Precise predictions from the MSSM

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Outline

specific class of SUSY models:

MINIMAL SUPERSYMMETRIC STANDARD MODEL (MSSM)

- Introduction
- Standard particles
- Higgs bosons
- SUSY particles
- Conclusions

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles:

$$\left[u,d,c,s,t,b\right]_{L,R} \quad \left[e,\mu,\tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u},\tilde{d},\tilde{c},\tilde{s},\tilde{t},\tilde{b}\right]_{L,R} \quad \left[\tilde{e},\tilde{\mu},\tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_L \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin } 1 \text{ / Spin } 0$$

$$\tilde{g} \quad \tilde{\chi}^{\pm}_{1,2} \quad \tilde{\chi}^{0}_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

masses and mixing of SUSY particles through soft-breaking

model parameters

- gaugino masses: M_1, M_2, M_3
- sfermion masses: $M_L, M_{\tilde{u}_R}, M_{\tilde{d}_R}$
  for each doublet of squarks and sleptons
- trilinear coupling: $A_{\tilde{f}}$ for each $\tilde{f}$
  $\rightarrow$ L-R sfermion mixing
- supersymmetric Higgsino mass parameter: μ
- Higgs sector parameters: $M_A, \quad \tan\beta = v_2/v_1$

- chargino masses: $m_{\tilde{\chi}^{\pm}_{1,2}}$ from M_2, μ

$$\begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}$$

- neutralino masses: $m_{\tilde{\chi}^{0}_{1,2,3,4}}$ from M_1, M_2, μ

$$\begin{pmatrix} M_1 & 0 & -M_Z s_W \cos\beta & M_Z s_W \sin\beta \\ 0 & M_2 & M_Z c_W \cos\beta & -M_Z c_W \sin\beta \\ -M_Z s_W \cos\beta & M_Z c_W \cos\beta & 0 & -\mu \\ M_Z s_W \sin\beta & -M_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}$$

- sfermion masses: $m_{\tilde{f}_{1,2}}$ from $M_L, M_{\tilde{f}_R}, A_f$

$$\begin{pmatrix} m_f^2 + M_L^2 + M_Z^2 c_{2\beta}(I_f^3 - Q_f s_W^2) & m_f(A_f - \mu\kappa) \\ m_f(A_f - \mu\kappa) & m_f^2 + M_{\tilde{f}_R}^2 + M_Z^2 c_{2\beta} Q_f s_W^2 \end{pmatrix}$$

with

$$\kappa = \{\cot\beta; \tan\beta\} \quad \text{for} \quad f = \{u, d\}$$

Models of SUSY breaking

generic MSSM: 105 parameters (masses, mixing angles, phases)

reduced to few parameters in specific models

mSUGRA: m_0, $m_{1/2}$, A_0, $\tan\beta$, $\mathrm{sign}(\mu)$

GMSB: $M_{\rm mess}$, $N_{\rm mess}$, $\tan\beta$, $\mathrm{sign}(\mu)$

AMSB: $m_{\rm aux}$, m_0, $\tan\beta$, $\mathrm{sign}(\mu)$

$\rightarrow$ mass parameters at the electroweak scale
$(M_1,\ M_2,\ M_3,\ \mu,\ M_{\tilde{f}_{L,R}},\ \ldots)$

Benchmark scenarios

"Snowmass points and slopes" (SPS), hep-ph/0202233

examples (mSUGRA):

- SPS1a: $m_0 = 100$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -100$, $\tan\beta = 10$, $\mu > 0$.

- SPS1b: $m_0 = 200$ GeV, $m_{1/2} = 400$ GeV, $A_0 = 0$, $\tan\beta = 30$, $\mu > 0$.

Precision analysis required for

- indirect tests of SUSY through
  $\rightarrow$ virtual SUSY effects in precision observables
- precision studies for SUSY particles
  $\rightarrow$ determination of masses & couplings
  $\rightarrow$ reconstruction of model parameters
- direct *versus* indirect tests
  $\rightarrow$ precision observables for precisely measured
  SUSY parameters
  $\rightarrow$ consistency check

Processes with external

(i) standard particles

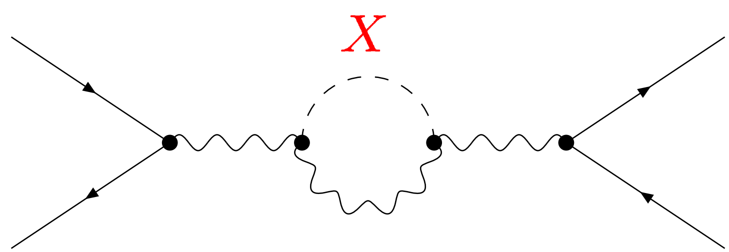
(ii) Higgs bosons, especially light Higgs h^0

(iii) SUSY particles

Standard particles

Test of theory at quantum level:

Sensitivity to loop corrections



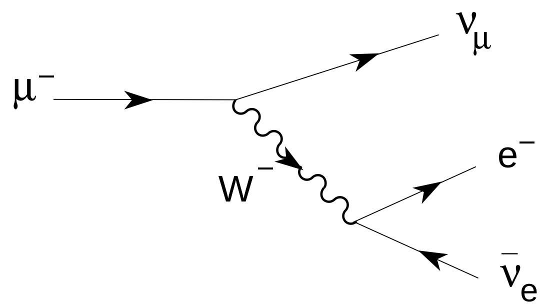
- μ lifetime: M_W, Δr, G_F
- Z observables: g_V, g_A, $\sin^2\theta_\mathrm{eff}$, Γ_Z, $\ldots$

[Heinemeyer, WH, Weiglein, Phys. Rep. 425 (2006) 265]

new: M_W with 2-loop improvements $\mathcal{O}(\alpha\alpha_s,\ \alpha_t^2,\ \alpha_b^2,\ \alpha_t\alpha_b)$ and complex parameters

[Heinemeyer, WH, Stöckinger, A. Weber, Weiglein, hep-ph/0604147]

$M_W - M_Z$ correlation

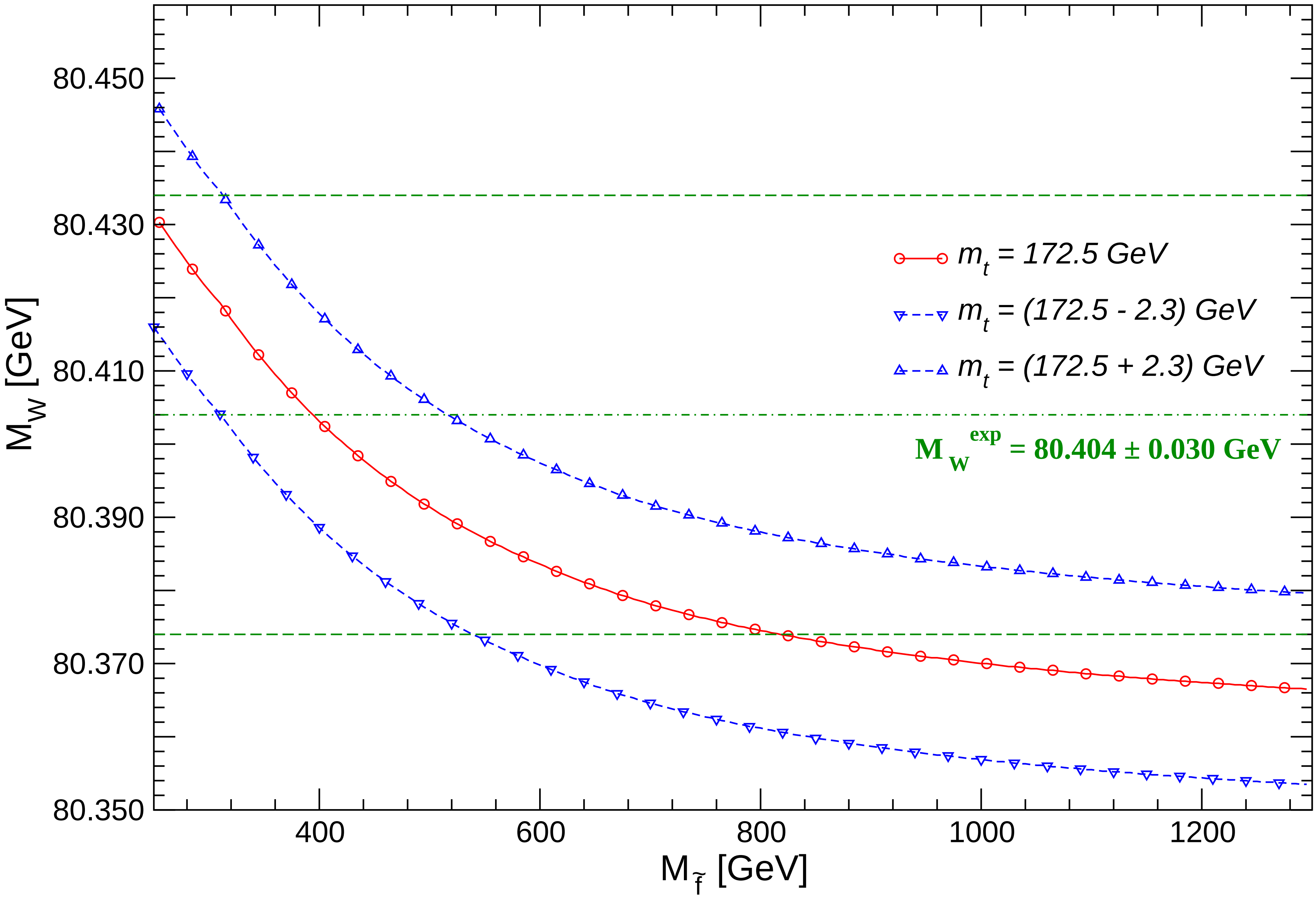


$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2\left(1 - M_W^2/M_Z^2\right)}\left(1 + \Delta r\right)$$

