Heterotic Brane World

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Based on work with S. Förste, P. Vaudrevange and A. Wingerter

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Outline

- Orbifold Compactification

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- Early work in 80's

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- Outlook

Orbifolds of heterotic string

Orbifold compactifications combine the

- success of Calabi-Yau compactification
- calculability of torus compactification

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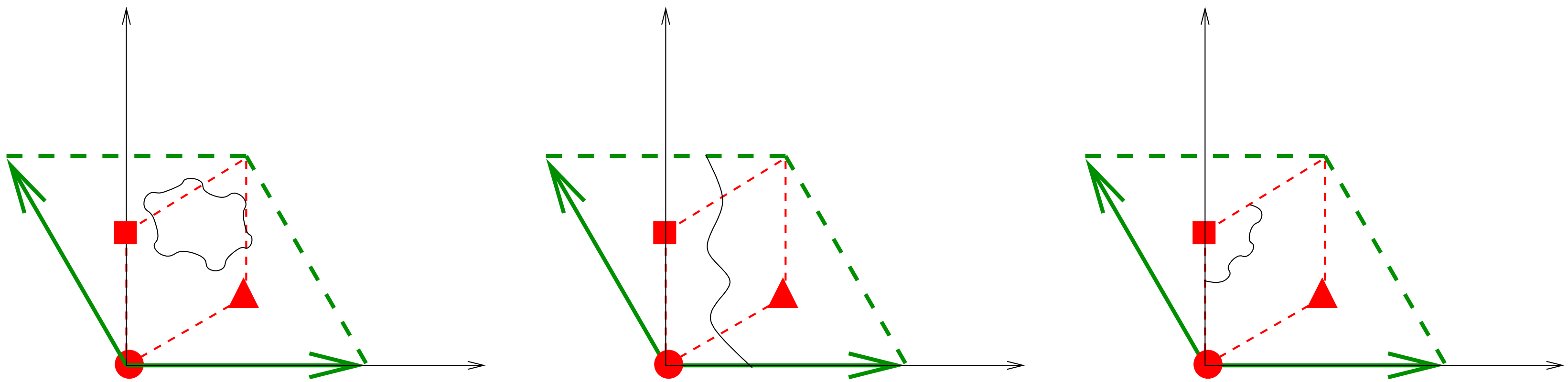
Fields can propagate

- Bulk ($d = 10$ untwisted sector)
- 3-Branes ($d = 4$ twisted sector fixed points)
- 5-Branes ($d = 6$ twisted sector fixed tori)

$\mathbb{Z}_3$ Example



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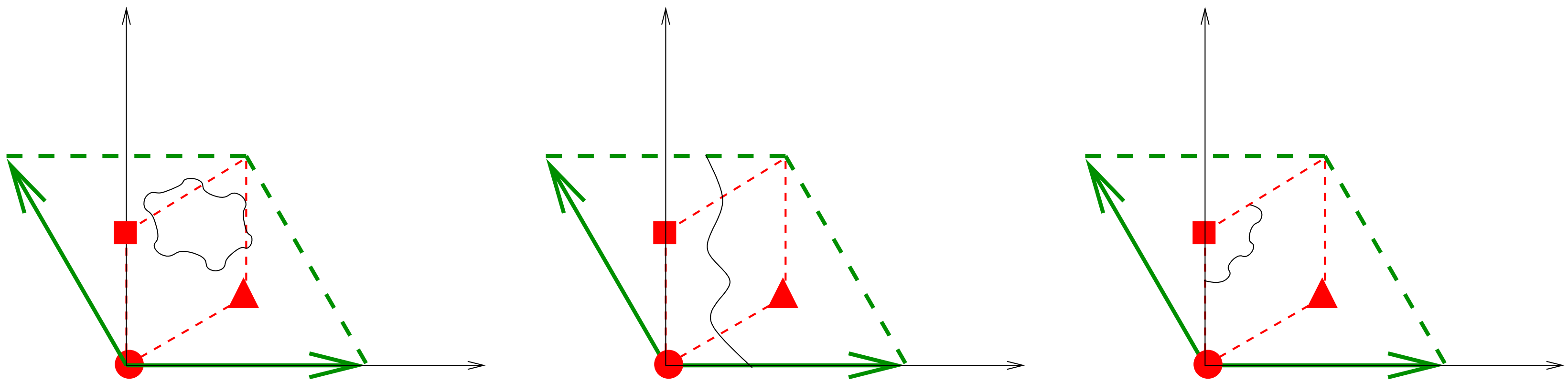

- Action of the space group on coordinates

$$X^i \to (\theta^k X)^i + n_\alpha e^i_\alpha, \quad k = 0, 1, 2, \quad i, \alpha = 1, \ldots, 6$$

- Embed twist in gauge degrees of freedom

$$X^I \to (\Theta^k X)^I \quad I = 1, \ldots, 16$$

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Very few inequivalent models

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Case	Shift V	Gauge Group	Gen.
1	$\left(\frac{1}{3},\frac{1}{3},\frac{2}{3},0^5\right)\left(0^8\right)$	$\mathsf{E}_6 \times \mathsf{SU}(3) \times \mathsf{E}_8'$	36
2	$\left(\frac{1}{3},\frac{1}{3},\frac{2}{3},0^5\right)\left(\frac{1}{3},\frac{1}{3},\frac{2}{3},0^5\right)$	$\mathsf{E}_6 \times \mathsf{SU}(3) \times \mathsf{E}_6' \times \mathsf{SU}(3)'$	9
3	$\left(\frac{1}{3},\frac{1}{3},0^6\right)\left(\frac{2}{3},0^7\right)$	$\mathsf{E}_7 \times \mathsf{U}(1) \times \mathsf{SO}(14)' \times \mathsf{U}(1)'$	0
4	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{2}{3},0^3\right)\left(\frac{2}{3},0^7\right)$	$\mathsf{SU}(9) \times \mathsf{SO}(14)' \times \mathsf{U}(1)'$	9

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as a result of the degeneracy of the matter multiplets at the 27 fixed points

