Establishing Einstein-Poldosky-Rosen Channels between Nanomechanics and Atomic Ensembles

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We suggest interfacing nanomechanical systems via an optical quantum bus to atomic ensembles, for which means of high precision state preparation, manipulation, and measurement are available. This allows, in particular, for a quantum nondemolition Bell measurement, projecting the coupled system, atomic-ensemble–nanomechanical resonator, into an entangled EPR state. The entanglement is observable even for nanoresonators initially well above their ground states and can be utilized for teleportation of states from an atomic ensemble to the mechanical system.

Opto- and electro-nanomechanical systems [1], representing cold high-$Q$ oscillators coupled to optical cavities or electrical circuits, are rapidly approaching the regime where quantum aspects are important [2–6]. It remains one of the key challenges of nanomechanics to develop both the tools for preparing and manipulating quantum states as superposition and entangled states, and to implement quantum measurements. Motivated by the remarkable achievements in the quantum control of atomic ensembles [7–10], which allow for high-fidelity preparation, readout, and laser manipulation of atomic states as long-lived quantum memory, we propose a quantum interface between atomic ensembles and optomechanical systems, where light plays the role of a quantum bus. This effort should be seen in the context of developing hybrid systems and interfaces where the goal is to combine the advantages of a solid state and atomic systems in compatible experimental setups.

Our goal is the creation of a robust EPR-type of entanglement [11] between collective spin variables of the atomic medium and a nanomechanical resonator. EPR entanglement is a key resource in many quantum information protocols, and, in particular, enables the manipulation and transfer of quantum states, e.g., by quantum teleportation. EPR states involve two systems, each described by a pair of continuous variables, say, $[X_m, P_m] = i$ and $[X_a, P_a] = i$, and exhibit a reduced so-called EPR variance in their correlations:

$$\Delta_{EPR} = \Delta(X_m + X_a)^2 + \Delta(P_m - P_a)^2 < 2.$$  (1)

The value of 2 refers to uncorrelated systems in their ground states and any value below 2 proves entanglement [12]. In the present case, canonical variables $X_m, P_m$ refer to the (dimensionless) position and momentum of the mechanical oscillator, while $X_a, P_a$ describe collective spin excitations in an atomic ensemble as follows: The fully polarized state of an ensemble of atoms, each with two stable ground states $|\pm\rangle$, is identified with the ground state of a harmonic oscillator, $|\ldots\rangle = |0\rangle$, and excited states are given by $|1\rangle = a\rangle|0\rangle$, etc., where $a^\dagger = N^{-1/2}J_\uparrow$, $N$ is the number of atoms, and $J_\uparrow = \sum_i |+\rangle\langle+|$ is a collective step-up operator. Accordingly, canonical operators correspond to scaled collective spin components $X_a = (N a^\dagger)^{-1/2}J_c$ and $P_a = (N a^\dagger)^{-1/2}J_y$. This is an excellent approximation for the high degree of polarization routinely achieved in current experiments [7–9].

The method to generate EPR correlations suggested here is based on a quantum nondemolition (QND) measurement [13] of commuting EPR observables $X_m + X_a$ and $P_m - P_a$. Using light as a meter system, as was demonstrated for two atomic ensembles by Julsgaard et al. [8], the QND measurement projects the hybrid nanomechanical-atomic system into a state with reduced variances in the EPR observables that obey (1). The method relies on the fact that both systems can be coupled to a cavity mode $[x_c, p_c] = i$ via structurally similar Hamiltonians:

$$H_{mc} = \frac{\omega_m}{2} (X_m^2 + P_m^2) + g X_m x_c,$$  (2)

$$H_{ac} = \frac{\Omega}{2} (X_a^2 + P_a^2) + G X_a x_c,$$  (3)

where $\omega_m$ is the mechanical frequency, $\Omega$ denotes the energy splitting of the two atomic levels, and $g$ and $G$ are coupling constants. The physical basis of (2) and (3) are, respectively, radiation pressure [1] and Faraday interaction [14], as will be detailed further below. The basic principle underlying the QND measurement is best explained by assuming for the moment that both systems are coupled to the same cavity mode and that we tune $G = g$ and $\Omega = -\omega_m$. The Hamiltonian is then the sum of (2) and (3), and in an interaction picture with respect to the harmonic oscillator terms the resulting Hamiltonian is

$$H_I = g [\cos(\omega_m t)x_c(X_m + X_a) + \sin(\omega_m t)x_c(P_m - P_a)].$$

Evidently, the relevant EPR observables are conserved...
quantities, and given that the cavity decay happens fast on a time scale of $g$, cosine and sine components of light coupled out of the cavity will read out these EPR observables, making them detectable in a homodyne measurement of light. Note that it is vital to choose $\Omega = -\omega_\text{in}$, i.e., to let the atomic ensemble realize a harmonic oscillator with negative mass, in order to have commuting EPR observables appearing in $H_f$ and, therefore, a QND interaction, a situation whose realization is not obvious for two nanomechanical oscillators.