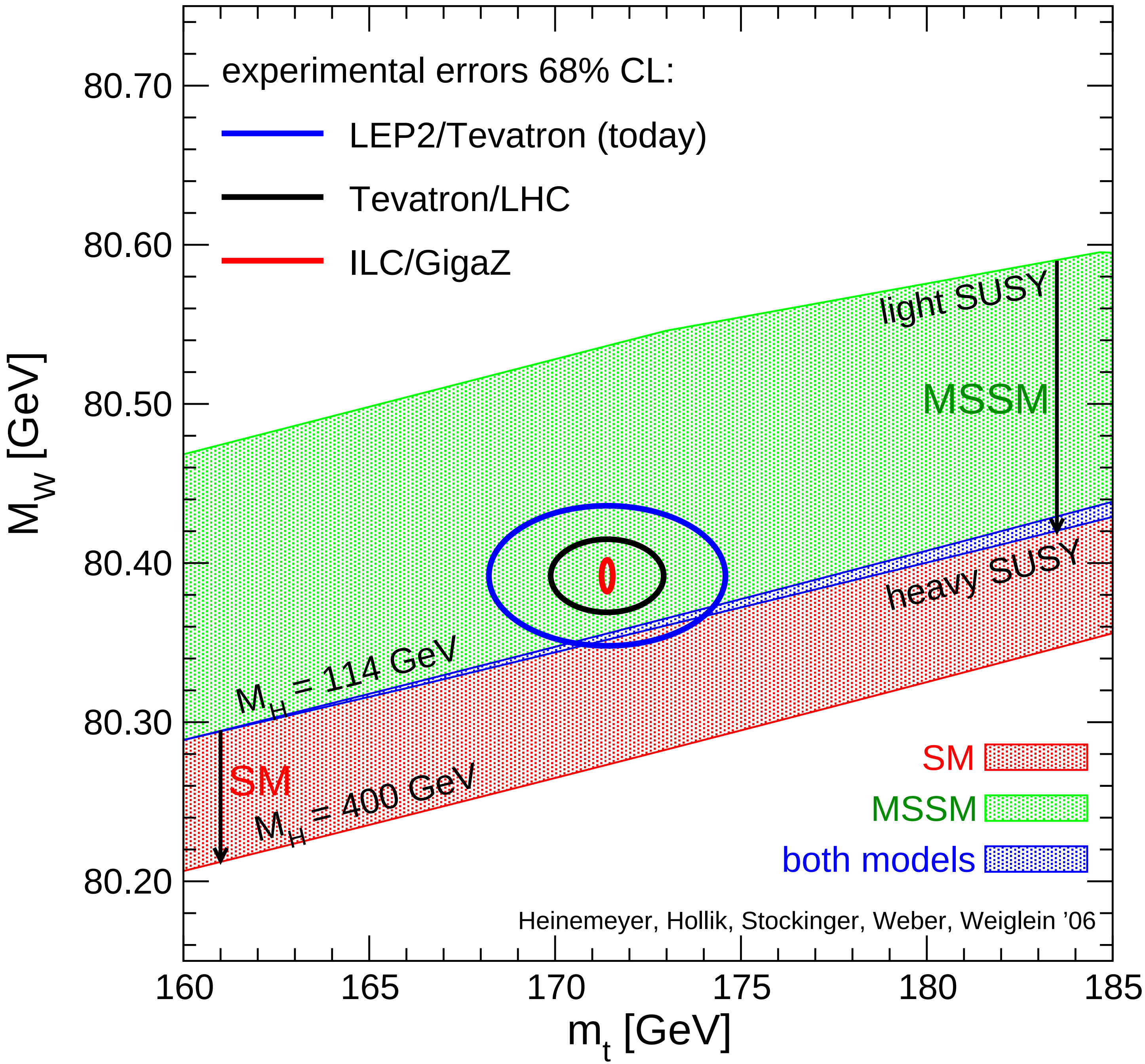
Δr : quantum correction, $\Delta r = \Delta r(m_t, X_{\mathrm{SUSY}})$

$\rightarrow \quad M_W = M_W(\alpha, G_F, M_Z, m_t, X_{\mathrm{SUSY}})$

X_{SUSY} = set of non-standard model parameters

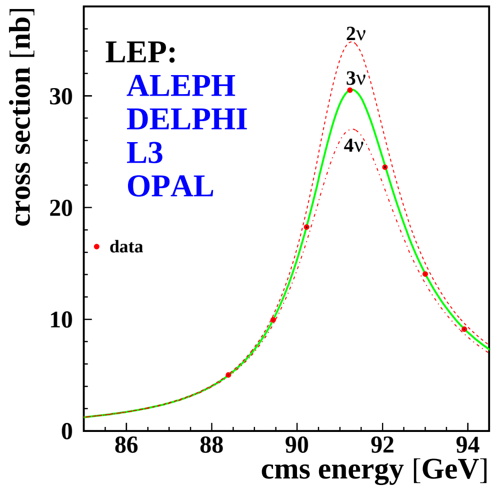


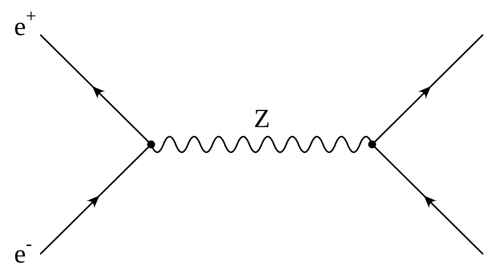
$$\tan\beta = 10, \quad M_A = \mu = M_2 = M_{\tilde{g}} = 300\,\mathrm{GeV}$$



experimental errors 68% CL:
LEP2/Tevatron (today)
Tevatron/LHC
ILC/GigaZ
light SUSY
MSSM
heavy SUSY
$M_H = 114$ GeV
SM
$M_H = 400$ GeV
SM
MSSM
both models
Heinemeyer, Hollik, Stockinger, Weber, Weiglein '06
80.70
80.60
80.50
80.40
80.30
80.20
M_W [GeV]
160
165
170
175
180
185
m_t [GeV]

Z resonance



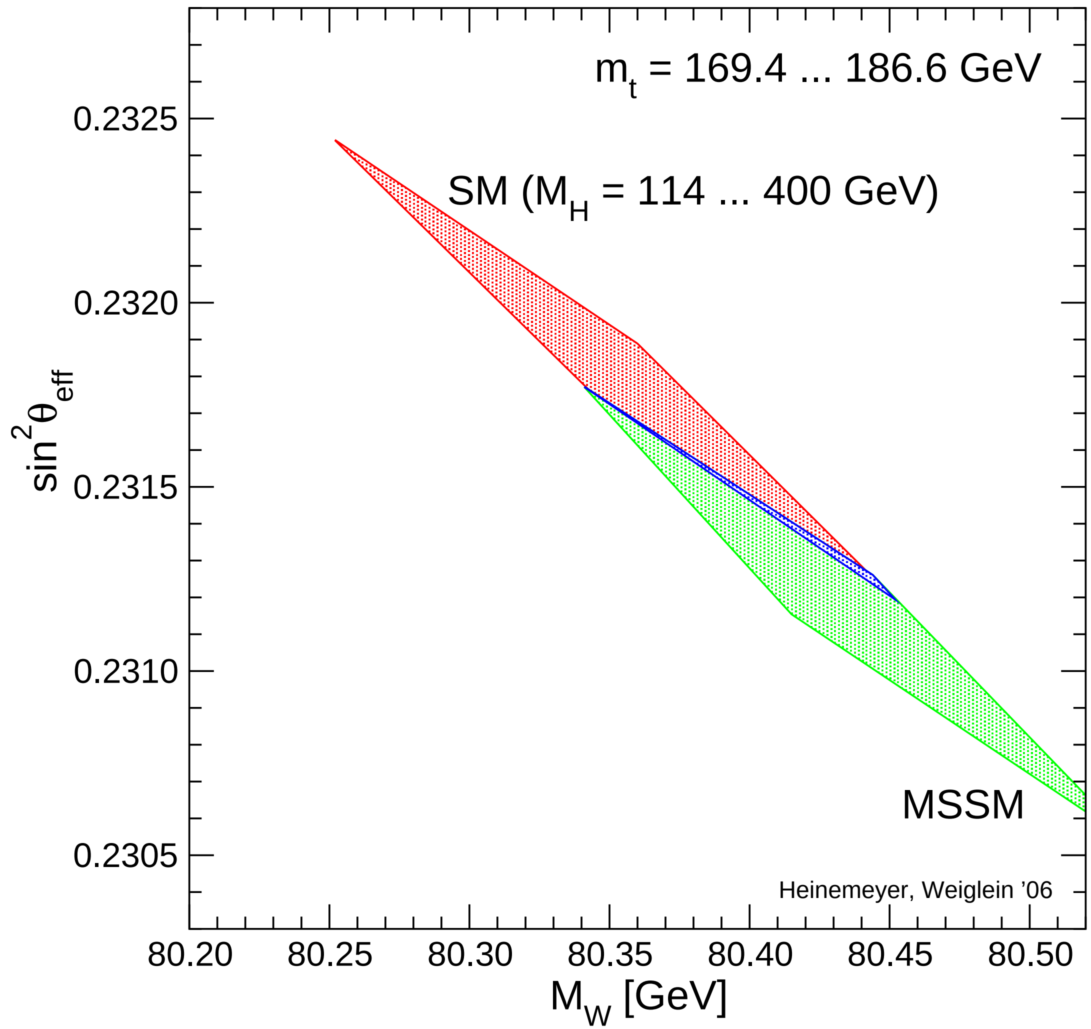


- effective Z boson couplings with higher-order $\Delta g_{V,A}$

$$g_V^f \longrightarrow g_V^f + \Delta g_V^f, \qquad g_A^f \longrightarrow g_A^f + \Delta g_A^f$$

- effective ew mixing angle (for $f = e$):

$$\sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \text{Re}\,\frac{g_V^e}{g_A^e}\right)$$



m_t = 169.4 ... 186.6 GeV
SM (M_H = 114 ... 400 GeV)
MSSM
Heinemeyer, Weiglein '06
$\sin^2\theta_{eff}$
0.2325
0.2320
0.2315
0.2310
0.2305
80.20
80.25
80.30
80.35
80.40
80.45
80.50
M_W [GeV]

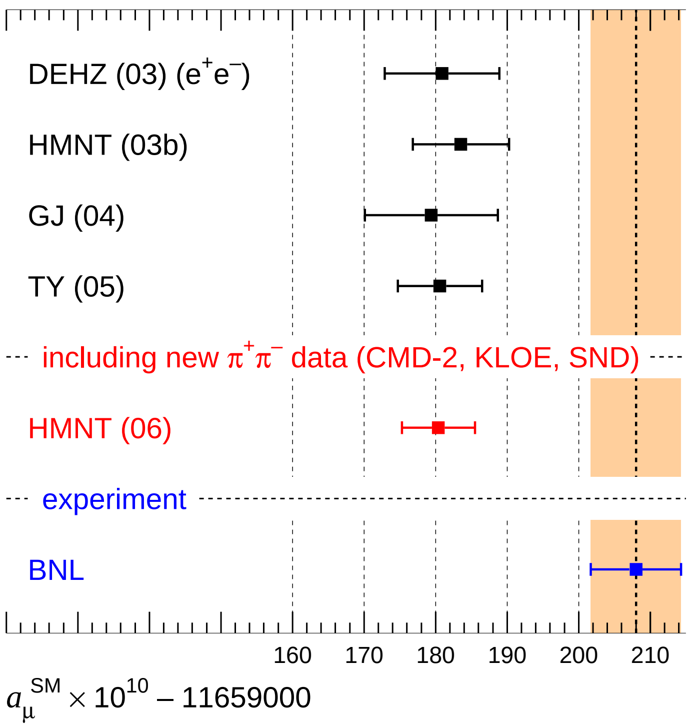
Anomalous g-factor of the muon

- Dirac theory: $g = 2$
- QED, 1-loop order: $g = 2 + \frac{\alpha}{\pi}$
- Standard Model prediction
  QED part: 4-loop (5-loop estimate)
  Electroweak part: 2-loop
- **Experiment 2004:** Brookhaven E821

$$a_\mu = \frac{g-2}{2} = 11659208(6) \cdot 10^{-10}$$

above the SM prediction

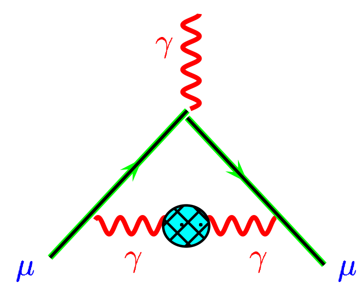
Theory versus experiment



Hagiwara, Martin, Nomura, Teubner

e^+e^- data based SM prediction: 3.4 σ below exp. value

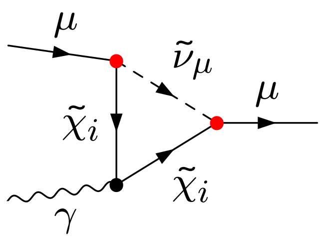
theory uncertainty from hadronic vacuum polarization

