit of product } i \]
\[ h_i = \text{inventory holding cost per unit of product } i \text{ per time period} \]
\[ A_i = \text{set-up cost for product } i \]
\[ G_t = \text{production hours available in period } t \]

\[ y_{it} = 1, \text{if set-up for product } i \text{ occurs in period } t (Q_{it} > 0) \]
Master Production Scheduling

**Make-To-Stock-Modeling**

\[
\begin{align*}
\min & \sum_{i=1}^{n} \sum_{t=1}^{T} (A_i y_{it} + h_i I_{it}) \\
I_{i,t-1} + Q_{it} - I_{it} & = D_{it} \quad \text{for all (i,t)} \\
\sum_{i=1}^{n} a_i Q_{it} & \leq G_t \quad \text{for all t} \\
Q_{it} - y_{it} \sum_{k=1}^{T} D_{ik} & \leq 0 \quad \text{for all (i,t)} \\
Q_{it} & \geq 0; I_{it} \geq 0; y_{it} \in \{0, 1\}
\end{align*}
\]
Master Production Scheduling

- **Assemble-To-Order Modeling**
  - **two master schedules**
    - MPS: forecast-driven
    - FAS: order driven
  - **overage costs**
    - Holding costs for modules and assembled products
  - **shortage costs**
    - Final product assembly based on available modules
    - No explicit but implicit shortage costs for modules
    - Final products: lost sales, backorders
Master Production Scheduling

- \( m \) module types and \( n \) product types
- \( Q_{kt} \) = quantity of module \( k \) produced in period \( t \)
- \( g_{kj} \) = number of modules of type \( k \) required to assemble order \( j \)

**Decision Variables:**
- \( I_{kt} \) = inventory of module \( k \) at the end of period \( t \)
- \( y_{jt} \) = 1, if order \( j \) is assembled and delivered in period \( t \); 0, otherwise
- \( h_k \) = holding cost
- \( \pi_{jt} \) = penalty costs, if order \( j \) is satisfied in period \( t \) and order \( j \) is due in period \( t' \) (\( t'<t \)); holding costs if \( t' > t \)
Assemble-To-Order Modeling

\[
\min \sum_{k=1}^{m} \sum_{t=1}^{L} h_k I_{kt} + \sum_{j=1}^{n} \sum_{t=1}^{L} \pi_{jt} y_{jt}
\]

subject to

\[
I_{kt} = I_{k,t-1} + Q_{kt} - \sum_{j=1}^{n} g_{kj} y_{jt} \quad \text{for all } (k, t)
\]

\[
\sum_{j=1}^{n} a_j y_{jt} \leq G_t \quad \text{for all } t
\]

\[
\sum_{t=1}^{L} y_{jt} = 1 \quad \text{for all } j
\]

\[
I_{kt} \geq 0; \quad y_{jt} \in \{0,1\} \quad \text{for all } (j, k, t)
\]
Master Production Scheduling

- **Capacity Planning**
  - Bottleneck in production facilities
  - Rough-Cut Capacity Planning (RCCP) at MPS level
  - feasibility
  - detailed capacity planning (CRP) at MRP level
  - both RCCP and CRP are only providing information
### Master Production Scheduling

#### MPS:

<table>
<thead>
<tr>
<th>Product</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>B</td>
<td>-</td>
<td>500</td>
<td>500</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>D</td>
<td>600</td>
<td>-</td>
<td>600</td>
<td>-</td>
</tr>
</tbody>
</table>

#### Bill of capacity (min)

<table>
<thead>
<tr>
<th></th>
<th>Assembly</th>
<th>Inspection</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>24</td>
<td>2.5</td>
</tr>
<tr>
<td>C</td>
<td>22</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>25</td>
<td>2.4</td>
</tr>
</tbody>
</table>

#### Capacity requires (hr)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Available capacity per week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assembly</td>
<td>1133</td>
<td>1083</td>
<td>1333!!</td>
<td>883</td>
<td>1200</td>
</tr>
<tr>
<td>Inspection</td>
<td>107</td>
<td>104</td>
<td>128!!</td>
<td>83</td>
<td>110</td>
</tr>
</tbody>
</table>