As will be detailed below, the QND measurement and EPR state preparation can actually be achieved in a cascaded quantum system according to Fig. 1, where the output light of the laser driven optomechanical system is coupled to an atomic ensemble at a (possibly) distant location, followed by a homodyne measurement. This setup has three remarkable features: First, this establishes distant EPR correlations, as is familiar from quantum communications with continuous variables systems, and avoids the requirement of holding the cloud of atoms close to the opto-nanomechanical system. Second, the present protocol remarkably does not require ground state cooling of the opto-nanomechanical system. Third, this setup also provides measurement and verification of a reduced EPR variance by simply repeating the protocol.

Recently, two proposals for hybrid quantum systems involving atoms and nanosystems were put forward [15,16]. Our proposal is distinctly different from the one by Treutlein et al. [16], which suggests direct coupling of a Bose-Einstein condensate to a magnetic island on a cantilever. Entanglement between short-lived electronically excited states of an atomic ensemble and a nanomechanical system, both being placed inside a cavity, has been very recently studied theoretically by Genes et al. [15].

The detailed treatment of light propagation and losses, as well as realistic conditions for the matching of time scales, will be the main content of the remaining parts of this Letter. We start with a brief derivation of the fundamental Hamiltonians (2) and (3), which are both well-established models in their respective fields; see [2–6,8–10], respectively. In the optomechanical system the fundamental interaction is based on radiation pressure [17] described by $V = g_0 a_i a^\dagger X_m$, where $g_0 = (\omega_\text{in}/L)\omega_c$ and $x_0$ is the mechanical oscillator ground state spread, $L$ the cavity length, and $\omega_c$ its frequency. If the cavity is driven by a resonant pulse of duration $\tau \gg 1/\gamma_c$, where $\gamma_c$ is the cavity decay rate, and power $P = N_\text{ph}\hbar \omega_c/\tau$ containing a total number $N_\text{ph}$ of photons, a steady state amplitude $\alpha = \sqrt{N_\text{ph}/\tau \gamma_c}$ builds up and the dynamics can be linearized giving an effective interaction $V_{\text{eff}} = g_x X_m$ as in (2), where $x_c$, $p_c$ describe fluctuations of the cavity field; see, e.g., [4]. From Hamiltonian (2) the evolution is given by $\dot{x}_c = -\gamma_c x_c - \sqrt{2}\gamma_c x_0 a^\dagger$, and $\dot{p}_c = -\gamma_c p_c - \sqrt{2}\gamma_c x_0 a - g X_m$, where $[x_0(t), p_0(t')] = i\delta(t - t')$ denotes vacuum noise. Assuming $\gamma_c \gg g, \omega_m$, we adiabatically eliminate the cavity mode and arrive at the cavity input-output relations $x_{\text{out}} = -x_{\text{in}}$ and $p_{\text{out}} = -p_{\text{in}} - g\sqrt{2}/\gamma_c X_m$ [18]. These expressions refer to field quadratures slowly varying around $\omega_c$. According to the last equation, the phase quadrature variance will be above shot noise due to correlations to the mechanical quadrature $X_m$.