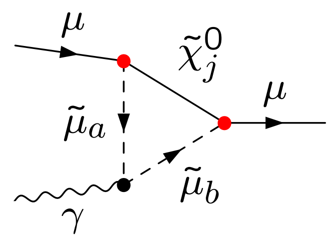


$g-2$ with supersymmetry

new contributions from virtual SUSY partners of $\mu,\ \nu_\mu$

and of $W^\pm,\ Z$





extra terms

$$+\quad \frac{\alpha}{\pi}\,\frac{m_\mu^2}{M_{\rm SUSY}^2}\cdot\frac{v_2}{v_1}$$

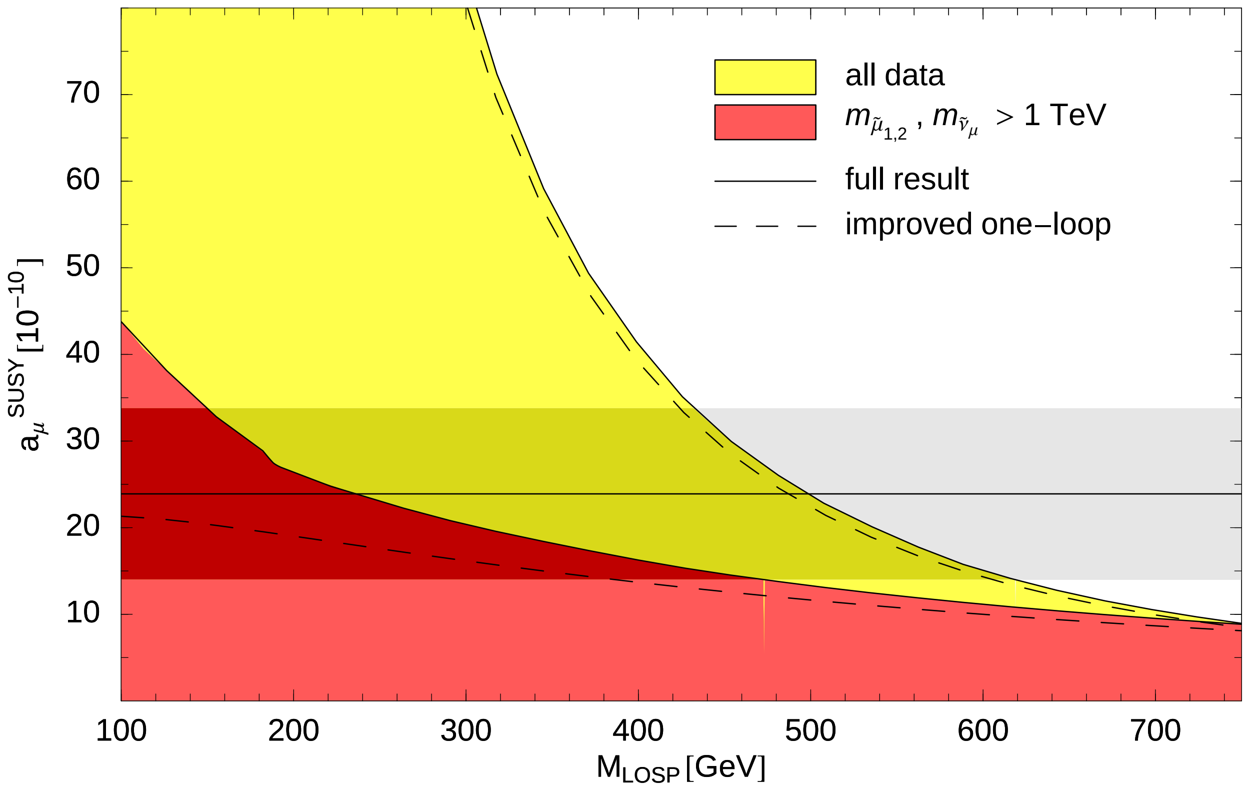
can provide missing contribution for

$$M_{\rm SUSY} = 200 - 600\,{\rm GeV}$$

2-loop calculation *[Heinemeyer, Stöckinger, ...]*

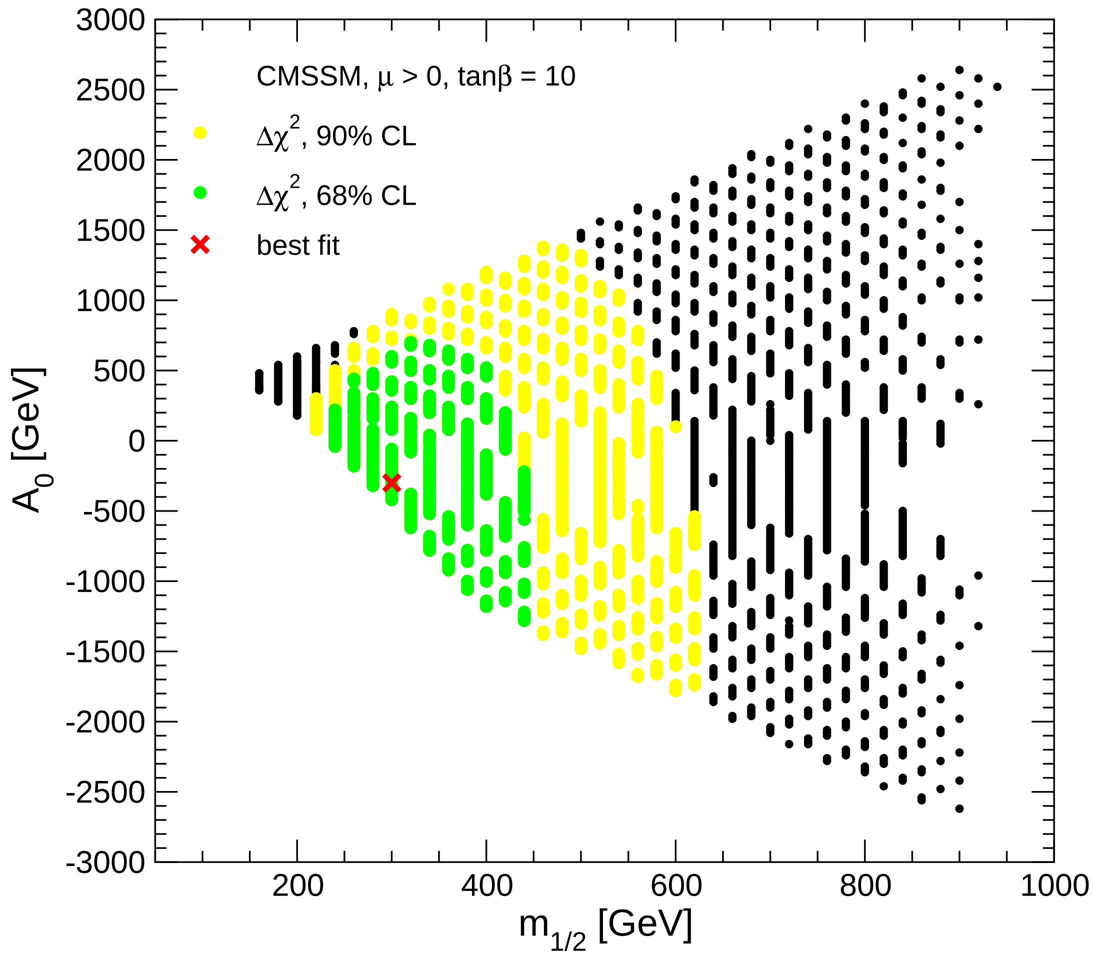
scan over SUSY parameters compatible with EW and $b \to s\gamma$ constraints $(\tan\beta = 50)$

[Stöckinger]



LOSP = lightest observable SUSY particle $(\chi_1^\pm, \chi_2^0, \cdots)$

fits in SUGRA model:



[Ellis, Heinemeyer, Olive, Weiglein]

Higgs bosons in the MSSM

MSSM Higgs potential contains two Higgs doublets:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$
$$+ \underbrace{\frac{g'^2 + g^2}{8}}\,(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}\,|H_1 \bar{H}_2|^2$$

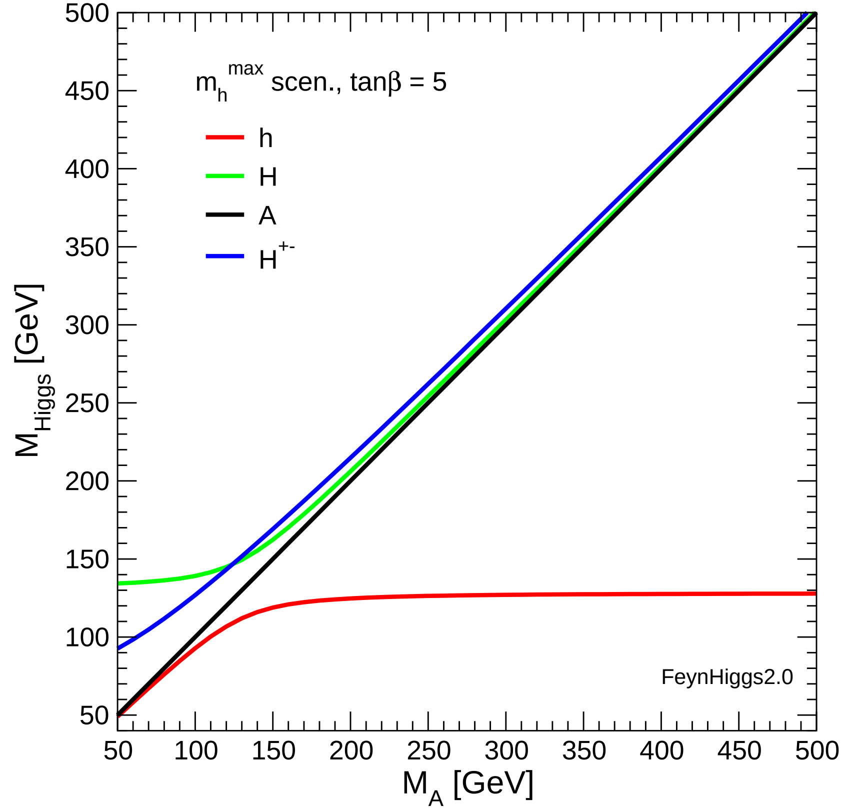
gauge couplings, in contrast to SM

Five physical states: $h^0, H^0, A^0, H^\pm$

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, $M_{\rm A}$

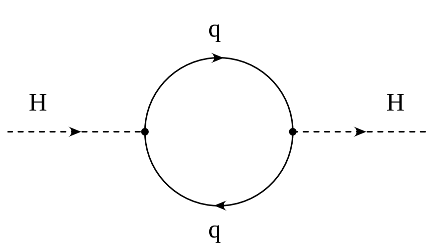
$\Rightarrow$ $m_{\rm h}$, $m_{\rm H}$, mixing angle α, $m_{{\rm H}^\pm}$: no free parameters