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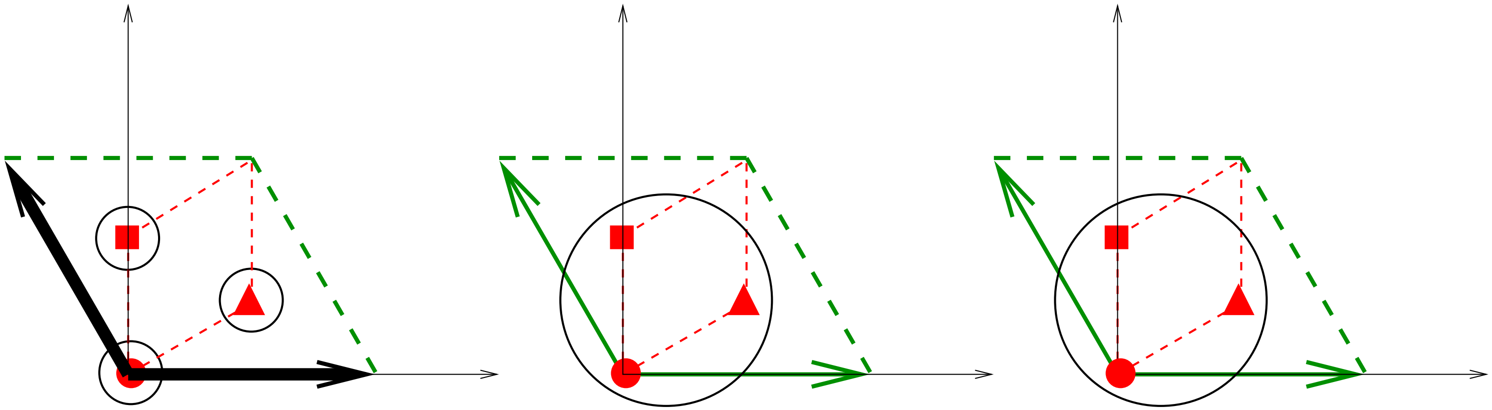
We need to lift this degeneracy ...

$\mathbb{Z}_3$ Orbifold with Wilson lines



Torus shifts embedded in gauge group as well

$$X^I \rightarrow X^I + V^I + n_\alpha A^I_\alpha$$

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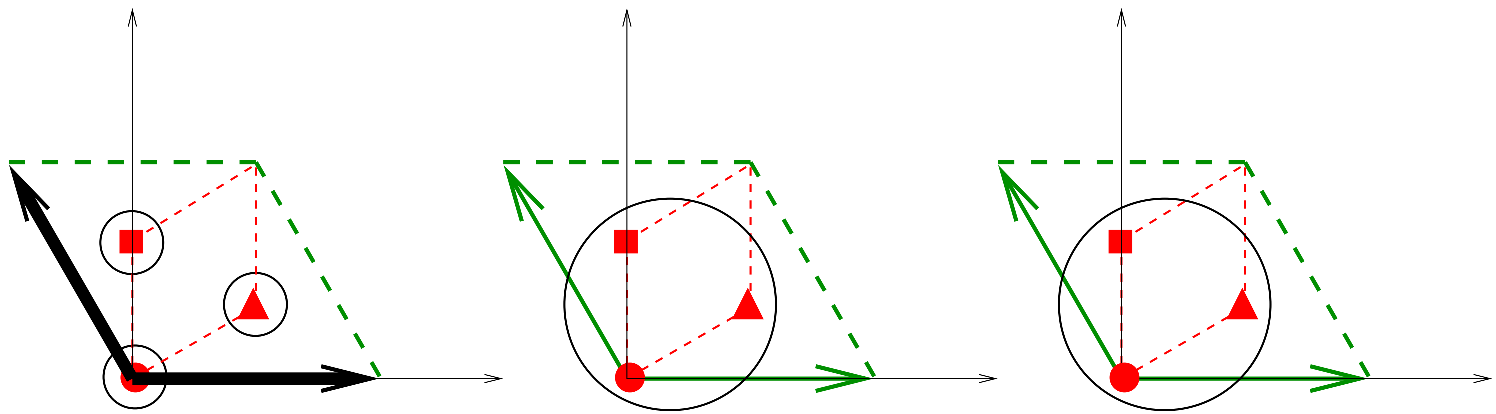

Torus shifts embedded in gauge group as well

$$X^I \rightarrow X^I + V^I + n_\alpha A_\alpha^I$$

- further gauge symmetry breakdown
- number of generations reduced

Early work on $\mathbb{Z}_3$ Orbifold

Successful model building with

- three families of quarks and leptons
- gauge group $SU(3) \times SU(2) \times U(1)^n$
- doublet-triplet splitting
- mechanism for Yukawa suppression

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Leads to a picture of "GUTs without GUT group"

- Incomplete gauge and Higgs multiplets
- Transparent geometric interpretation
- Unification of gauge couplings ($\sin^2 \theta_W$)

Things to improve

$\mathbb{Z}_3$ orbifold example is simple, but too rigid

- only fixed points and no fixed tori
- difficult to get “normal” grand unified picture
- no large string threshold corrections
- continuous Wilson lines too “distructive”
- value of $\sin^2\theta_W$

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This was the situation around 1988-89.

It gave a picture of unification that is somewhat different form the conventional one....

Work in the 90's

- some continuation on orbifold constructions, though not very specific
- fermionic formulation of heterotic string with very specific (semi) realistic models
- Type IIB orientifolds
- D brane constructions
- intersecting branes

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This gives a vast variety of models,
both with and without supersymmetry in $d = 4$ ....
small or large compactified dimensions ....

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Can we incorporate this into a string theory description?

Five golden rules

- Family as spinor of SO(10)
- Incomplete multiplets
- $N = 1$ superymmetry in $d = 4$
- Repetition of families from geometry
- Discrete symmetries of stringy origin

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We need more general constructions to incorporate this program into string theory .....

Candidates

In ten space-time dimensions.....

- Type I SO(32)
- Type II orientifolds
- Heterotic SO(32)
- Heterotic $E_8 \times E_8$
- Intersecting Branes $U(N)^M$

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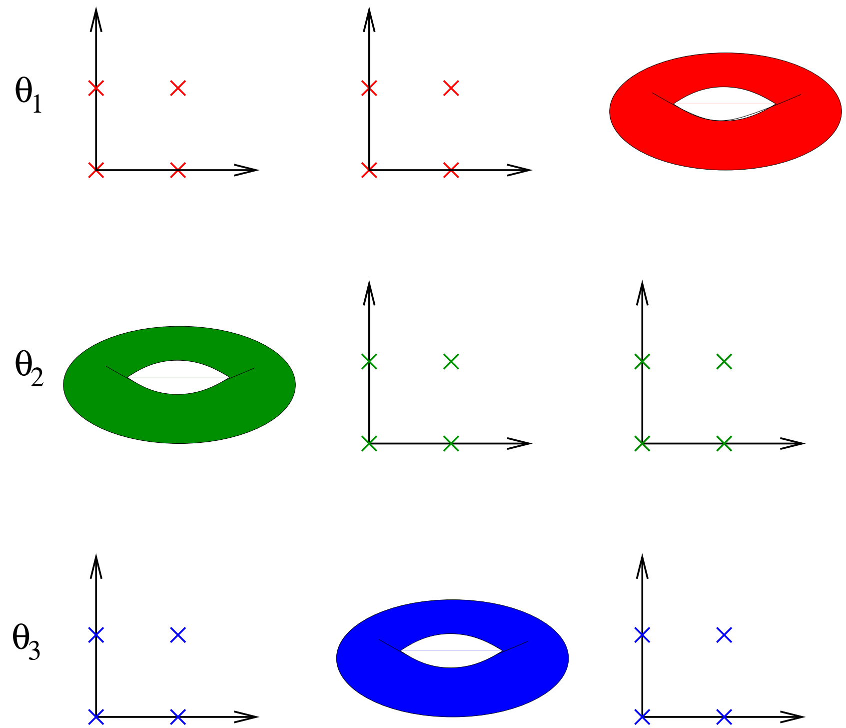
....or in eleven