As indicated in Fig. 1(a) the beam leaving the cavity interacts with an ensemble of $N_{\text{at}}$ atoms in free space, with a relevant level scheme shown in Fig. 1(b). For simplicity we will give a derivation of (3) for atoms inside a low finesse cavity and derive from it input-output relations, as was done above. The same input-output relations can be shown to hold in free space [10] and provide an excellent description of experiments [8,9]. The level configuration in Fig. 1(b) gives rise to the Hamiltonian $H \approx J_z S_z$, where $S_z = -i(a^\dagger_\text{in}a_\text{out} - a^\dagger_\text{out}a_\text{in})/2$ is a Stokes vector defined for two polarization modes $|a_j, a_j^\dagger\rangle = \delta_{ij}(i, j = y, z)$. This so-called Faraday interaction describes mutual polarization rotation of the atomic spins and the cavity field. If the $z$-polarized cavity mode is coherently driven, such that $\langle a_z \rangle = i|a_\text{in}|$, one can approximate $S_z \approx |a_\text{in}|(a_y + a_y^\dagger)/2 \approx x_c$. Using canonical variables $X_a, P_a$ for collective spin components $J_x, J_y$ as explained above results in an interaction $H \approx X_a x_c$. For nondegenerate ground states there will be, in addition, a free Hamiltonian $H_0 = \Omega J_z = -(\Omega/2)(X_a^2 + P_a^2)$, where the minus sign is due to the fact that atoms are pumped to the energetically higher lying state; cf. Fig. 1(b). Overall, we arrive at a Hamiltonian (3) with $\Omega \rightarrow -\Omega$. Adiabatic elimination of the cavity mode, just as for the mechanical system, will yield the input-output relations $x_{\text{out}} = -x_{\text{in}}$ and $p_{\text{out}} = -p_{\text{in}} - \kappa\sqrt{2/\tau}X_a$. In free space, $\tau$ is the pulse length and...
\( \kappa^2 = (\sigma \Gamma / \Delta \Delta)^2 N_\text{at} N_\text{ph} \), where \( \Gamma \) is the spontaneous decay rate, \( \Delta \) the detuning, \( \sigma \) the scattering, and \( \Delta \) the beam cross section [10].

We now assume that the cavity output provides the input to the light-atoms interaction such that \( x'_{\text{in}} = -x_{\text{out}} \) and \( p'_{\text{in}} = -p_{\text{out}} \). This requires that the coherent pulse at frequency \( \omega_c \) is rotated in polarization by 90° and phase shifted by \( \pi / 2 \) relative to its sideband components at \( \omega_c \pm \omega_m \), which can be achieved by separating the optical carrier and the sidebands with an auxiliary cavity and then performing the required rotations and shifts. Note that because the sidebands will be measured by homodyning with the carrier, a well-feasible extinction ratio for the carrier at the percent level is sufficient [19]. We now assume a matching of time scales by requiring

\[
\kappa / \sqrt{\tau} = g / \sqrt{\gamma_c}.
\]

which can be fulfilled experimentally as indicated below. Under these conditions the overall input-output relations become

\[
x'_{\text{out}} = -x_{\text{in}}, \quad p'_{\text{out}} = -p_{\text{in}} - \kappa \sqrt{2 / \tau} (X_m + X_a). \tag{5}
\]

In order to achieve a QND measurement of EPR variables \( X_m + X_a \) and \( P_m - P_a \), these variables have to be free of the backaction of light and light has to read out both. This is indeed the case. From the discussion above it follows straight forwardly that the mechanical oscillator evolves as \( X_m = \omega_m P_m \) and \( P_m = -\omega_m X_m - g x_c = -\omega_m X_m + g \sqrt{2 / \gamma_c} x_{\text{out}} \), where in the last equality the cavity mode was again adiabatically eliminated. In these equations we neglect the decay of the oscillator, which is justified if the whole protocol happens on a time scale \( \sim \tau \) such that \( \tau \gamma_m \tilde{n}_\text{th} \ll 1 \). Here \( \gamma_m \) is the mechanical damping rate, and \( \tilde{n}_\text{th} = k_B T / h \omega_m \) is the mean occupation in thermal equilibrium at temperature \( T \). If this condition is met, decay can be treated perturbatively, as will be done below. Transverse atomic spin components evolve as \( X_m = -\Omega P_a \) and \( P_a = \Omega X_a + \kappa \sqrt{2 / \tau} x_{\text{in}} \).

Decoherence due to spontaneous emission can be kept small for sufficient optical depth [10,14] and will be included perturbatively further below. Using again condition (4) and taking in addition \( \omega_m = \Omega \), we finally arrive at

\[
\frac{d}{dt} (X_m + X_a) = \Omega (P_m - P_a) \quad \text{and} \quad \frac{d}{dt} (P_m - P_a) = -\Omega (X_m + X_a).
\]

In this closed set of equations of motion for commuting EPR observables \( X_m + X_a \) and \( P_m - P_a \), backaction effects of light cancel out by interference. This establishes the QND character of the present interactions. The QND signal lies essentially in the Fourier components at frequency \( \Omega \) of the in-quadrature component \( p'_{\text{out}} \).