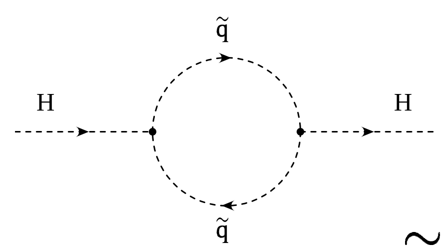
Spectrum of Higgs bosons in the MSSM (example)



large M_A: h^0 like SM Higgs boson $\sim$ decoupling regime

m_h^0 strongly influenced by quantum effects, *e.g.*





$\sim \alpha_t$

determination of masses and couplings at higher order

- physical states $h,\ H,\ A,\ H^{\pm}$
- conventional input: M_A, $\tan\beta = v_2/v_1$

dressed $h,\ H$ propagators, renormalized self-energies $\hat{\Sigma}$

$$\left(\Delta_{\text{Higgs}}\right)^{-1} = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_H(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{Hh}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_h(q^2) \end{pmatrix}$$

- det = 0 $\rightarrow$ $m_{h,H}^{\text{pole}}$
- diagonalization $\rightarrow$ effective couplings (α_{eff})

1-loop: complete

2-loop:

– QCD corrections $\sim \alpha_s \alpha_t,\ \alpha_s \alpha_b$

– Yukawa corrections $\sim \alpha_t^2$

present theoretical uncertainty:

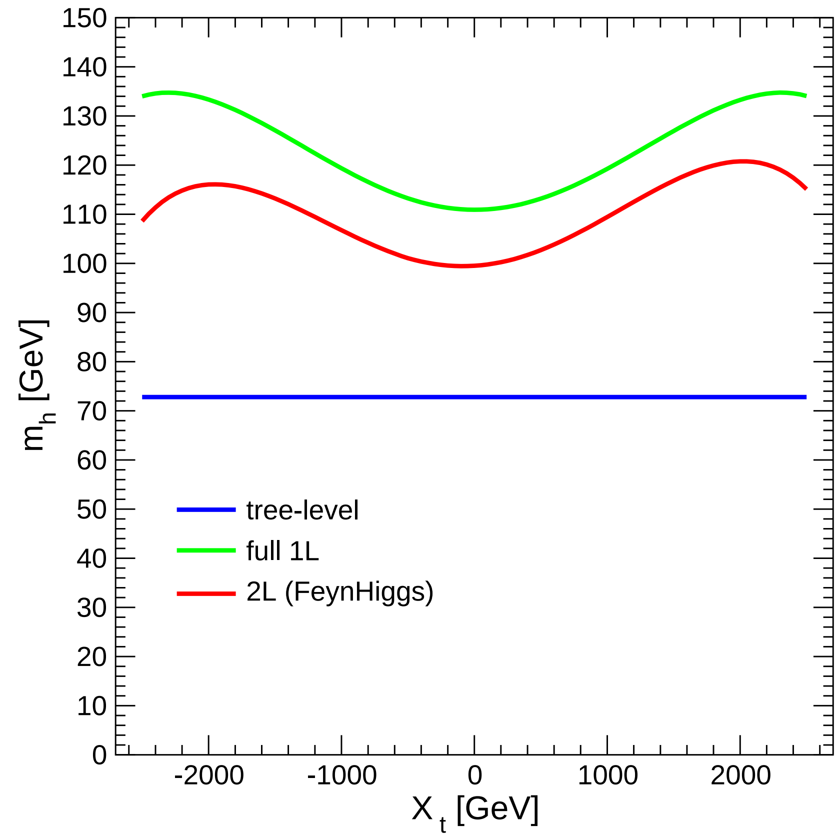
$\boldsymbol{\delta m_h} \simeq$ 3-4 GeV

[Degrassi, Heinemeyer, WH, Slavich, Weiglein]

new version

`FeynHiggs2.5`

m_{h^0} prediction at different levels of accuracy:

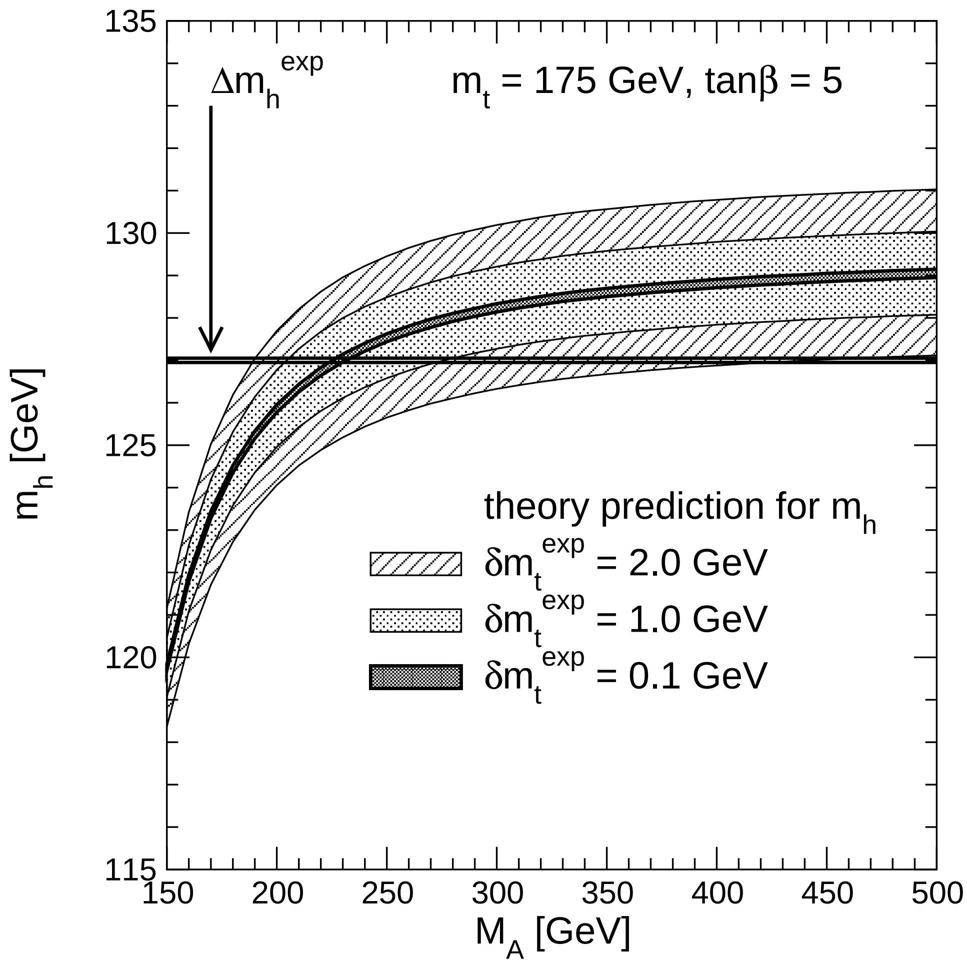


$\tan\beta = 3, \quad M_{\tilde{Q}} = M_A = 1$ TeV, $\quad m_{\tilde{g}} = 800$ GeV

X_t : top-squark mixing parameter

$$\mathrm{X_t} = \mathrm{A_t} - \mu \cot\beta$$

[Heinemeyer, Kraml, Porod, Weiglein]



dependent on all SUSY particles and masses/mixings through Higgs self-energies

Recent developments:

1. Counterterms at two-loop order

   ST identities valid in dimensional reduction (DR)

   DR scheme consistent with symmetric counterterms

   [WH, Stöckinger]

2. $\mathcal{O}(\alpha_s \alpha_b)$ beyond m_b^{eff} approximation

   $m_b^{\text{eff}} = \frac{m_b}{1+\Delta m_b} \quad \text{in} \quad \alpha_b$ Yukawa coupling

   $\Delta m_b =$ non-decoupling SUSY contribution $\sim \alpha_s\, \mu\, \tan\beta$

   [Heinemeyer, WH, Rzehak, Weiglein]

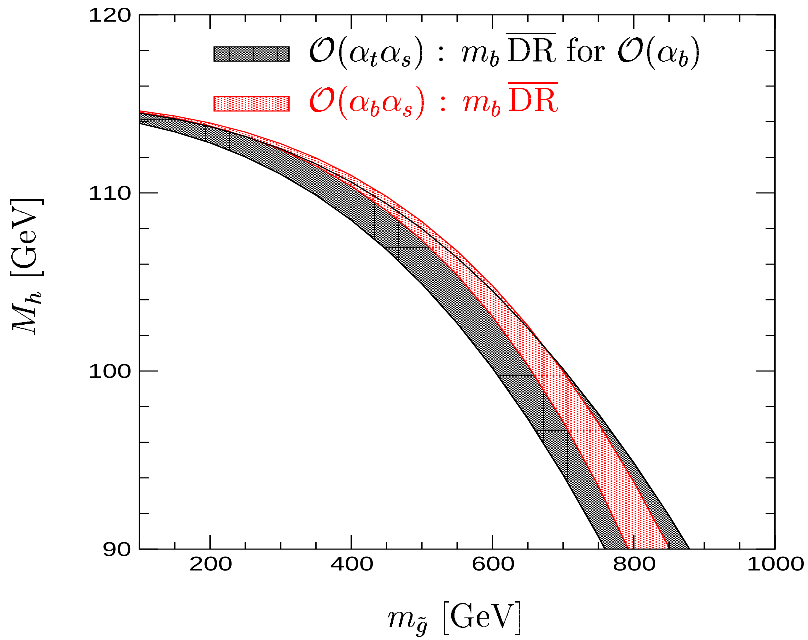
   small shifts $\sim$ few GeV, but stabilizes prediction

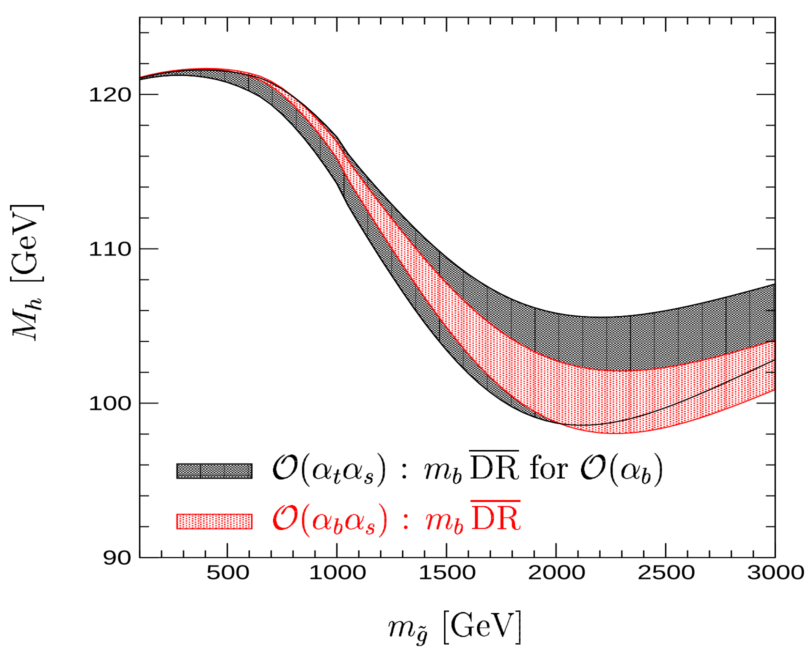
3. MSSM with complex parameters

   tree level: CP conserving Higgs sector

   loop level: CP violation $\leftarrow$ other sectors

   [Frank, Heinemeyer, WH, Rzehak, Weiglein]





$$m_t/2 < \mu^{\overline{\mathrm{DR}}} < 2\,m_t$$

$$M_A = \begin{cases} 120\,\mathrm{GeV} \\ 700\,\mathrm{GeV} \end{cases}$$

The Higgs sector of the cMSSM at the loop-level:

Complex parameters enter via loop corrections:

– μ : Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings $\Rightarrow$ $X_{t,b,\tau} = A_{t,b,\tau} - \mu^* \{\cot\beta, \tan\beta\}$ complex

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– M_3 : gluino mass parameter

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

Result:

$$(A, H, h) \to (h_3, h_2, h_1)$$

with

$$m_{h_3} > m_{h_2} > m_{h_1}$$

$\Rightarrow$ strong changes in Higgs couplings to SM gauge bosons and fermions

Inclusion of higher-order corrections:

$\rightarrow$ Feynman-diagrammatic approach

Propagator / mass matrix with higher-order corrections:

$$\begin{pmatrix} q^2 - M_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H, A)$: renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0$ $\Rightarrow$ $\mathcal{CP}$V, $\mathcal{CP}$-even and $\mathcal{CP}$-odd fields can mix

propagator matrix Δ:

$$\boxed{\Delta^{-1}(q^2) = q^2\,\mathbf{1} - \mathbf{m}^2_{\text{tree}} + \hat{\mathbf{\Sigma}}(q^2)}$$

renormalized self energies:

$$\hat{\Sigma} = \Sigma + \text{counter terms}$$

- renormalization of tadpoles:

  $T_h + \delta T_h = 0, \quad T_H + \delta T_H = 0, \quad T_A + \delta T_A = 0$

- renormalization of $M_{H^\pm}$: $\quad \delta M^2_{H^\pm} = \Sigma_{H^\pm}(M^2_{H^\pm})$

  on-shell conditon for pole mass

- renormalization of $\tan\beta$:

$$\tan\beta = \frac{v_2}{v_1} \quad \to \quad \sqrt{\frac{Z_{H_2}}{Z_{H_1}}} \cdot \frac{v_2 + \delta v_2}{v_1 + \delta v_1}$$

$$= \quad \frac{v_2}{v_1}\left(1 + \delta Z_{H_2} - \delta Z_{H_1} + \frac{\delta v_2}{v_2} - \frac{\delta v_1}{v_1}\right)$$

$$\overline{DR} \qquad\qquad = 0$$

propagator matrix Δ:

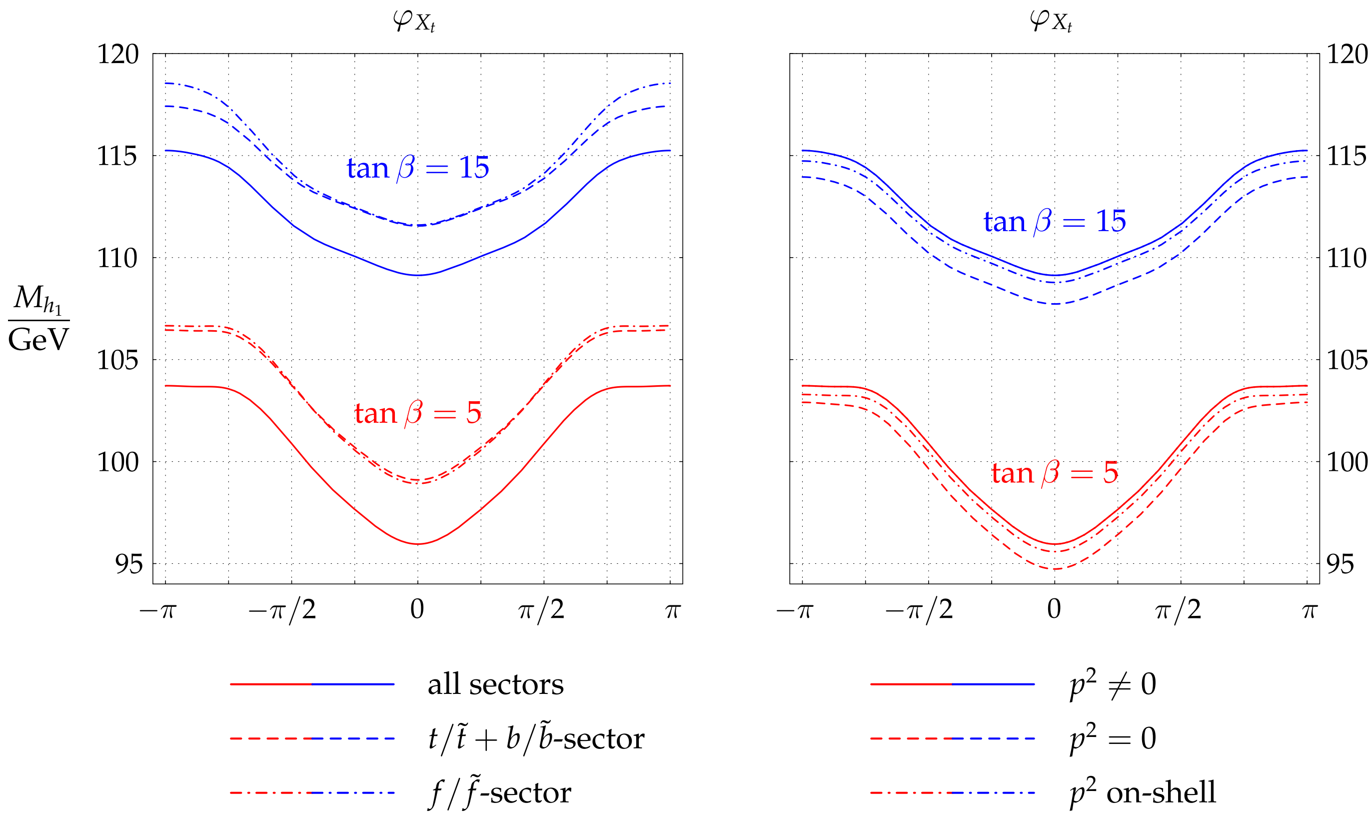
$$\boxed{\Delta^{-1}(p^2) = p^2\,\mathbf{1} - \mathbf{m}^2_{\rm tree} + \hat{\mathbf{\Sigma}}(p^2)}$$

- $\hat{\mathbf{\Sigma}}(p^2)$ contain imaginary parts
- Higgs boson masses are complex poles:
  $s_0 = M^2 - \mathrm{i} M\Gamma$
- zeros of determinant: $\det[\Delta^{-1}(s_0)] = 0$

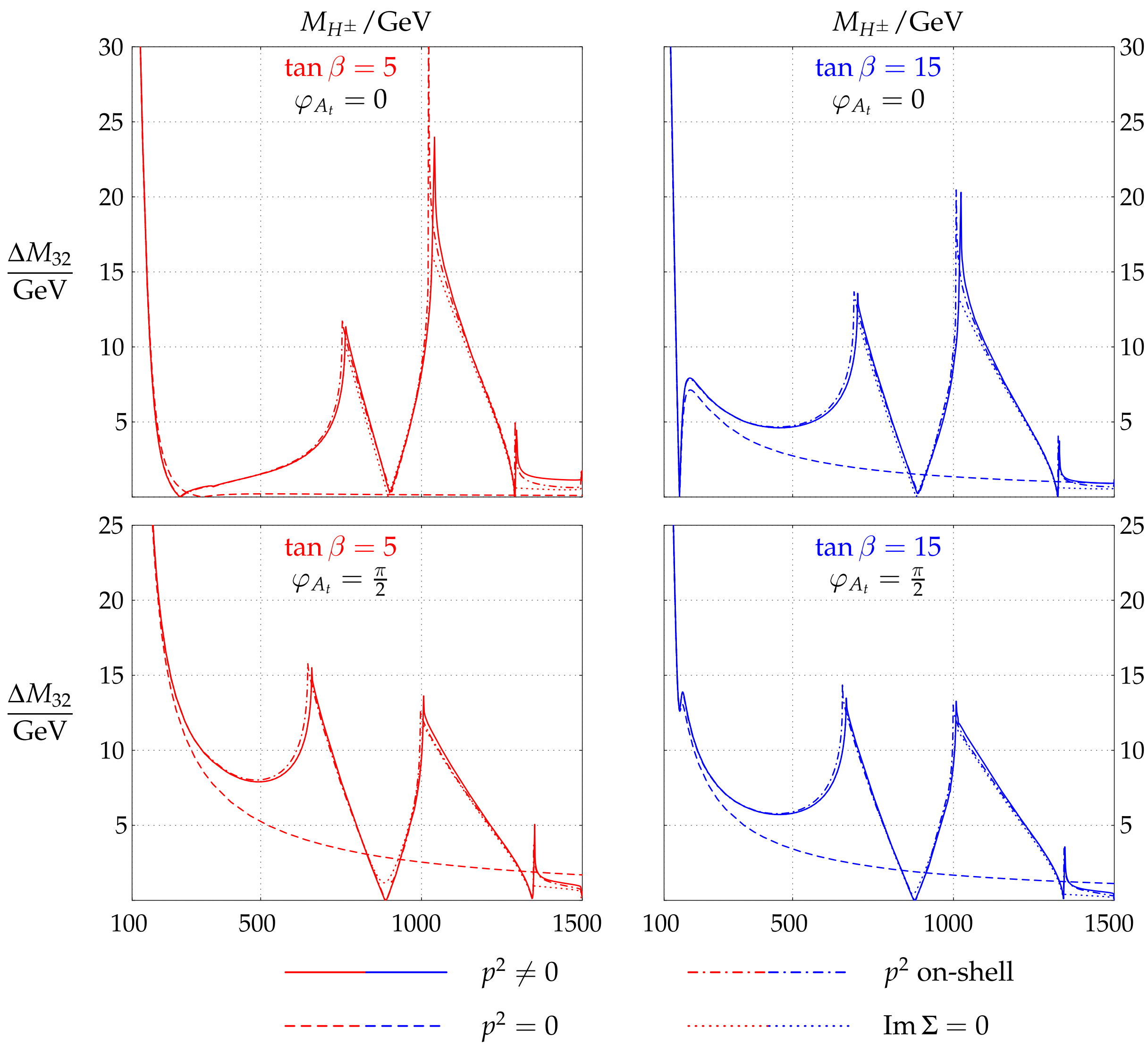
$p^2 = 0$ approximation: $$\boxed{\mathbf{M}^2 \;\simeq\; \mathbf{m}^2_{\rm tree} - \hat{\mathbf{\Sigma}}(0)}$$

diagonalized by orthogonal matrix **R**

on-shell approximation: $\hat{\Sigma}_{ii}(m_i^2),\ \hat{\Sigma}_{ij}(\frac{m_i^2+m_j^2}{2})$