- Horava-Witten heterotic M-theory
- Type IIA on manifolds with G_2 holonomy

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold Example



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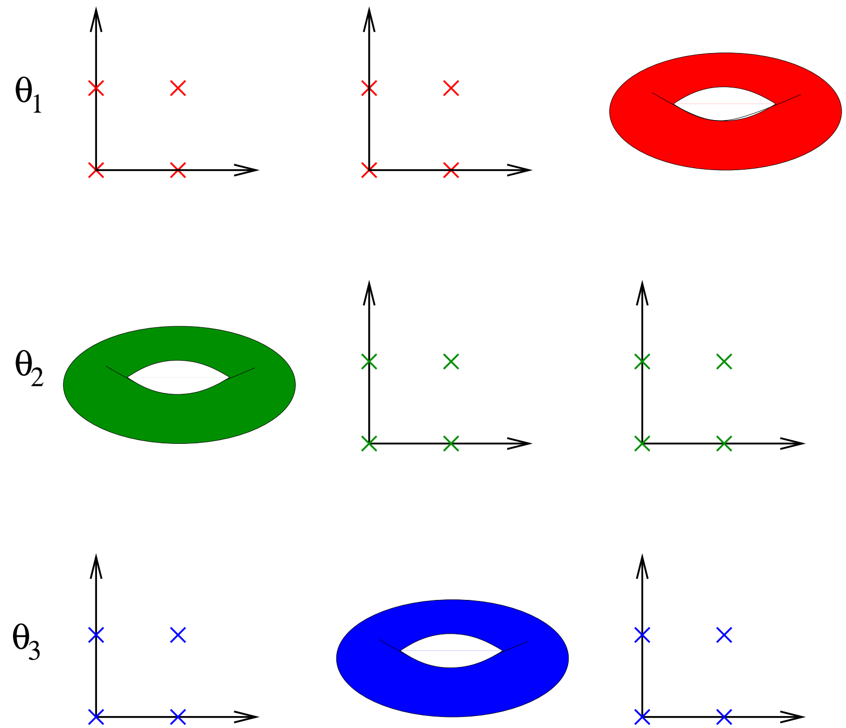
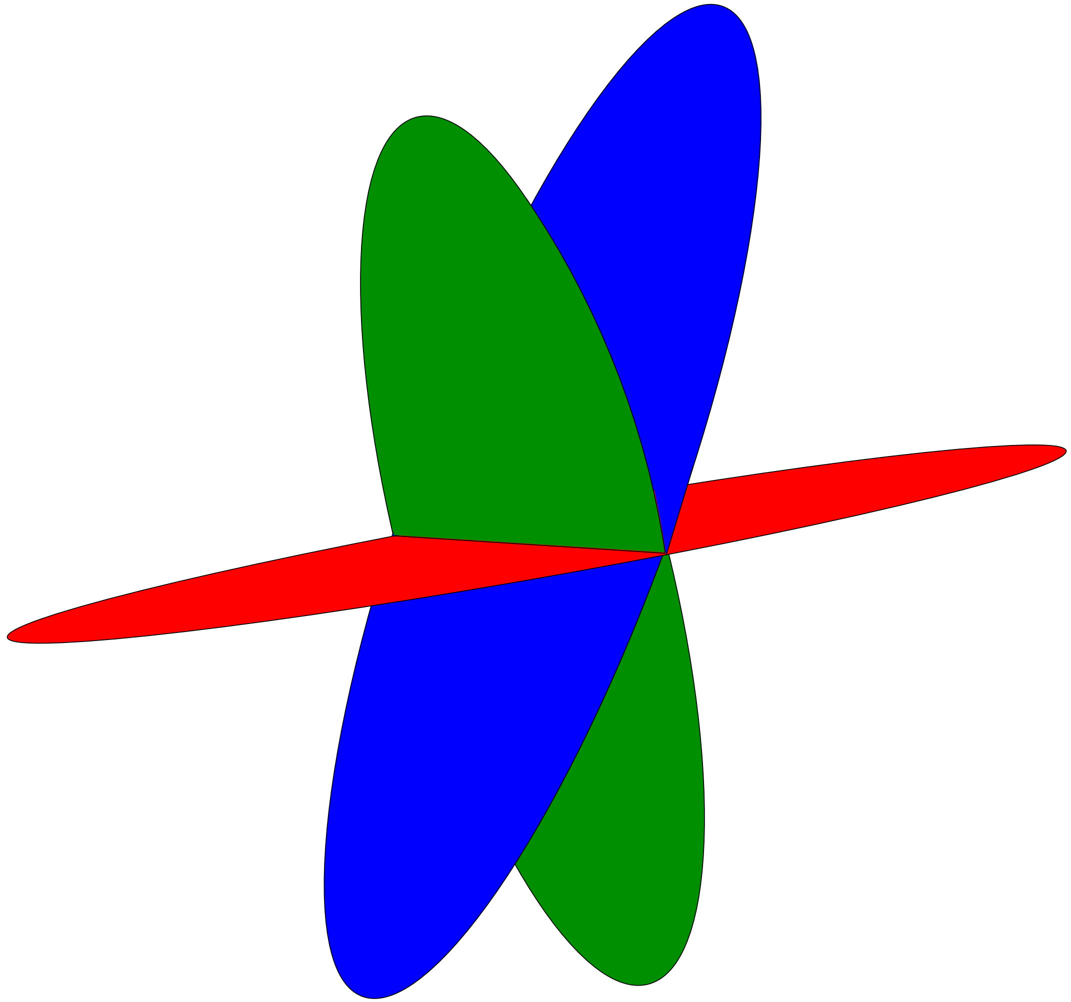


3 twisted sectors (with 16 fixed tori in each) lead to a geometrical picture of ....