- **weekly capacity requirements?**
- **Assembly**: $1000 \times 20 + 1500 \times 22 + 600 \times 25 = 68000$ min = 1133,33 hr
- **Inspection**: $1000 \times 2 + 1500 \times 2 + 600 \times 2.4 = 6440$ min = 107,33 hr etc.
- **available capacity per week** is 1200 hr for the assembly work center and 110 hours for the inspection station;
Master Production Scheduling

- Infinite capacity planning (information providing)
- Finding a feasible cost optimal solution is a NP-hard problem

- If no detailed bill of capacity is available: capacity planning using overall factors (globale Belastungsfaktoren)
  - Required input:
    - MPS
    - Standard hours of machines or direct labor required
    - Historical data on individual shop workloads (%)

- Example from Günther/Tempelmeier
  - C133.3: overall factors
## Master Production Scheduling

**Capacity planning using overall factors**

<table>
<thead>
<tr>
<th>product</th>
<th>week</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>A</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>product</th>
<th>work on critical machine</th>
<th>work on non-critical machine</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

**Historic capacity requirements on critical machines:**
- 40% on machine a
- 60% on machine b
Master Production Scheduling

in total 500 working units are available per week, 80 on machine a and 120 on machine b;

Solution:
overall factor = time per unit x historic capacity needs

product A:
machine a: 1 x 0,4 = 0,4
machine b: 1 x 0,6 = 0,6

product B:
machine a: 4 x 0,4 = 1,6
machine b: 4 x 0,6 = 2,4
# Master Production Scheduling

## Capacity Requirements: Product A

<table>
<thead>
<tr>
<th>Machine</th>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td>40</td>
<td>32</td>
<td>48</td>
<td>40</td>
<td>48</td>
<td>24</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td>60</td>
<td>48</td>
<td>72</td>
<td>60</td>
<td>72</td>
<td>36</td>
</tr>
<tr>
<td>other</td>
<td></td>
<td>200</td>
<td>160</td>
<td>240</td>
<td>200</td>
<td>240</td>
<td>120</td>
</tr>
</tbody>
</table>

## Capacity Requirements: Product B

<table>
<thead>
<tr>
<th>Machine</th>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td>64</td>
<td>-</td>
<td>96</td>
<td>-</td>
<td>64</td>
<td>-</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td>96</td>
<td>-</td>
<td>144</td>
<td>-</td>
<td>96</td>
<td>-</td>
</tr>
<tr>
<td>other</td>
<td></td>
<td>80</td>
<td>-</td>
<td>120</td>
<td>-</td>
<td>80</td>
<td>-</td>
</tr>
</tbody>
</table>
## Master Production Scheduling

### Total capacity requirements

<table>
<thead>
<tr>
<th>Machine</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>104</td>
<td>32</td>
<td>144</td>
<td>40</td>
<td>112</td>
<td>24</td>
</tr>
<tr>
<td>b</td>
<td>156</td>
<td>48</td>
<td>216</td>
<td>60</td>
<td>168</td>
<td>36</td>
</tr>
<tr>
<td>Other</td>
<td>280</td>
<td>160</td>
<td>360</td>
<td>200</td>
<td>320</td>
<td>120</td>
</tr>
</tbody>
</table>
Master Production Scheduling

The diagram illustrates the capacity requirements for different categories over the weeks. Here are the details:

- **Category a**: Maximum 80 units per week.
- **Category b**: Maximum 120 units per week.
- **Other**: Maximum 300 units per week.

The bars show the actual capacity requirements for each week, which are as follows:

- **Week 1**: Category a = 100 units, Category b = 200 units, Other = 50 units.
- **Week 2**: Category a = 50 units, Category b = 100 units, Other = 150 units.
- **Week 3**: Category a = 30 units, Category b = 150 units, Other = 200 units.
- **Week 4**: Category a = 80 units, Category b = 120 units, Other = 250 units.
- **Week 5**: Category a = 100 units, Category b = 180 units, Other = 230 units.
- **Week 6**: Category a = 70 units, Category b = 100 units, Other = 220 units.
Master Production Scheduling

- **Capacity Modeling**
  - heuristic approach for finite-capacity-planning
  - based on input/output analysis
  - relationship between capacity and lead time