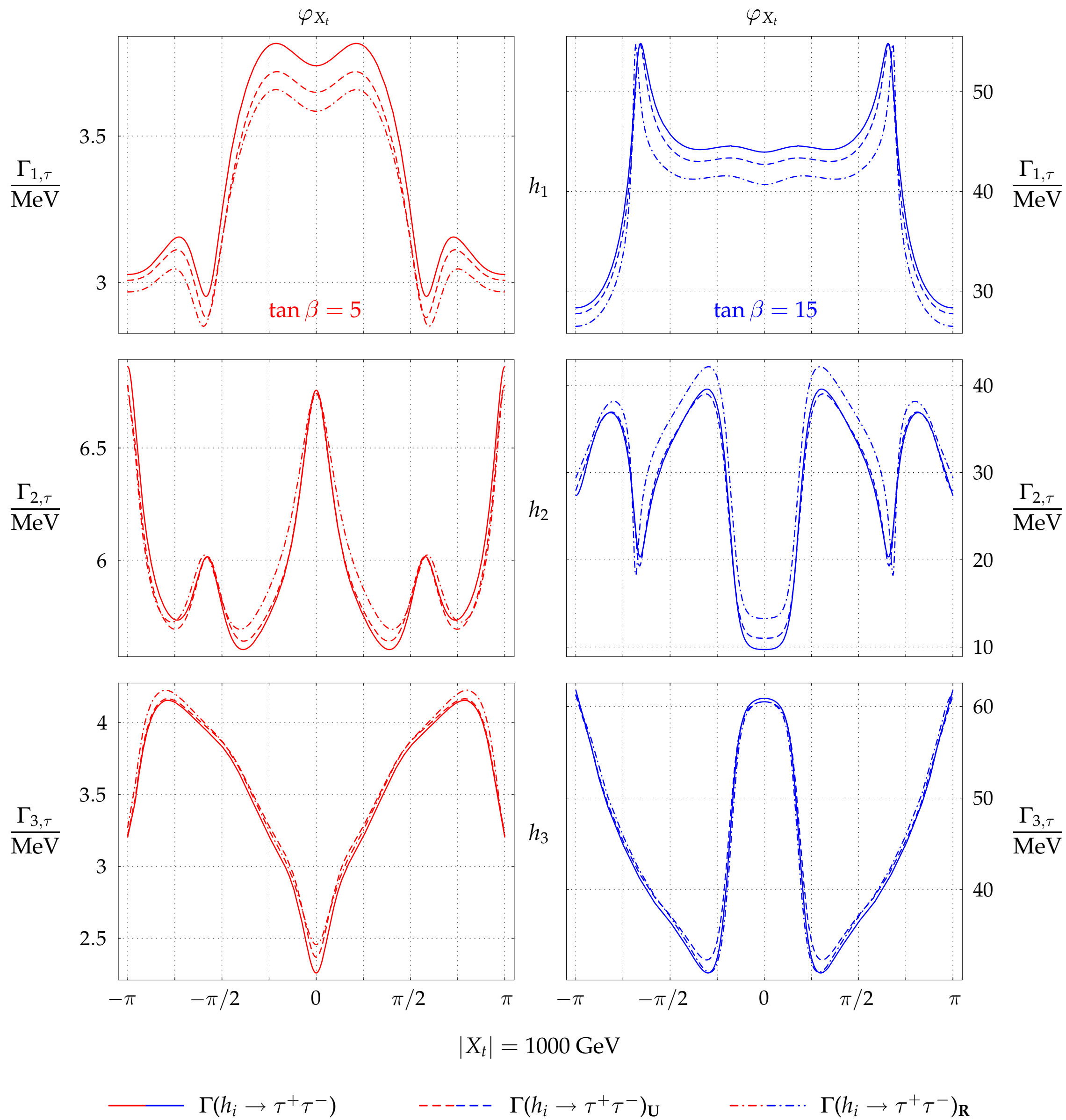
[Frank, Hahn, Heinemeyer, WH, Rzehak, Weiglein]



$M_{H^\pm}$ /GeV
$M_{H^\pm}$ /GeV
tan $\beta = 5$
$\varphi_{A_t} = 0$
tan $\beta = 15$
$\varphi_{A_t} = 0$
tan $\beta = 5$
$\varphi_{A_t} = \frac{\pi}{2}$
tan $\beta = 15$
$\varphi_{A_t} = \frac{\pi}{2}$
$\frac{\Delta M_{32}}{\text{GeV}}$
$\frac{\Delta M_{32}}{\text{GeV}}$
30
25
20
15
10
5
100
500
1000
1500
$p^2 \neq 0$
p^2 on-shell
$p^2 = 0$
$\mathrm{Im}\,\Sigma = 0$

implications for couplings

example: $h_i \to \tau^+ \tau^-$



present status:

effective potential approximation + RGE

[Carena, Ellis, Pilaftsis, Wagner]

complete at one-loop order

[Frank, Hahn, Heinemeyer, WH, Rzehak, Weiglein]

leading two-loop contributions of $\mathcal{O}(\alpha_s \alpha_t)$

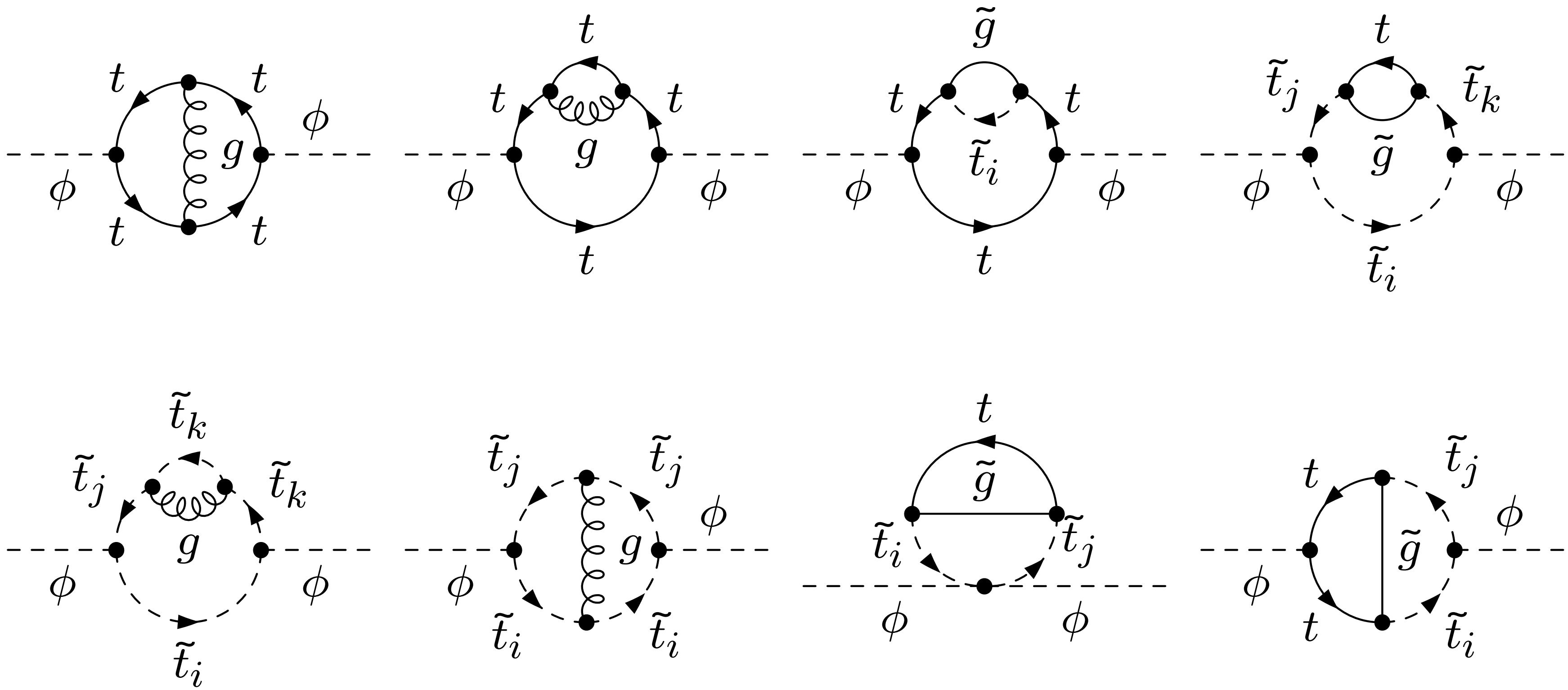
[Rzehak, PhD thesis]

for Higgs phenomenology with CP violation see

CERN 2006-009, hep-ph/0608079 *[S. Kraml et al. (Conv.)]*

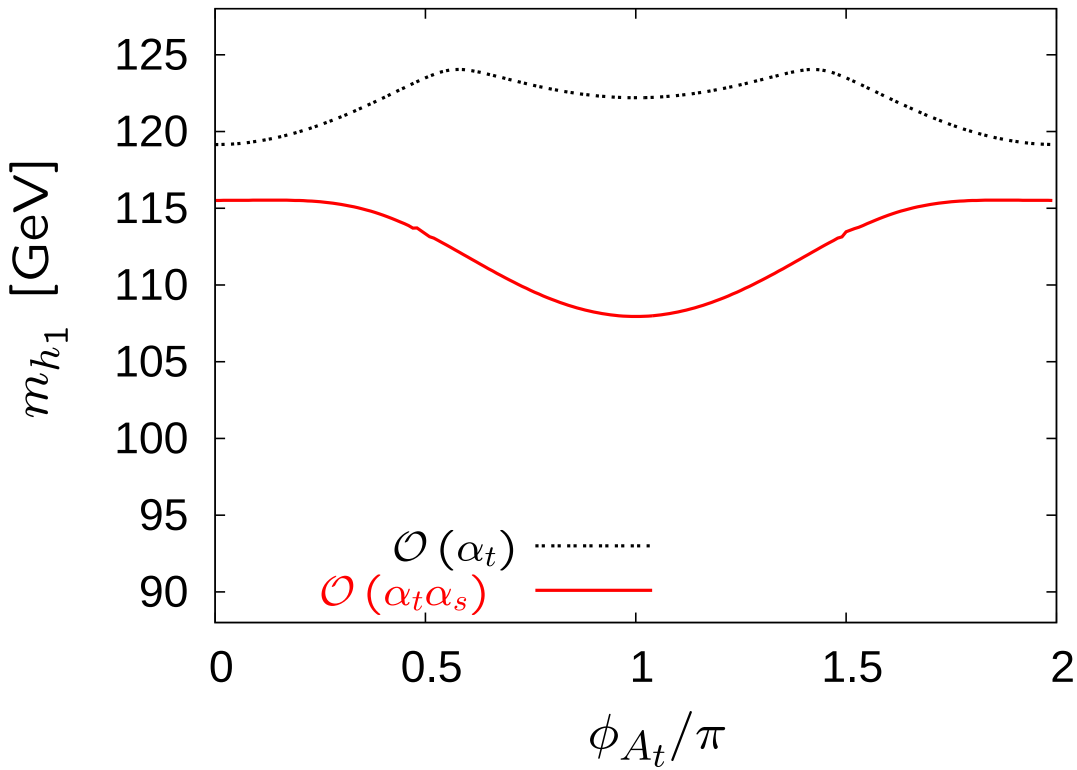
Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