For the normalized observables

\[
p_{\text{out}}^\text{cos} = -\sqrt{2 / \tau} \int_0^\tau dt \cos (\Omega t) p'_{\text{out}}(t) \quad \text{and} \quad p_{\text{out}}^\text{sin} = -\sqrt{2 / \tau} \int_0^\tau dt \sin (\Omega t) p'_{\text{out}}(t),
\]

one readily derives from (5) the input-output relations \( p_{\text{out}}^\text{cos} = p_{\text{in}}^\text{cos} + \kappa (X_m + X_a) \) and \( p_{\text{out}}^\text{sin} = p_{\text{in}}^\text{sin} + \kappa (P_m - P_a) \), with appropriate definitions for the in-components \( x_{\text{in}}^{\text{cos(sin)}} \). We assume here \( \Omega \tau \gg 1 \) such that cosine and sine components can be taken as independent variables.

A measurement of \( x_{\text{out}}^{\text{cos(sin)}} \) leaves the mechanical resonator and the collective spin in a state with reduced EPR variance (1), conditioned on the respective measurement results \( \xi_{\text{out}}^{\text{cos(sin)}} \) of \( x_{\text{out}}^{\text{cos(sin)}} \). An unconditionally reduced variance can be achieved by a feedback operation on the atomic spin, e.g., via fast rf pulses causing an appropriate tilt of the spin, generating a displacement \( X_m = -g \xi_{\text{cos}} \), \( P_a \rightarrow P_a - g \xi_{\text{sin}} \) with a suitable gain \( g \). In the ensemble average, this generates a state whose statistics is described by the input-output relations [20]

\[
(X_m + X_a)_{\text{out}} = (X_m + X_a)_{\text{in}} - g p_{\text{out}}^\text{cos},
\]

\[
\quad = (1 - g \kappa)(X_m + X_a)_{\text{in}} - g p_{\text{in}}^\text{cos}, \quad (6)
\]

\[
(P_m - P_a)_{\text{out}} = (1 - g \kappa)(P_m - P_a)_{\text{in}} - g p_{\text{in}}^\text{sin}.
\]

Our aim is to minimize this variance with respect to the feedback gain \( g \). As initial conditions we assume that light modes are in vacuum, the collective spin is in its ground (fully polarized) state, and the mechanical resonator is in a thermal state with mean occupation \( \tilde{n}_i = \tilde{n}_\text{th} \). Note that in principle \( \tilde{n}_i \) can be reduced by initial laser cooling. For optimal gain the minimal EPR variance is

\[
\Delta_{\text{EPR}} = \frac{2}{1 + \tilde{n}_i} + 2 \kappa^2,
\]

which is the main result of this Letter.

According to Eq. (1) this is an entangled state if the right-hand side of (7) falls below 2. As \( 2[(1 + \tilde{n}_i) + 2 \kappa^2]^{-1} \leq \kappa^{-2} \), there is no fundamental limit on observable entanglement due to initial thermal occupation \( \tilde{n}_i \) of the mechanical system. Thus, even for moderate values of \( \kappa \approx 0.5 \), achievable as outlined below, the present protocol produces an entangled state independent of the initial thermal occupation of the nanomechanical resonator.

We turn to the discussion of losses and decoherence. The dominant impinging effects are as follows: (i) number mismatch in Eq. (4); (ii) loss of light, detection inefficiency, and spontaneous emission in light-atom interactions; (iii) thermalization of the mechanical oscillator.

Regarding (i), it is straightforward to derive that a nonzero mismatch \( \epsilon = (\kappa - \lambda \sqrt{\gamma_c} \lambda) / (\kappa + \lambda \sqrt{\gamma_c} \lambda) \) will give rise to additional terms in the variance of EPR variables (7) scaling in leading order as \( \{ \epsilon \kappa (\tilde{n}_i + 2) \}^2 \). For \( \kappa \gg 1 \) a mismatch of \( \epsilon \approx 1 / 10 \tilde{n}_i \) is tolerable. This poses a practical limit to the initial thermal occupation of the nanomechanical resonator. Effects due to processes (ii) and (iii) can be treated perturbatively as linear losses, as we exemplify for damping of the resonator: During the interaction the state of the resonator will undergo damping at a rate \( \gamma_m \) and provided that \( \gamma_m \tau \ll 1 \), e.g., Eq. (6), will generalize to

\[
\langle X_m + X_a \rangle_{\text{out}} = \langle X_m + X_a \rangle_{\text{in}} + \sqrt{\gamma_m} t f_{X_m} - g p_{\text{out}}^\text{cos} \quad \langle X_a \rangle_{\text{out}} = \langle X_a \rangle_{\text{in}} - g p_{\text{out}}^\text{sin}.
\]

where \( f_{X_m} \) is a Langevin operator for thermal noise, \( \langle f_{X_m}^2 \rangle = (\tilde{n}_\text{th} + 1) \). The variance will thus receive an addi-