- $G_t =$ work center capacity
- $R_t =$ work released to the center in period $t$
- $Q_t =$ production (output) from the work center in period $t$
- $W_t =$ work in process in period $t$
- $U_t =$ queue at the work center measured at the beginning of period $t$, prior to the release of work
- $L_t =$ lead time at the work center in period $t$
Master Production Scheduling

\[ Q_t = \min\{G, U_{t-1} + R_t\} \]
\[ U_t = U_{t-1} + R_t - Q_t \]
\[ W_t = U_{t-1} + R_t = U_t + Q_t \]
\[ L_t = \frac{W_t}{G} \]

- **Lead time is not constant**
- **assumptions:**
  - constant production rate
  - any order released in this period is completed in this period
**Master Production Scheduling**

**Example**

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{t}$ (hr/week)</td>
<td></td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>$R_{t}$ (hours)</td>
<td></td>
<td>20</td>
<td>30</td>
<td>60</td>
<td>20</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>$Q_{t}$ (hours)</td>
<td></td>
<td>30</td>
<td>30</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>$U_{t}$ (hours)</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>8</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>$W_{t}$ (hours)</td>
<td></td>
<td>30</td>
<td>30</td>
<td>60</td>
<td>44</td>
<td>48</td>
<td>52</td>
</tr>
<tr>
<td>$L_{t}$ (weeks)</td>
<td></td>
<td>0.83</td>
<td>0.83</td>
<td>1.67</td>
<td>1.22</td>
<td>1.33</td>
<td>1.44</td>
</tr>
</tbody>
</table>
Material Requirements Planning

- **Inputs**
  - master production schedule
  - inventory status record
  - bill of material (BOM)

- **Outputs**
  - planned order releases
    - purchase orders(supply lead time)
    - workorders(manufacturing lead time)
Material Requirements Planning

Legend:
- **S/A** = subassembly
- **PP** = purchased part
- **MP** = manufactured part
- **RM** = raw material

<table>
<thead>
<tr>
<th>Level 4</th>
<th>Level 3</th>
<th>Level 2</th>
<th>Level 1</th>
<th>Level 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM</td>
<td>MP</td>
<td>PP</td>
<td>S/A</td>
<td>End-Item</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Legend:
- **part #**
- **quantity**
Material Requirements Planning

MRP Process

- goal is to find net requirements (trigger purchase and work orders)
- explosion
  - Example:
    - MPS, 100 end items
    - yields gross requirements
- netting
  - Net requirements = Gross requirements - on hand inventory - quantity on order
  - done at each level prior to further explosion
- offsetting
  - the timing of order release is determined
- lotsizing
  - batch size is determined
Material Requirements Planning

Example 7-6

- Telephone (1)
  - Hand Set Assembly (11)
    - Microphone S/A (111)
    - Receiver S/A (112)
    - Housing S/A (121)
    - Tapping Screw (115)
    - Upper Cover (113)
    - Lower Cover (114)
  - Base Assembly (12)
    - Board Pack S/A (122)
    - Key Pad (1211)
    - Key Pad Cord (1212)
  - Hand Set Cord (13)
    - Rubber Pad (123)
    - Tapping Screw (124)
Material Requirements Planning

PART 11 (gross requirements given)
net requirements?
Planned order release?

Net requ.(week 2) = 600 – (1600 + 700) = -1700 => Net requ.(week2) = 0
Net requ.(week 3) = 1000 – (1700 + 200) = -900 => Net requ.(week3) = 0
Net requ.(week 4) = 1000 – 900 = 100 etc.

<table>
<thead>
<tr>
<th>week</th>
<th>current</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>gross requirements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scheduled receipts</td>
<td>1200</td>
<td>1600</td>
<td>1700</td>
<td>900</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>net requirements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>planned order release</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Assumptions:
lot size: 3000
lead time: 2 weeks

<table>
<thead>
<tr>
<th></th>
<th>current</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>gross requirements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scheduled receipts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>projected inventory</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>balance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>net requirements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>planned receipts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>planned order</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>release</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Assumptions:
  - Lot size: 3000
  - Lead time: 2 weeks