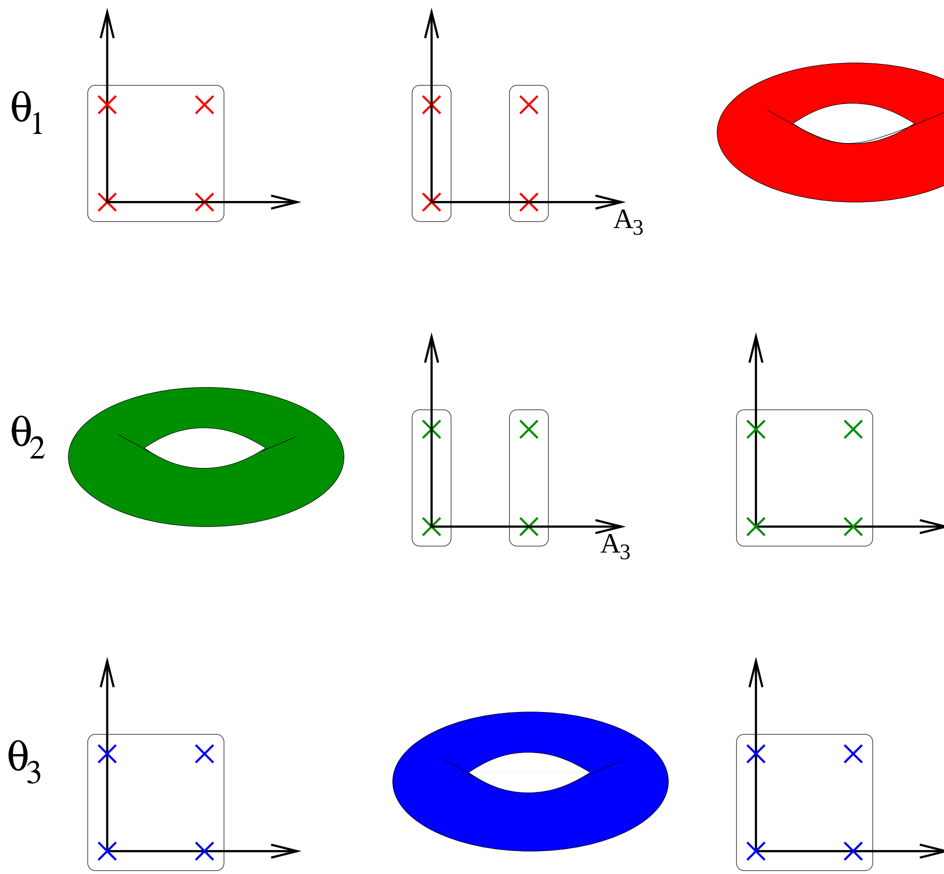
Intersecting Branes



$\mathbb{Z}_2 \times \mathbb{Z}_2$ classification

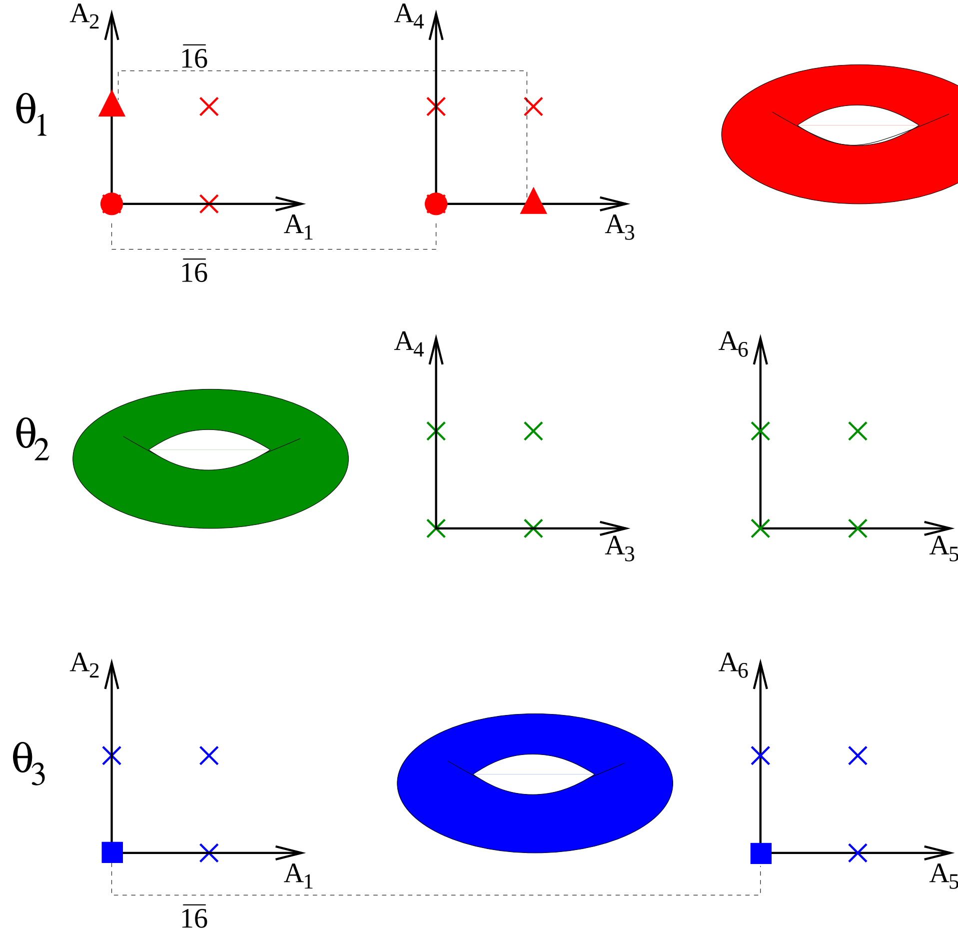
Case	Shifts	Gauge Group	Gen.
1	$\left(\frac{1}{2}, -\frac{1}{2}, 0^6\right) \left(0^8\right)$ $\left(0, \frac{1}{2}, -\frac{1}{2}, 0^5\right) \left(0^8\right)$	$\mathsf{E}_6 \times \mathsf{U}(1)^2 \times \mathsf{E}_8'$	48
2	$\left(\frac{1}{2}, -\frac{1}{2}, 0^6\right) \left(0^8\right)$ $\left(0, \frac{1}{2}, -\frac{1}{2}, 0^4, 1\right) \left(1, 0^7\right)$	$\mathsf{E}_6 \times \mathsf{U}(1)^2 \times \mathsf{SO}(16)'$	16
3	$\left({\frac{1}{2}}^2, 0^6\right) \left(0^8\right)$ $\left(\frac{5}{4}, {\frac{1}{4}}^7\right) \left(\frac{1}{2}, \frac{1}{2}, 0^6\right)$	$\mathsf{SU}(8) \times \mathsf{U}(1) \times \mathsf{E}_7' \times \mathsf{SU}(2)'$	16
4	$\left({\frac{1}{2}}^2, 0^5, 1\right) \left(1, 0^7\right)$ $\left(0, \frac{1}{2}, -\frac{1}{2}, 0^5\right) \left(-\frac{1}{2}, {\frac{1}{2}}^3, 1, 0^3\right)$	$\mathsf{E}_6 \times \mathsf{U}(1)^2 \times \mathsf{SO}(8)'^2$	0
5	$\left(\frac{1}{2}, -\frac{1}{2}, -1, 0^5\right) \left(1, 0^7\right)$ $\left(\frac{5}{4}, {\frac{1}{4}}^7\right) \left(\frac{1}{2}, \frac{1}{2}, 0^6\right)$	$\mathsf{SU}(8) \times \mathsf{U}(1) \times \mathsf{SO}(12)' \times \mathsf{SU}(2)'^2$	0