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tional term $\gamma_m \tau \langle \hat{n}_{1m} \rangle + 1$, such that we have to require $\tau \ll 1/\gamma_m \tau \langle \hat{n}_{1m} \rangle = Q_m h / k_B T$ for a quality factor $Q_m = \omega_m / \gamma_m$. In a similar vein one can treat losses (ii), which have the advantageous property that the corresponding noise is, to a good approximation, vacuum noise. That is, a photon loss by a fraction $\varepsilon$ will cause the final EPR variance to be $\Delta_{\text{EPR}} \rightarrow (1 - \varepsilon) \Delta_{\text{EPR}} + 2\varepsilon$. This will reduce but not remove the entanglement created by this protocol.

The suggested protocol can be realized with current technology. We consider two possible setups in which the nanomechanical resonator is used as either one of the mirrors of a Fabry-Perot cavity [2] or a dispersive element in a rigid cavity [6]. Assuming that $\kappa \approx 1$ and that condition (4) can be matched within an error of $\varepsilon = 0.01$, we need $\hat{n}_i \leq 30$. This can be achieved for high $\omega_m$ or the low temperature, by applying additional laser cooling. As an example for the two cases we assume a moving micromirror with $\omega_m/2\pi = 5$ MHz, mass $m = 10^{-12}$ kg, and quality factor $Q_m \geq 5 \times 10^5$ operated at $T = 0.2$ K (resulting in $\hat{n}_i = 850$, which requires modest laser cooling by a factor of 30), and a small (dispersively coupled) nanomembrane with $\omega_m/2\pi = 30$ MHz, $m = 10^{-14}$ kg, and $Q_m \geq 10^5$ operated at $T = 0.04$ K ([$\hat{n}_i = 30$]. Mechanical quality and temperature limit the interaction time to $\tau \ll 20 \mu s$, which is principle sufficient for entanglement of room temperature atoms and certainly enough in the case of cold atoms. For the laser-cooled micromirror (4) can be achieved with a finesse $F = 4500$ and power $P = 100$ $\mu W$. Adiabatic elimination of the cavity mode finally poses an upper bound $L \leq 300$ $\mu m$ on the cavity length. For the nanomembrane a modest finesse $F = 1100$ is already sufficient at a pump power $P = 100$ $\mu W$ and cavity length $L \leq 250$ $\mu m$.

Finally, the generated entanglement can serve as a basis for teleporting quantum states of a collective spin onto a nanomechanical system. The protocol proceeds in three steps. First, an entangled state characterized by Eq. (7) is created and a second, additional atomic ensemble is prepared in a coherent state with amplitudes $\langle X_{a2} \rangle, \langle P_{a2} \rangle$. Second, following the approach of [8], a QND Bell measurement of $\langle X_{a2} + X_0 \rangle$ and $\langle P_{a2} - P_0 \rangle$ is performed on the two atomic ensembles. Third, the measurement result is used in feedback on the mechanical system, via, e.g., piezoelectric or radiation pressure displacement. This completes the teleportation and generates a state $\rho_{\text{fin}} = X_m + g [\rho_{\text{in}}^{\text{os}} + \kappa_{\text{QND}}(X_{a2} + X_0)] = X_m + X_0 + X_{a2}$ and $\rho_{\text{fin}} = P_m + g [\rho_{\text{in}}^{\text{os}} + \kappa_{\text{QND}}(P_{a2} - P_0)] = P_m - P_0 + P_{a2}$.

Here $\kappa_{\text{QND}}$ and $g$ denote the strength of QND interaction and feedback gain in the Bell measurement on the two atomic ensembles. The second equalities of both lines are valid in the asymptotic limit $\kappa_{\text{QND}} \rightarrow \infty$, $g \rightarrow 0$ while $\kappa_{\text{QND}} g = 1$, which essentially requires a large optical depth [10,14]. Amplitudes are thus transmitted correctly, $\langle X_m \rangle = \langle X_{a2} \rangle$ and $\langle P_m \rangle = \langle P_{a2} \rangle$, and the amount of added noise is given by $\Delta_{\text{EPR}}/2$ in Eq. (7), as, e.g., $\Delta (X_{a2}^{\text{fin}})^2 = \Delta (X_{a2})^2 + \Delta (X_m + X_0)^2$ and equivalently for $\Delta (P_{a2}^{\text{fin}})^2$. For $\kappa \approx 1$ this is approximately one unit of vacuum noise in each variable, corresponding to a fidelity of $2/3$. We note that this implies the intriguing possibility to cool a mechanical resonator by teleporting the ground state onto it.

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