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>700</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td>1600</td>
<td>1700</td>
<td>900</td>
<td>2900</td>
<td>900</td>
<td>1900</td>
<td>2900</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Projected inventory balance:
- Week 1: 1200
- Week 2: 1600
- Week 3: 1700
- Week 4: 900
- Week 5: 2900
- Week 6: 900
- Week 7: 1900
- Week 8: 2900

Net requirements:
- Week 1: 100
- Week 2: 2000
- Week 3: 2000
- Week 4: 2000
- Week 5: 2000
- Week 6: 2000

Planned receipts:
- Week 1: 3000
- Week 2: 3000
- Week 3: 3000
- Week 4: 3000
- Week 5: 3000
- Week 6: 3000

Planned order release:
- Week 1: 3000
- Week 2: 3000
- Week 3: 3000
- Week 4: 3000
- Week 5: 3000
- Week 6: 3000

Note: The table data represents the requirements planning for a product with a lot size of 3000 and a lead time of 2 weeks.
Material Requirements Planning

- Multilevel explosion

<table>
<thead>
<tr>
<th>part number</th>
<th>description</th>
<th>Qty</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>base assembly</td>
<td>1</td>
</tr>
<tr>
<td>121</td>
<td>housing S/A</td>
<td>1</td>
</tr>
<tr>
<td>123</td>
<td>rubber pad</td>
<td>4</td>
</tr>
<tr>
<td>1211</td>
<td>key pad</td>
<td>1</td>
</tr>
</tbody>
</table>

- lead time is one week
- lot for lot for parts 121, 123, 1211
- part 12: fixed lot size of 3000
<table>
<thead>
<tr>
<th>Part 12</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>current</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>gross requirements</td>
<td></td>
<td>600</td>
<td>1000</td>
<td>1000</td>
<td>2000</td>
<td>2000</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>scheduled receipts</td>
<td></td>
<td></td>
<td>400</td>
<td>400</td>
<td>400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>projected inventory balance</td>
<td></td>
<td>800</td>
<td>1200</td>
<td>1000</td>
<td>400</td>
<td>2400</td>
<td>400</td>
<td>1400</td>
</tr>
<tr>
<td>net requirements</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>planned receipts</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3000</td>
<td>0</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>planned order release</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3000</td>
<td>0</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>Part 121</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>current</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>gross requirements</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3000</td>
<td>0</td>
<td>3000</td>
<td><em>3000</em></td>
</tr>
<tr>
<td>scheduled receipts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>projected inventory balance</td>
<td></td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>net requirements</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>planned receipts</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3000</td>
</tr>
<tr>
<td>planned order release</td>
<td></td>
<td>0</td>
<td>2500</td>
<td>0</td>
<td>3000</td>
<td>3000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Part 123</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>current</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>gross requirements</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>12000</td>
<td>0</td>
<td>12000</td>
<td>12000</td>
</tr>
<tr>
<td>scheduled receipts</td>
<td></td>
<td></td>
<td>10000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>projected inventory balance</td>
<td></td>
<td>15000</td>
<td>15000</td>
<td>25000</td>
<td>13000</td>
<td>13000</td>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>net requirements</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>planned receipts</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>11000</td>
</tr>
<tr>
<td>planned order release</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>11000</td>
<td>0</td>
</tr>
<tr>
<td>Part 1211</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>current</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>gross requirements</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3000</td>
<td>0</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>scheduled receipts</td>
<td></td>
<td></td>
<td>1500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>projected inventory balance</td>
<td></td>
<td>1200</td>
<td>2700</td>
<td>200</td>
<td>200</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>net requirements</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>planned receipts</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2800</td>
<td>3000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>planned order release</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2800</td>
<td>3000</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Material Requirements Planning

- **MRP Updating Methods**
  - MRP systems operate in a dynamic environment
  - regeneration method: the entire plan is recalculated
  - net change method: recalculates requirements only for those items affected by change

<table>
<thead>
<tr>
<th>Product</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>350</td>
<td>-</td>
<td>-</td>
<td>350</td>
<td>500</td>
<td>-</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>C</td>
<td>1000</td>
<td>-</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>-</td>
<td>800</td>
<td>1000</td>
</tr>
<tr>
<td>D</td>
<td>-</td>
<td>300</td>
<td>200</td>
<td>-</td>
<td>-</td>
<td>300</td>
<td>200</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Product</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>300</td>
<td>-100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>150</td>
<td>200</td>
<td>-200</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Net Change for February**