$\mathbb{Z}_2 \times \mathbb{Z}_2$ with Wilson lines



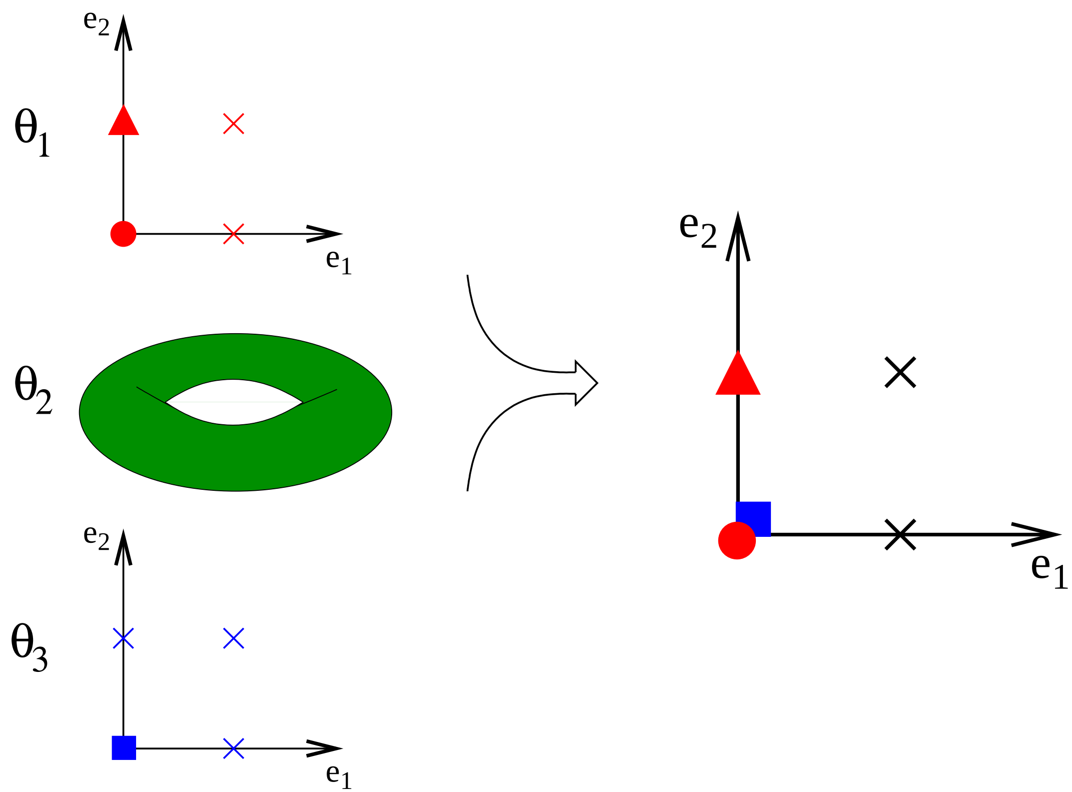
Again, Wilson lines can lift the degeneracy....

Three family $SO(10)$ toy model



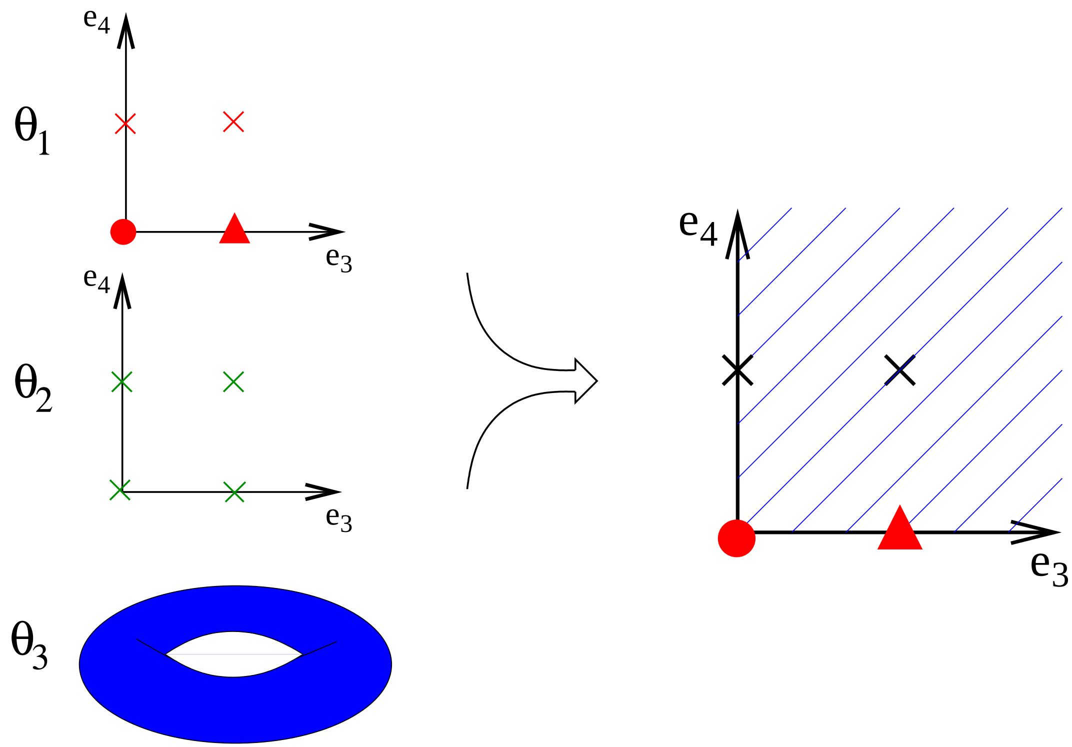
Localization of families at various fixed tori

Zoom on first torus ...



Interpretation as 6-dim. model with 3 families on branes

second torus ...



... 2 families on branes, one in (6d) bulk ...

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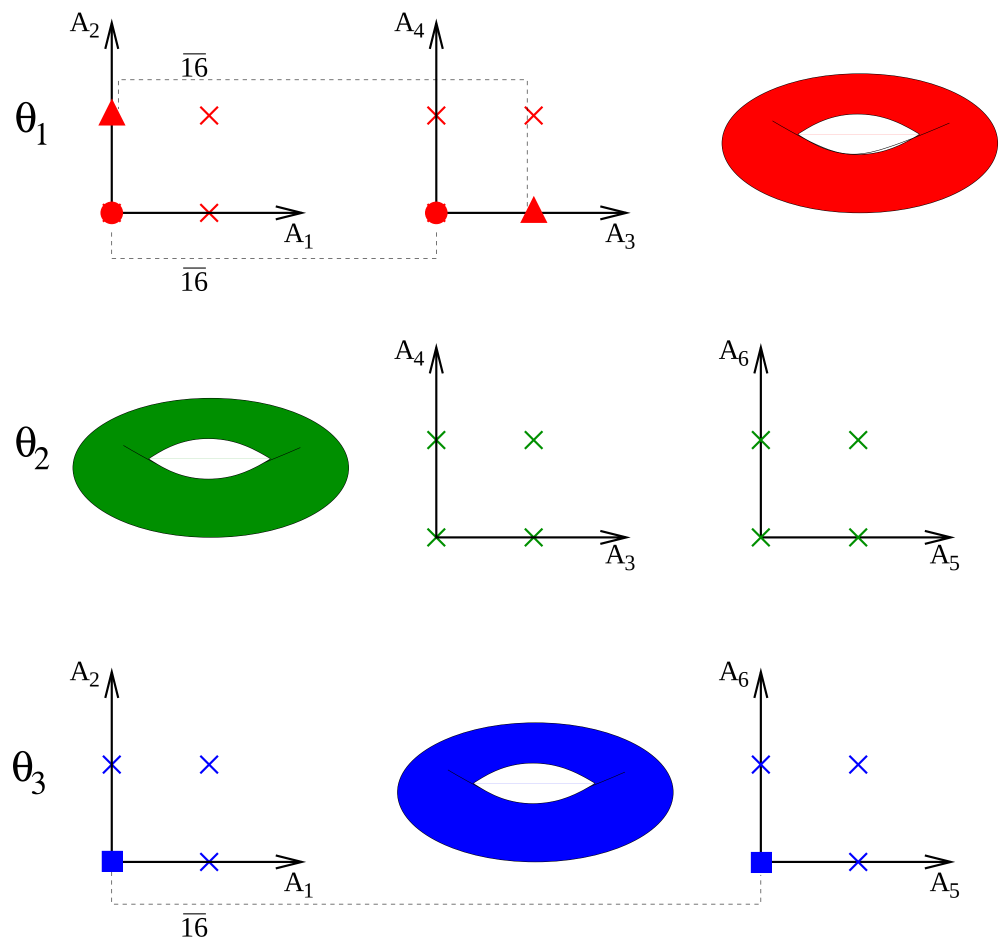