$\phi = h, H, A$

m_{h_1} as a function of ϕ_{A_t}:

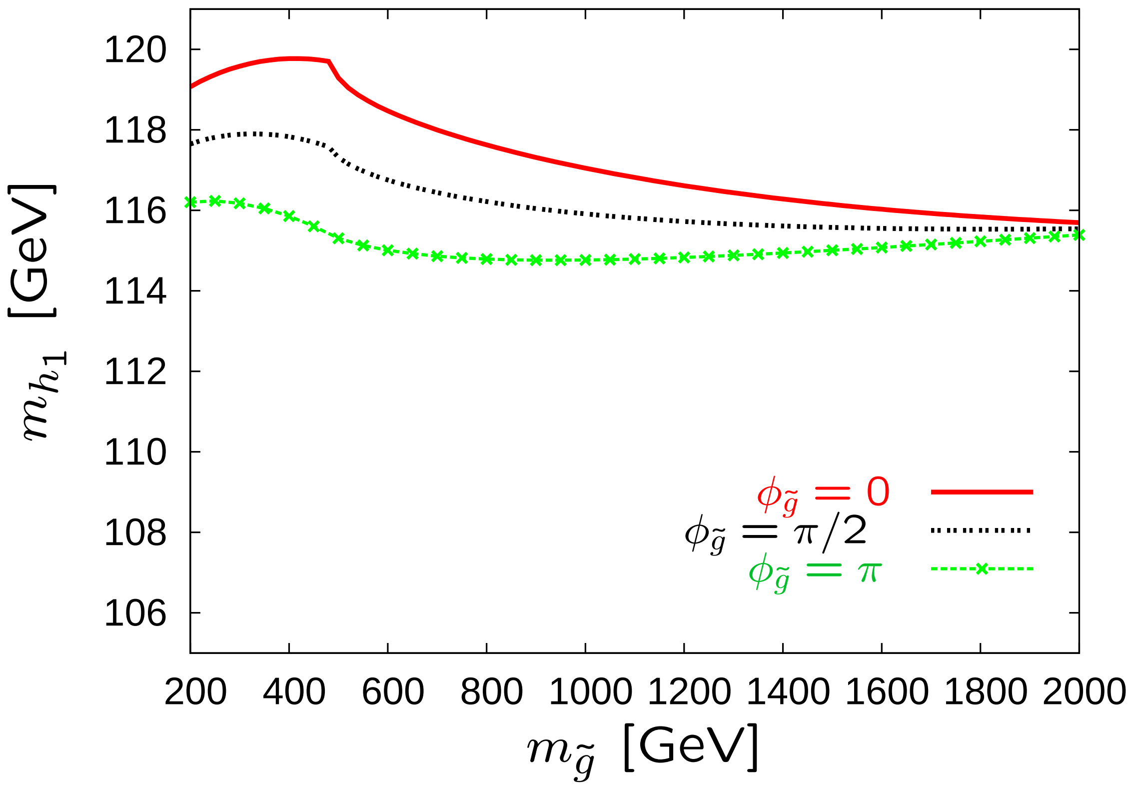


$M_{\mathrm{SUSY}} = 1000$ GeV
$|A_t| = 2000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 150$ GeV

OS renormalization

$\Rightarrow$ modified dependence on ϕ_{A_t} at the 2-loop level

m_{h_1} as a function of $\phi_{\tilde{g}}$:



$M_{\text{SUSY}} = 500$ GeV
$A_t = 1000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

$\Rightarrow$ threshold at $m_{\tilde{g}} = m_{\tilde{t}} + m_t$

$\Rightarrow$ large effects around threshold

$\Rightarrow$ phase dependence has to be taken into account

SUSY particles

$\boxed{\text{from experiment:}}$

$\longrightarrow$ precision analyses of masses and couplings LHC$\oplus$ILC

$\boxed{\text{from theory:}}$

$\longrightarrow$ accurate theoretical predictions to match exp. data

$\longrightarrow$ loop contributions to Lagrangian param $\leftrightarrow$ observables

$\longrightarrow$ reconstruction of fundamental SUSY parameters and breaking mechanism

$\longrightarrow$ RGEs for extrapolation to high scales

- **chargino/neutralino sector**
  complete at one loop:
  renormalization and mass spectrum
  pair production and decay processes
  [Fritzsche, WH] *[Eberl, Majerotto, Öller]*

- **sfermion sector**
  renormalization and mass spectrum
  [WH, Rzehak]
  sfermion pair production in e^+e^- collisions
  complete at one-loop

[Arhrib, WH]	squarks, sleptons
[Kovarik, Weber, Eberl, Majerotto]	squarks
[Freitas, Miller, von Manteuffel, Zerwas]	sleptons

- **sfermion decays into fermions and -inos**
  complete at one-loop
  [Guasch, WH, Solà]

Basis for precision calculations

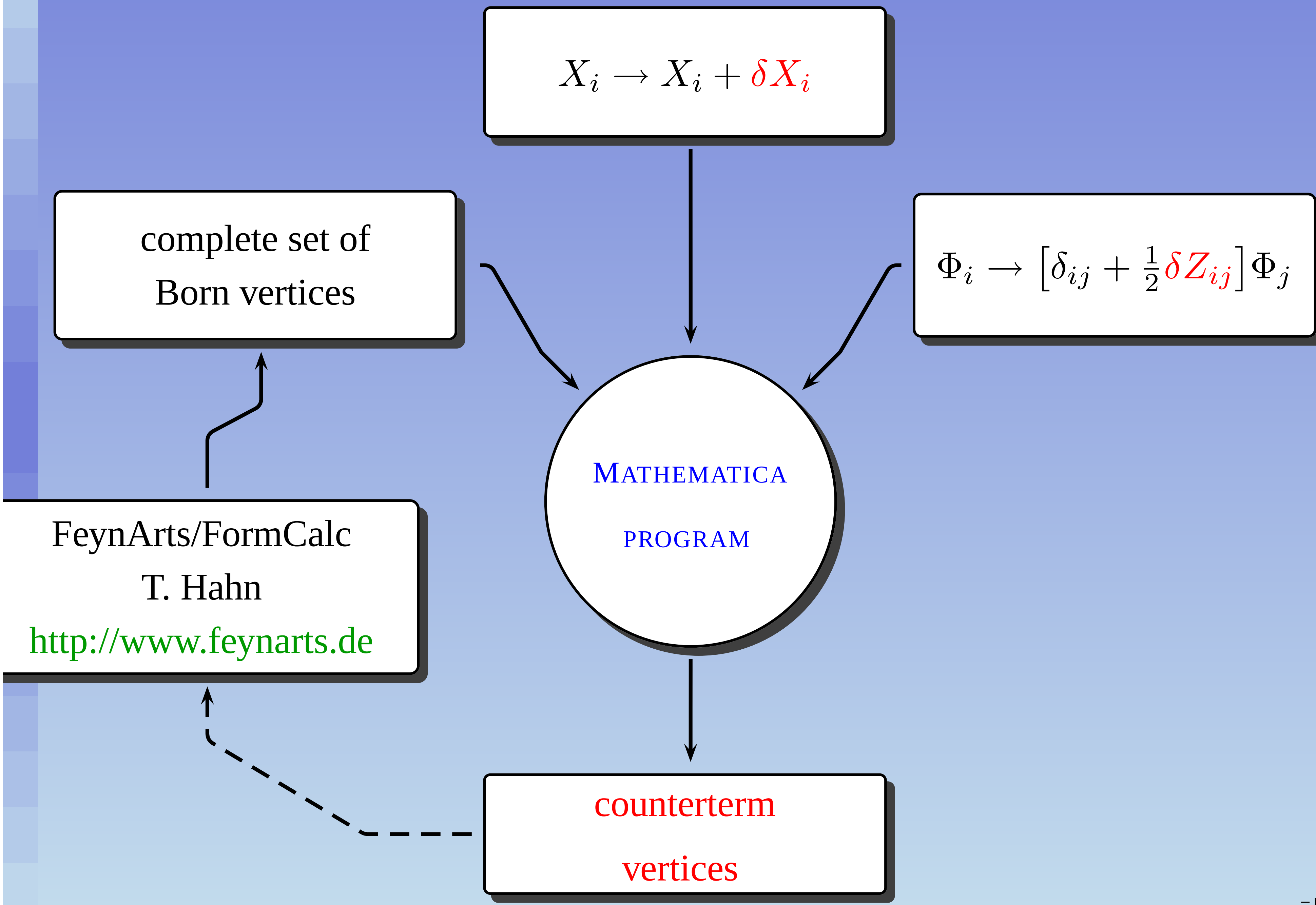
- complete Feynman rules $\longrightarrow$ FeynArts
  [Hahn, Schappacher]
- complete set of counter terms
  automatic generation $\longrightarrow$ FeynArts *[Fritzsche]*
- real photon bremsstrahlung

Renormalization schemes

- on-shell scheme:
  renormalization conditions for pole masses
  [WH, Kraus, Roth, Rupp, Sibold, Stöckinger]
- $\overline{\mathrm{DR}}$ scheme:
  CTs = singular parts in dimensional reduction

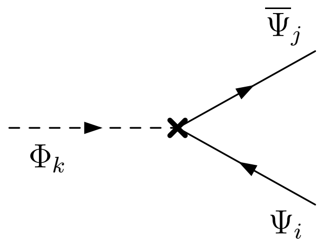
SUSY parameters different in $\overline{\mathrm{DR}}$ and on-shell

Automatic generation of CTs



example:

FFS

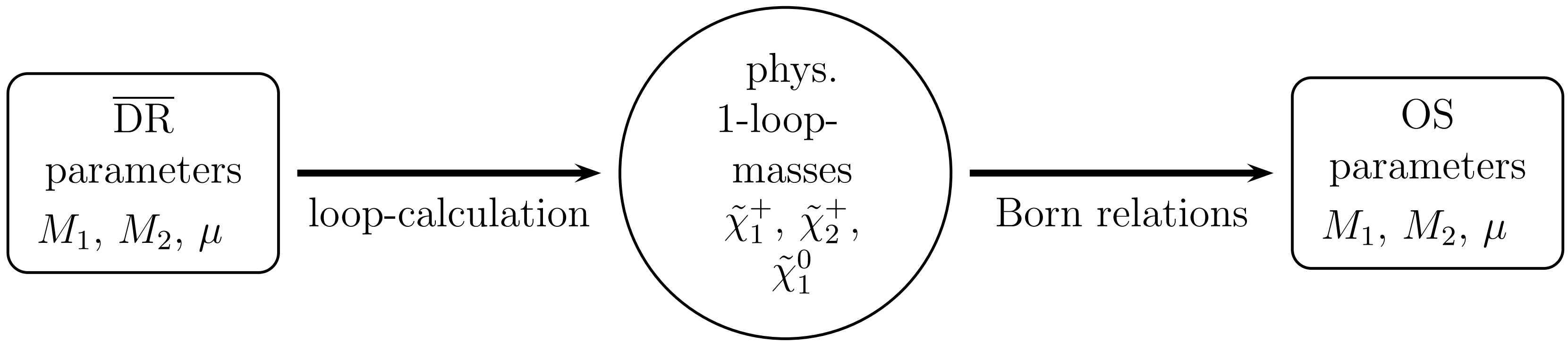


$$= ie\left[\vec{C}_{\rm FFS}\left(\overline{\Psi}_j, \Psi_i, \Phi_k\right)\cdot\begin{pmatrix}\omega_{\rm L}\\ \omega_{\rm R}\end{pmatrix}\right]$$