<table>
<thead>
<tr>
<th>Product</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>300</td>
<td>-100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>150</td>
<td>200</td>
<td>-200</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Material Requirements Planning

- **Additional Netting procedures**
  - **implode:**
    - opposite of explosion
    - finds common item
  - **combining requirements:**
    - process of obtaining the gross requirements of a common item
  - **pegging:**
    - identify the item’s end product
    - useful when item shortages occur
Material Requirements Planning

Lot Sizing in MRP
- minimize set-up and holding costs

- can be formulated as MIP

- a variety of heuristic approaches are available

- simplest approach: use independent demand procedures (e.g. EOQ) at every level
Material Requirements Planning

MIP Formulation

Indices:

- $i = 1...P$: label of each item in BOM (assumed that all labels are sorted with respect to the production level starting from the end-items)
- $t = 1...T$: period $t$
- $m = 1...M$: resource $m$

Parameters:

- $\Gamma(i)$: set of immediate successors of item $i$
- $\Gamma^{-1}(i)$: set of immediate predecessors of item $i$
- $s_i$: setup cost for item $i$
- $c_{ij}$: quantity of item $i$ required to produce item $j$
- $h_i$: holding cost for one unit of item $i$
- $a_{mi}$: capacity needed on resource $m$ for one unit of item $i$
- $b_{mi}$: capacity needed on resource $m$ for the setup process of item $i$
- $L_{mt}$: available capacity of resource $m$ in period $t$
- $oc_m$: overtime cost of resource $m$
- $G$: large number, but as small as possible (e.g. sum of demands)
- $D_{it}$: external demand of item $i$ in period $t$
Decision variables:

- $x_{it}$: delivered quantity of item $i$ in period $t$
- $l_{it}$: inventory level of item $i$ at the end of period $t$
- $O_{mt}$: overtime hours required for machine $m$ in period $t$
- $y_{it}$: binary variable indicating if item $i$ is produced in period $t$ ($=1$) or not ($=0$)

Equations:

- $\min \sum_{i=1}^{P} \sum_{t=1}^{T} (s_i y_{it} + h_i I_{it}) + \sum_{t=1}^{T} \sum_{m=1}^{M} o_c m O_{mt}$

- $I_{i,t} = I_{i,t-1} + x_{i,t} - \sum_{j \in \Gamma(i)} c_{ij} x_{jt} - D_{it} \quad \forall i, t$

- $x_{it} - G y_{it} \leq 0 \quad \forall i, t$

- $\sum_{i=1}^{P} (a_{mi} x_{it} + b_{mi} y_{it}) \leq L_{mt} + O_{mt} \quad \forall m, t$

- $x_{it}, I_{it}, O_{mt} \geq 0, \quad y_{it} \in \{0, 1\} \quad \forall i, m, t$
Material Requirements Planning

- **Multi-Echelon Systems**
  - Multi-echelon inventory
  - each level is referred as an **echelon**
  - “total inventory in the system varies with the number of stocking points”
  - Modell (Freeland 1985):
    - demand is insensitive to the number of stocking points
    - demand is normally distributed and divided evenly among the stocking points,
    - demands at the stocking points are independent of one another
    - a (Q,R) inventory policy is used
    - β-Service level (fill rate) is applied
    - Q is determined from the EOQ formula
Material Requirements Planning

- Reorder point in (Q,R) policies:
  - $i$: total annual inventory costs (%)
  - $c$: unit costs
  - $A$: ordering costs
  - $T$: lead time
  - $\sigma_T$: variance of demand in lead time