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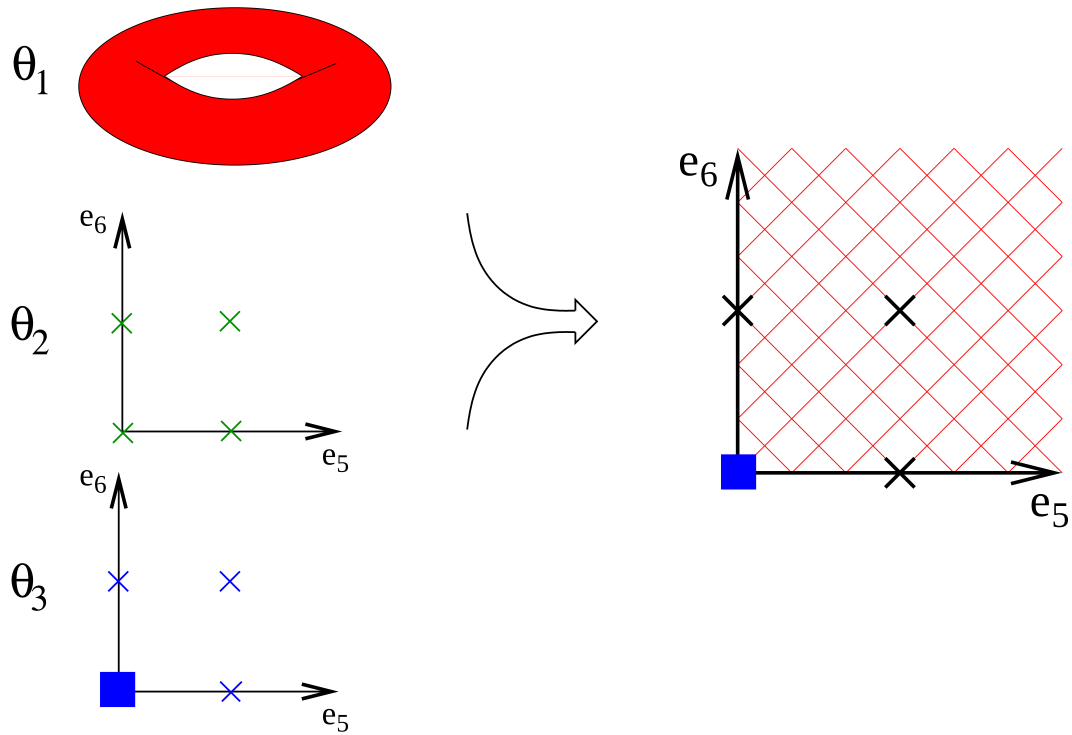
third torus



... 1 family on brane, two in (6d) bulk.