- Given a fill rate $\beta$, choose $z(\beta)$ such that:

$$L(z) = \int_{z}^{\infty} (y - z) \phi(y) dy = \frac{(1 - \beta)Q}{\sigma_T}$$

$\phi$: density of $N(0,1)$ distribution; $L(z)$: standard loss function
<table>
<thead>
<tr>
<th>$Z$</th>
<th>.00</th>
<th>.02</th>
<th>.04</th>
<th>.06</th>
<th>.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>.3989</td>
<td>.3890</td>
<td>.3793</td>
<td>.3697</td>
<td>.3602</td>
</tr>
<tr>
<td>0.10</td>
<td>.3509</td>
<td>.3418</td>
<td>.3329</td>
<td>.3240</td>
<td>.3154</td>
</tr>
<tr>
<td>0.20</td>
<td>.3069</td>
<td>.2986</td>
<td>.2904</td>
<td>.2824</td>
<td>.2745</td>
</tr>
<tr>
<td>0.30</td>
<td>.2668</td>
<td>.2592</td>
<td>.2518</td>
<td>.2445</td>
<td>.2374</td>
</tr>
<tr>
<td>0.40</td>
<td>.2304</td>
<td>.2236</td>
<td>.2170</td>
<td>.2104</td>
<td>.2040</td>
</tr>
<tr>
<td>0.50</td>
<td>.1978</td>
<td>.1917</td>
<td>.1857</td>
<td>.1799</td>
<td>.1742</td>
</tr>
<tr>
<td>0.60</td>
<td>.1687</td>
<td>.1633</td>
<td>.1580</td>
<td>.1528</td>
<td>.1478</td>
</tr>
<tr>
<td>0.70</td>
<td>.1429</td>
<td>.1381</td>
<td>.1335</td>
<td>.1289</td>
<td>.1245</td>
</tr>
<tr>
<td>0.80</td>
<td>.1202</td>
<td>.1160</td>
<td>.1120</td>
<td>.1080</td>
<td>.1042</td>
</tr>
<tr>
<td>0.90</td>
<td>.1004</td>
<td>.0968</td>
<td>.0933</td>
<td>.0899</td>
<td>.0866</td>
</tr>
<tr>
<td>1.00</td>
<td>.0833</td>
<td>.0802</td>
<td>.0772</td>
<td>.0742</td>
<td>.0714</td>
</tr>
<tr>
<td>1.10</td>
<td>.0686</td>
<td>.0660</td>
<td>.0634</td>
<td>.0609</td>
<td>.0585</td>
</tr>
<tr>
<td>1.20</td>
<td>.0561</td>
<td>.0539</td>
<td>.0517</td>
<td>.0496</td>
<td>.0475</td>
</tr>
<tr>
<td>1.30</td>
<td>.0456</td>
<td>.0437</td>
<td>.0418</td>
<td>.0401</td>
<td>.0383</td>
</tr>
<tr>
<td>1.40</td>
<td>.0367</td>
<td>.0351</td>
<td>.0336</td>
<td>.0321</td>
<td>.0307</td>
</tr>
<tr>
<td>1.50</td>
<td>.0293</td>
<td>.0280</td>
<td>.0268</td>
<td>.0256</td>
<td>.0244</td>
</tr>
<tr>
<td>1.60</td>
<td>.0233</td>
<td>.0222</td>
<td>.0212</td>
<td>.0202</td>
<td>.0192</td>
</tr>
<tr>
<td>1.70</td>
<td>.0183</td>
<td>.0174</td>
<td>.0166</td>
<td>.0158</td>
<td>.0150</td>
</tr>
<tr>
<td>1.80</td>
<td>.0143</td>
<td>.0136</td>
<td>.0129</td>
<td>.0122</td>
<td>.0116</td>
</tr>
<tr>
<td>1.90</td>
<td>.0110</td>
<td>.0104</td>
<td>.0099</td>
<td>.0094</td>
<td>.0089</td>
</tr>
<tr>
<td>2.00</td>
<td>.0084</td>
<td>.0080</td>
<td>.0075</td>
<td>.0071</td>
<td>.0067</td>
</tr>
<tr>
<td>2.10</td>
<td>.0063</td>
<td>.0060</td>
<td>.0056</td>
<td>.0053</td>
<td>.0050</td>
</tr>
<tr>
<td>2.20</td>
<td>.0047</td>
<td>.0044</td>
<td>.0042</td>
<td>.0039</td>
<td>.0037</td>
</tr>
<tr>
<td>2.30</td>
<td>.0036</td>
<td>.0034</td>
<td>.0032</td>
<td>.0030</td>
<td>.0028</td>
</tr>
<tr>
<td>2.40</td>
<td>.0027</td>
<td>.0026</td>
<td>.0024</td>
<td>.0023</td>
<td>.0022</td>
</tr>
<tr>
<td>2.50</td>
<td>.0021</td>
<td>.0018</td>
<td>.0017</td>
<td>.0016</td>
<td>.0016</td>
</tr>
</tbody>
</table>
Material Requirements Planning