Model building

We can easily find

- models with gauge group $SU(3) \times SU(2) \times U(1)$
- 3 families of quarks and leptons
- doublet-triplet splitting
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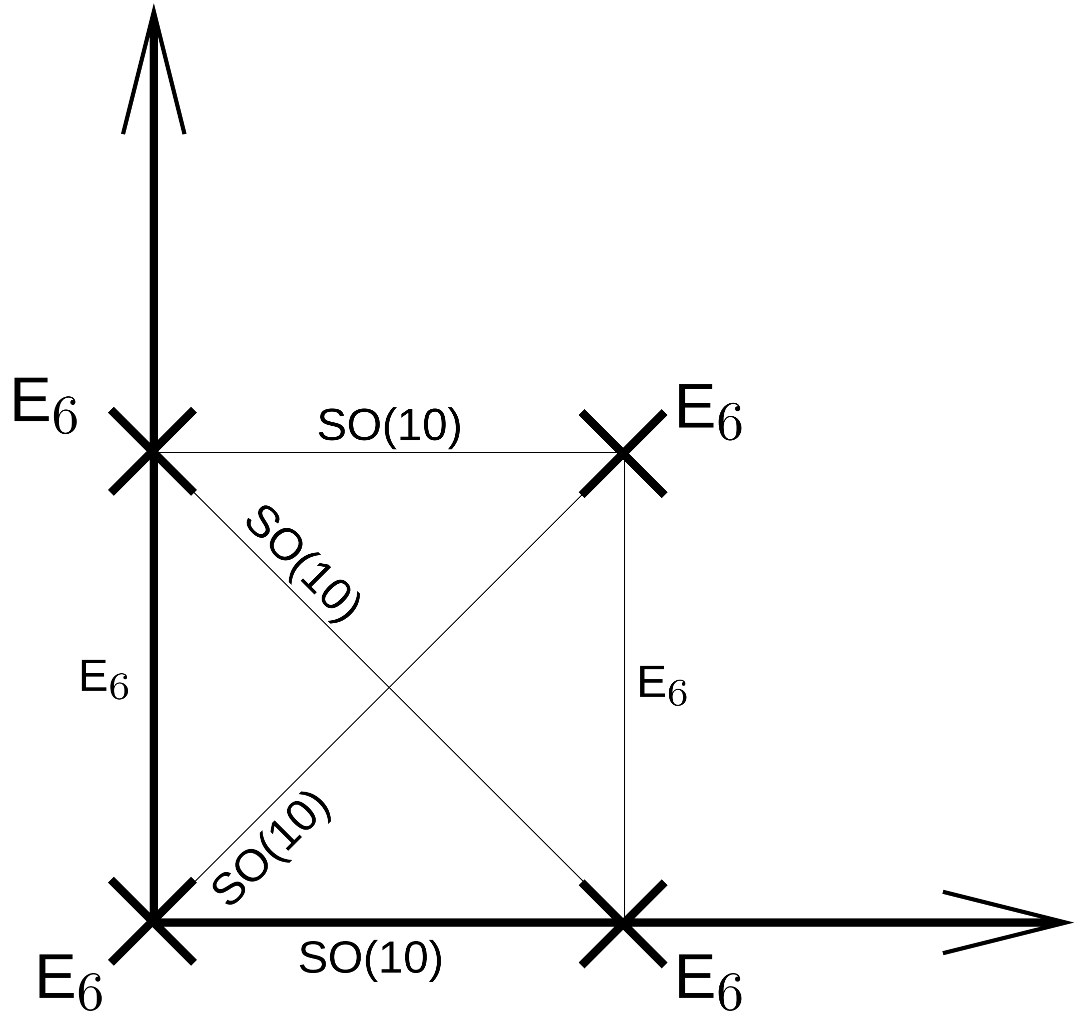
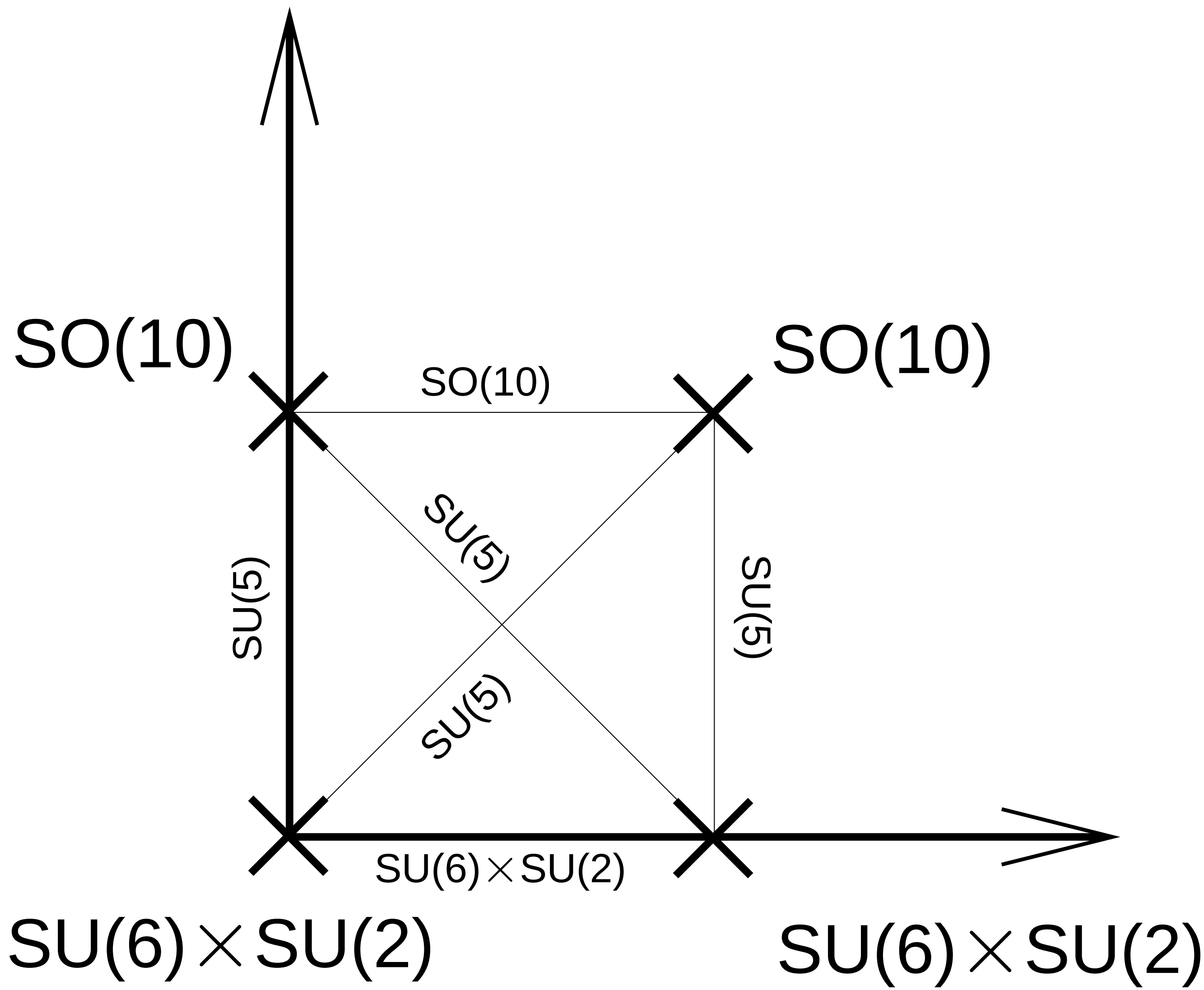
Key properties of the models depend on geometry

- family symmetries
- texture of Yukawa couplings
- number of families
- gauge group on branes

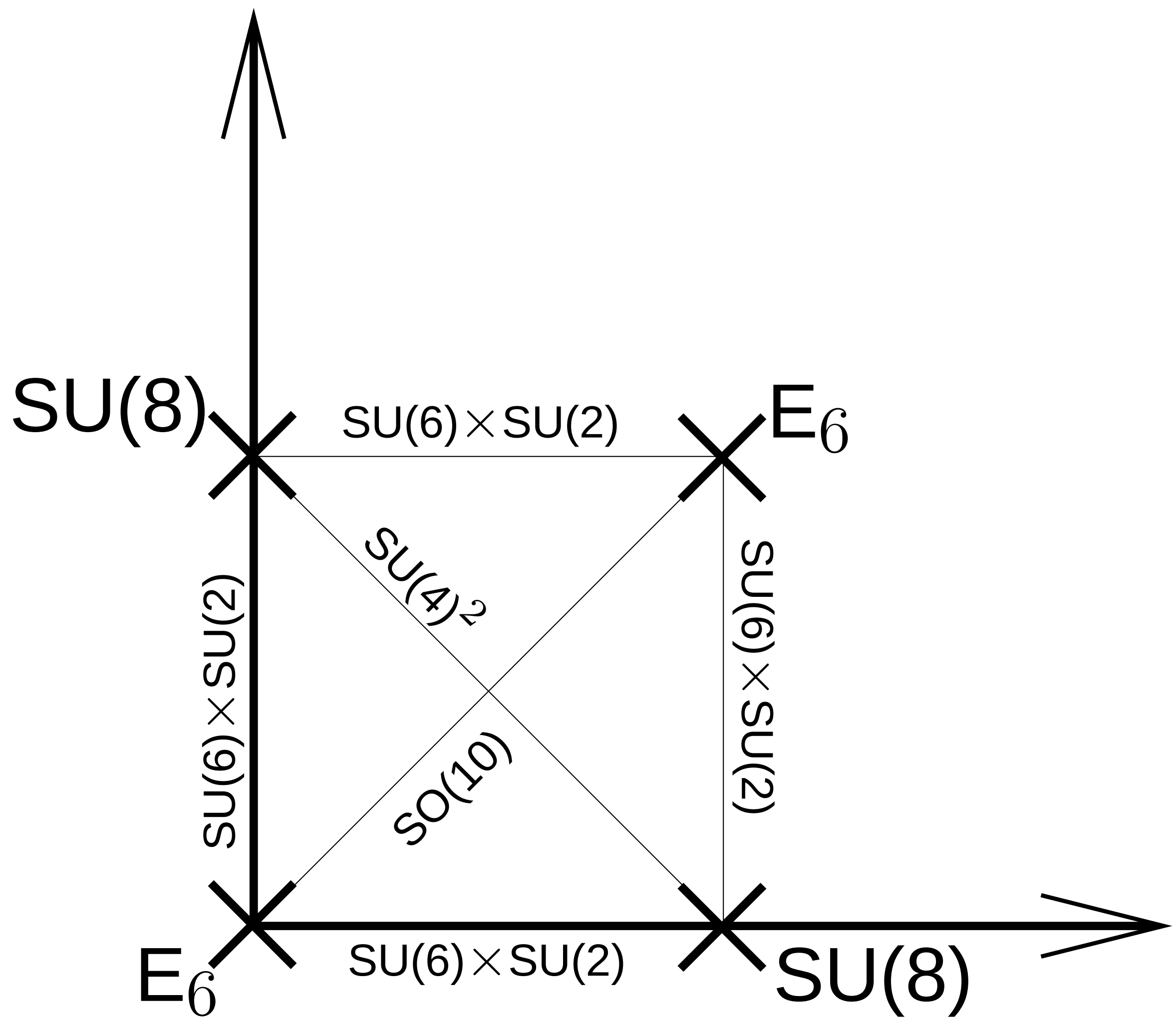
Gauge group geography SO(10)