- **Safety stock:** 
  \[ s = z \cdot \sigma_r \]

- **Reorder point:** 
  \[ R = \overline{D}_r + z \cdot \sigma_r \]

- **Order quantity:** 
  \[ Q = EOQ = \sqrt{\frac{2AD}{ic}} \]

- **Average inventory:** 
  \[ \overline{I}(1) = \frac{Q}{2} + s \]
  \[ \overline{I}(n) = \text{average inventory for } n \text{ stocking points} \]
  \[ \overline{I}(1) = \frac{1}{2} \sqrt{\frac{2AD}{ic}} + z \sigma_r \]
Material Requirements Planning

for two stocking points:
demand at each point: $D/2$
variance of lead-time demand: $\sigma_r^2 / 2$
standard deviation is: $\sigma_r / \sqrt{2}$

average inventory at each stocking point is:

$$\frac{1}{2} \sqrt{\frac{2AD/2}{\text{ic}}} + \frac{z\sigma_r}{\sqrt{2}} = \frac{1}{\sqrt{2}} (Q/2 + s)$$
Material Requirements Planning

the average inventory for two stocking point is:

\[ \bar{I}(2) = 2 \left[ \frac{1}{\sqrt{2}} (Q/2 + s) \right] = \sqrt{2} (Q/2 + s) = \sqrt{2} \cdot \bar{I}(1) \]

\[ \bar{I}(n) = \sqrt{n} \cdot \bar{I}(1) \]

for each level the safety stock is: \( s/\sqrt{n} \)

the total safety stock is \( \sqrt{n} \cdot s \)
Material Requirements Planning

Example: At the packaging department of a sugar refinery:

A very-high-grade powdered sugar:

Sugar-refining lead time is five days;
Production lead time (filling time) is negligible;
Annual demand: \( D = 800 \) tons and \( \sigma = 2.5 \)
Lead-time demand is normally distributed with \( D_T = 16 \) tons and \( \sigma_T = 3.54 \) tons
Fill rate = 95%
\( A = $50, \ c = $4000, \ i = 20\% \)
Material Requirements Planning

Inventory at level 0 and 1? Safety stock?

\[ Q = \sqrt{\frac{2 \times AD}{ic}} = \sqrt{\frac{2 \times 50 \times 800}{800}} = 10 \text{ tons} \]

\[ \beta = 0.95 \Rightarrow z = 0.71 \]

\[ s = z\sigma = 0.71 \times 3.54 = 2.51 \text{ tons} \]

Suppose we keep inventory in level 0 only, i.e., \( n = 1 \):

\[ I(1) = \frac{Q}{2} + s = \frac{10}{2} + 2.51 = 7.51 \text{ tons} \]

Suppose inventory is maintained at both level 0 and level 1, i.e., \( n = 2 \):

\[ I(2) = \sqrt{2I(1)} = 10.62 \text{ tons} \]

The safety stock in each level is going to be:

\[ \frac{s}{\sqrt{2}} = \frac{2.51}{\sqrt{2}} = 1.77 \text{tons} \]
Material Requirements Planning

MRP as Multi-Echelon Inventory Control

- continuous-review type policy (Q,R)
- hierarchy of stocking points (installation)
- installation stock policy
- echelon stock (policy): installation inventory position plus all downstream stock

MRP:
- rolling horizon
- level by level approach
- bases ordering decisions on projected future installation inventory level
Material Requirements Planning

- All demands and orders occur at the beginning of the time period
- Orders are initiated immediately after the demands, first for the final items and then successively for the components
- All demands and orders are for an integer number of units
- $T =$ planning horizon
- $\tau_i =$ lead time for item $i$
- $s_i =$ safety stock for item $i$
- $R_i =$ reorder point for item $i$
- $Q_i =$ fixed order quantity of item $i$
- $D_{it} =$ external requirements of item $i$ in period $t$
Installation stock policies \((Q,R_i)\) for MRP:

- a production order is triggered if the installation stock minus safety stock is insufficient to cover the requirements over the next \(\tau_i\) periods
- an order may consist of more than one order quantity \(Q\)

- if lead time \(\tau_i = 0\), the MRP is equal to an installation stock policy.
- safety stock = reorder point
Material Requirements Planning

- **Echelon stock policies \((Q,R^e)\) for MRP:**
  - Consider a serial assembly system
  - Installation 1 is the downstream installation (final product)
  - the output of installation \(i\) is the input when producing one unit of item \(i-1\) at the immediate downstream installation
  - \(w_i\) = installation inventory position at installation \(i\)
  - \(l_i\) = echelon inventory position at installation \(i\) (at the same moment)

\[
l_i = w_i + w_{i-1} + \ldots + w_1
\]

- a multi-echelon \((Q,R)\) policy is denoted by \((Q_i,R_i^e)\)
- \(R_i^e\) gives the reorder point for echelon inventory at \(i\)
Material Requirements Planning

\[ R_1^e = s_1 + D \tau_1 \]
\[ R_i^e = s_i + D \tau_i + R_{i-1}^e + Q_{i-1} \]

**Example:**

- **Two-level system, 6 periods**
  - \( I_1^0 = 18, \ I_2^0 = 38, \ R_1^e = 20, \ R_2^e = 34, \ Q_1 = 10, \ Q_2 = 30 \)
  - \( D = 2 \) (Item 1), \( \tau_1 = 1, \ \tau_2 = 2 \)
### Material Requirements Planning

#### Demand and Production Levels

<table>
<thead>
<tr>
<th>Period</th>
<th>Item 1</th>
<th>Item 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Demand</td>
<td>Level w1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>28</td>
</tr>
</tbody>
</table>

*Suppose now that five units were demanded in period 2:*

<table>
<thead>
<tr>
<th>Period</th>
<th>Item 1</th>
<th>Item 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Demand</td>
<td>Level w1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>23</td>
</tr>
</tbody>
</table>
Material Requirements Planning

- **Lot Size and Lead Time**
  - lead time is affected by capacity constraints
  - lot size affects lead time

- **batching effect**
  - an increase in lot size should increase lead time

- **saturation effect**
  - when lot size decreases, and set-up is not reduced, lead time will increase

- expected lead time can be calculated using models from queueing theory (M/G/1)
Material Requirements Planning

\[ L = \text{lead time} \]
\[ L = \frac{(\lambda / \mu)^2 + \lambda^2 \sigma^2}{2\lambda(1 - \lambda / \mu)} + \frac{1}{\mu} \]

\[ \lambda = \text{mean arrival rate} \]
\[ \mu = \text{mean service rate} \]
\[ \sigma^2 = \text{service time variance} \]
Material Requirements Planning

\[ D_j = \text{demand per period for product } j \]
\[ t_j = \text{unit - production time for product } j \]
\[ S_j = \text{set - up time for product } j \]
\[ Q_j = \text{lotsize for product } j \]

mean arrival rate of batches :

\[ \lambda = \sum_{j=1}^{n} \lambda_j = \sum_{j=1}^{n} \frac{D_j}{Q_j} \]

mean service time :

\[ \frac{1}{\mu} = \frac{\sum_{j=1}^{n} \lambda_j (S_j + t_j Q_j)}{\sum_{j=1}^{n} \lambda_j} \]

service - time variance :

\[ \sigma^2 = \frac{\sum_{j=1}^{n} \lambda_j (S_j + t_j Q_j)^2}{\sum_{j=1}^{n} \lambda_j} - \left( \frac{1}{\mu} \right)^2 \]
Material Requirements Planning

![Graph showing the relationship between lead time, optimum, lot size, combined effect, batching effect, and saturation effect.](image-url)
Material Planning

- Work to do: 7.7ab, 7.8, 7.10, 7.11, 7.14 (additional information: available hours: 225 (Paint), 130 (Mast), 100 (Rope)), 7.15, 7.16, 7.17, 7.31-7.34