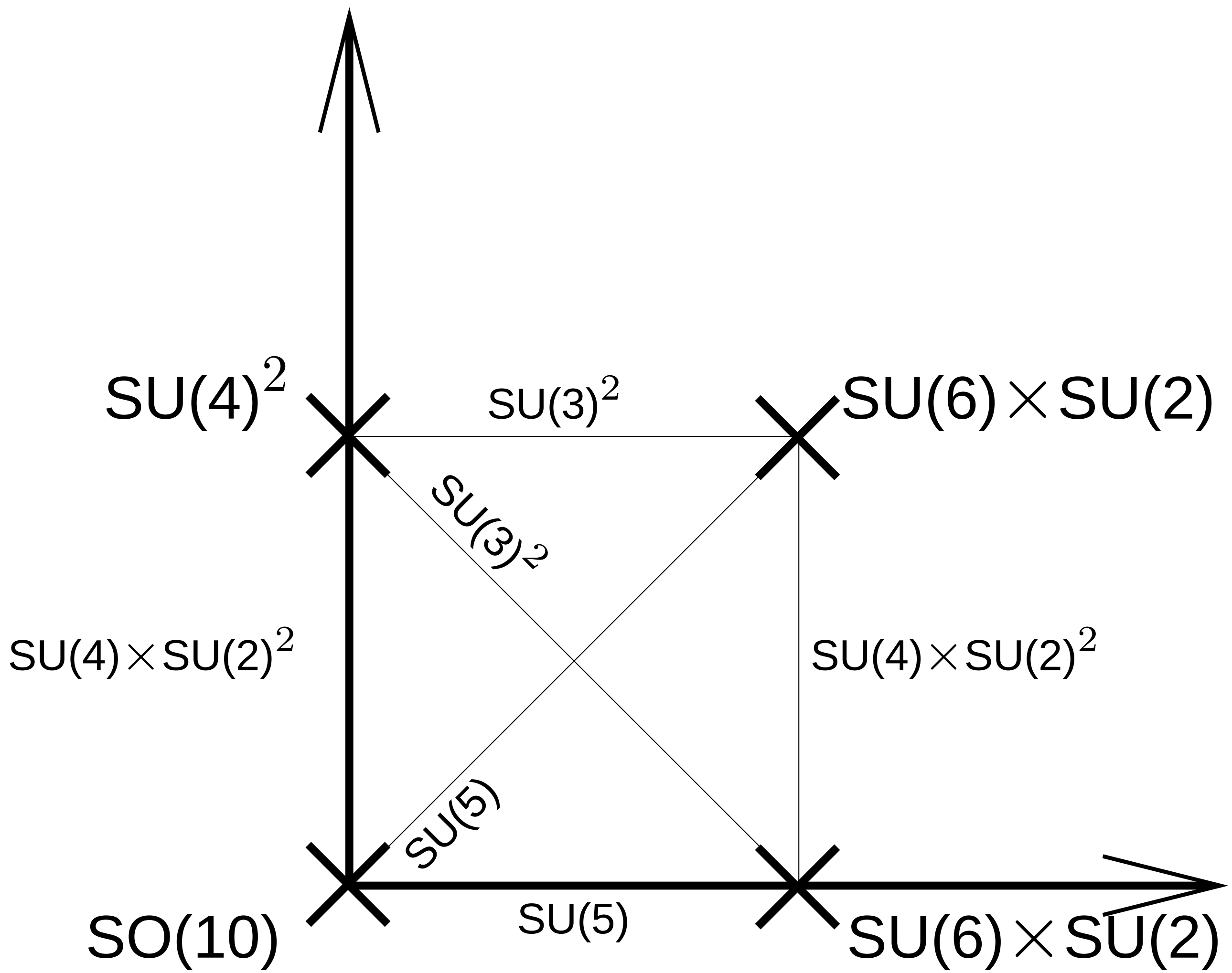
Gauge group geography SU(5)



Gauge group geography: Pati-Salam



Gauge geography: Standard Model



The Memory of SO(10)

- $SO(10)$ is realized in the higher dimensional theory
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Still there could be remnants of $SO(10)$ symmetry

- 16 of SO(10) at some branes
- correct hypercharge normalization
- R-parity
- family symmetries

that are very useful for realistic model building ...

Proton decay

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Thus the proton could be practically stable!

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- presence of fixed tori allows for large threshold corrections at the high scale to match string and unification scale
- gauge-Yukawa unification from SO(10) memory for third family (on an SO(10) brane)
- no gauge-Yukawa unification for first and second family required

Yukawa textures and family symmetries

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- Exponential suppression if fields at distant branes
- neutrino masses need some bulk mixing (one might need anti-families)
- GUT relations could be partially present, depending on the nature of the brane (e.g. SO(10) brane)
- family symmetries arise if different fields live on the same brane

Conclusion

$E_8 \times E_8$ heterotic compactifications might lead to models that incorporate all the successful ingredients of grand unified theories, while avoiding the problematic ones

- spinor representations of SO(10)
- geometric origin of (three) families
- incomplete multiplets
- supersymmetric unification
- R-parity
- “absence” of proton decay
- gauge-Yukawa unification (partial GUT relations)
- discrete family symmetries