- $$\vec{C}_{\rm FFS}\left(\overline{e_{\rm j}}, \tilde{\chi}^0_{\rm i}, \tilde{e}^{\rm s}_{\rm k}\right) = \frac{\delta_{\rm jk}}{2\sqrt{2}\,c_\beta^2\,c_w^3\,M_W^3\,s_w^2}\times$$

$$\begin{bmatrix}
-c_\beta\,c_w^2\,M_W^2\,s_w\sum_{n=1}^{4}\left[\delta Z_{\tilde{\chi}^0}\right]_{n{\rm i}}\left(c_w\,m_{e_{\rm j}}\,N^*_{n3}\,U^{\tilde{e}_{\rm j}*}_{\rm s1} + 2\,c_\beta\,M_W\,s_w\,N^*_{n1}\,U^{\tilde{e}_{\rm j}*}_{\rm s2}\right) - \\
c_\beta\,c_w^2\,M_W^2\,s_w\sum_{n=1}^{2}\left[\delta Z^{\rm k}_{\tilde{e}}\right]_{n{\rm s}}\left(c_w\,m_{e_{\rm j}}\,N^*_{\rm i3}\,U^{\tilde{e}_{\rm j}*}_{n1} + 2\,c_\beta\,M_W\,s_w\,N^*_{\rm i1}\,U^{\tilde{e}_{\rm j}*}_{n2}\right) - \\
\left(c_w^3\,N^*_{\rm i3}\,U^{\tilde{e}_{\rm j}*}_{\rm s1}\right)\Big\{2\,c_\beta\,M_W^2\,s_w\,\delta m^{\rm e}_{\rm j} + m_{e_{\rm j}}\Big[-2\,\delta c_\beta\,M_W^2\,s_w + c_\beta\,M_W^2\,s_w\,\left[\delta Z^{\rm R}_{\rm e}\right]^*_{\rm jj} + \\
c_\beta\,\left(-\delta M_W^2\,s_w - 2\,M_W^2\,\left(\delta s_w - \delta Z_e\,s_w\right)\right)\Big]\Big\} - \\
\left(2\,c_w^2\,\delta Z_e + 2\,\delta s_w\,s_w + c_w^2\,\left[\delta Z^{\rm R}_{\rm e}\right]^*_{\rm jj}\right)\left(2\,c_\beta^2\,M_W^3\,s_w^2\,N^*_{\rm i1}\,U^{\tilde{e}_{\rm j}*}_{\rm s2}\right) \\
\\
c_\beta\,c_w^2\,M_W^2\,s_w\sum_{n=1}^{4}\left[\delta Z_{\tilde{\chi}^0}\right]^*_{n{\rm i}}\left(\left(s_w\,N_{n1} + c_w\,N_{n2}\right)\left(c_\beta\,M_W\,U^{\tilde{e}_{\rm j}*}_{\rm s1}\right) - c_w\,m_{e_{\rm j}}\,N_{n3}\,U^{\tilde{e}_{\rm j}*}_{\rm s2}\right) + \\
c_\beta\,c_w^2\,M_W^2\,s_w\sum_{n=1}^{2}\left[\delta Z^{\rm k}_{\tilde{e}}\right]_{n{\rm s}}\left(\left(s_w\,N_{\rm i1} + c_w\,N_{\rm i2}\right)\left(c_\beta\,M_W\,U^{\tilde{e}_{\rm j}*}_{n1}\right) - c_w\,m_{e_{\rm j}}\,N_{\rm i3}\,U^{\tilde{e}_{\rm j}*}_{n2}\right) - \\
\left(c_w^3\,N_{\rm i3}\,U^{\tilde{e}_{\rm j}*}_{\rm s2}\right)\Big\{2\,c_\beta\,M_W^2\,s_w\,\delta m^{\rm e}_{\rm j} + m_{e_{\rm j}}\Big[-2\,\delta c_\beta\,M_W^2\,s_w + c_\beta\,M_W^2\,s_w\,\left[\delta Z^{\rm L}_{\rm e}\right]^*_{\rm jj} + \\
c_\beta\,\left(-\delta M_W^2\,s_w - 2\,M_W^2\,\left(\delta s_w - \delta Z_e\,s_w\right)\right)\Big]\Big\} + \\
\left(c_\beta^2\,M_W^3\,U^{\tilde{e}_{\rm j}*}_{\rm s1}\right)\Big[\left(2\,c_w^2\,\delta Z_e + 2\,\delta s_w\,s_w + c_w^2\,\left[\delta Z^{\rm L}_{\rm e}\right]^*_{\rm jj}\right)\left(s_w^2\,N_{\rm i1}\right) + \\
\left(s_w\,\left[\delta Z^{\rm L}_{\rm e}\right]^*_{\rm jj} - 2\,\left(\delta s_w - \delta Z_e\,s_w\right)\right)\left(c_w^3\,N_{\rm i2}\right)\Big]
\end{bmatrix}$$

$\times$



pole masses $\longleftrightarrow$ on-shell parameters

$$
\begin{aligned}
M_2^2 + \mu^2 + 2M_W^2 &= m^2_{\tilde{\chi}_1^+} + m^2_{\tilde{\chi}_2^+} \\
\left(M_2\,\mu - 2M_W^2 \sin\beta\,\cos\beta\right)^2 &= m^2_{\tilde{\chi}_1^+}\, m^2_{\tilde{\chi}_2^+} \quad .
\end{aligned}
$$

$$
\begin{aligned}
M_1 = {} & \Big[-M_2\mu M_Z^2 \sin 2\beta + \left[\mu M_Z^2 \sin 2\beta - M_2\left(\mu^2 + M_Z^2 s_W^2\right)\right] m_{\tilde{\chi}_1^0} \\
& + \left[\mu^2 + M_Z^2\right] m^2_{\tilde{\chi}_1^0} + M_2 m^3_{\tilde{\chi}_1^0} - m^4_{\tilde{\chi}_1^0}\Big] \\
& \times \left[\mu M_Z^2 c_W^2 \sin 2\beta - M_2\mu^2 + \left[\mu^2 + M_Z^2 c_W^2\right] m_{\tilde{\chi}_1^0} + M_2 m^2_{\tilde{\chi}_1^0} - m^3_{\tilde{\chi}_1^0}\right]^{-1}
\end{aligned}
$$

$\overline{\mathrm{DR}}$ parameters (SPS1a′)

$$
\begin{aligned}
\tan\beta &= 10 &;\quad \mu &= 402.87\ \mathrm{GeV}\\
M_{A^0} &= 431.02\ \mathrm{GeV} &;\quad M_1 &= 103.22\ \mathrm{GeV}\\
M_3 &= 572.33\ \mathrm{GeV} &;\quad M_2 &= 193.31\ \mathrm{GeV}
\end{aligned}
$$

$$
\begin{aligned}
A_{u,c} &= -784.7\ \mathrm{GeV} &;\quad A_t &= -535.4\ \mathrm{GeV}\\
A_{d,s} &= -1025.7\ \mathrm{GeV} &;\quad A_b &= -938.5\ \mathrm{GeV}\\
A_{e,\mu} &= -449.0\ \mathrm{GeV} &;\quad A_\tau &= -445.5\ \mathrm{GeV}
\end{aligned}
$$

$$
\begin{aligned}
m_{\tilde{l}}^{1,2} &= 181.3\ \mathrm{GeV} &;\quad m_{\tilde{l}}^3 &= 179.5\ \mathrm{GeV}\\
m_{\tilde{e},\tilde{\mu}} &= 115.6\ \mathrm{GeV} &;\quad m_{\tilde{\tau}} &= 109.8\ \mathrm{GeV}\\
m_{\tilde{q}}^{1,2} &= 526.9\ \mathrm{GeV} &;\quad m_{\tilde{q}}^3 &= 471.3\ \mathrm{GeV}\\
m_{\tilde{u},\tilde{c}} &= 507.7\ \mathrm{GeV} &;\quad m_{\tilde{t}} &= 384.6\ \mathrm{GeV}\\
m_{\tilde{d},\tilde{s}} &= 505.5\ \mathrm{GeV} &;\quad m_{\tilde{t}} &= 501.3\ \mathrm{GeV}
\end{aligned}
$$

on-shell parameters

$$
\begin{aligned}
\tan\beta &= 10 &;\quad \mu &= 399.26\ \mathrm{GeV}\\
M_{A^0} &= 431.02\ \mathrm{GeV} &;\quad M_1 &= 100.11\ \mathrm{GeV}\\
M_3 &= 612.85\ \mathrm{GeV} &;\quad M_2 &= 197.55\ \mathrm{GeV}
\end{aligned}
$$

$$
\begin{aligned}
A_{u,c} &= -784.7\ \mathrm{GeV} &;\quad A_t &= -535.4\ \mathrm{GeV}\\
A_{d,s} &= -1025.7\ \mathrm{GeV} &;\quad A_b &= -938.5\ \mathrm{GeV}\\
A_{e,\mu} &= -449.0\ \mathrm{GeV} &;\quad A_\tau &= -445.5\ \mathrm{GeV}
\end{aligned}
$$

$$
\begin{aligned}
m_{\tilde{l}}^1 &= 184.12\ \mathrm{GeV} &;\quad m_{\tilde{l}}^2 &= 184.11\ \mathrm{GeV} &;\quad m_{\tilde{l}}^3 &= 182.18\ \mathrm{GeV}\\
m_{\tilde{e}} &= 118.02\ \mathrm{GeV} &;\quad m_{\tilde{\mu}} &= 117.99\ \mathrm{GeV} &;\quad m_{\tilde{\tau}} &= 111.29\ \mathrm{GeV}\\
m_{\tilde{q}}^1 &= 565.97\ \mathrm{GeV} &;\quad m_{\tilde{q}}^2 &= 565.91\ \mathrm{GeV} &;\quad m_{\tilde{q}}^3 &= 453.05\ \mathrm{GeV}\\
m_{\tilde{u}} &= 546.78\ \mathrm{GeV} &;\quad m_{\tilde{c}} &= 546.84\ \mathrm{GeV} &;\quad m_{\tilde{t}} &= 460.52\ \mathrm{GeV}\\
m_{\tilde{d}} &= 544.95\ \mathrm{GeV} &;\quad m_{\tilde{s}} &= 544.97\ \mathrm{GeV} &;\quad m_{\tilde{t}} &= 538.13\ \mathrm{GeV}
\end{aligned}
$$

SPA
SUPERSYMMETRY PARAMETER ANALYSIS

The SPA project is a joint study of theorists and experimentalists working on LHC and Linear Collider phenomenology. The study focuses on the supersymmetric extension of the Standard Model. The main targets are

- High-precision determination of the supersymmetry Lagrange parameters at the electroweak scale
- Extrapolation to a high scale to reconstruct the fundamental parameters and the mechanism for supersymmetry breaking

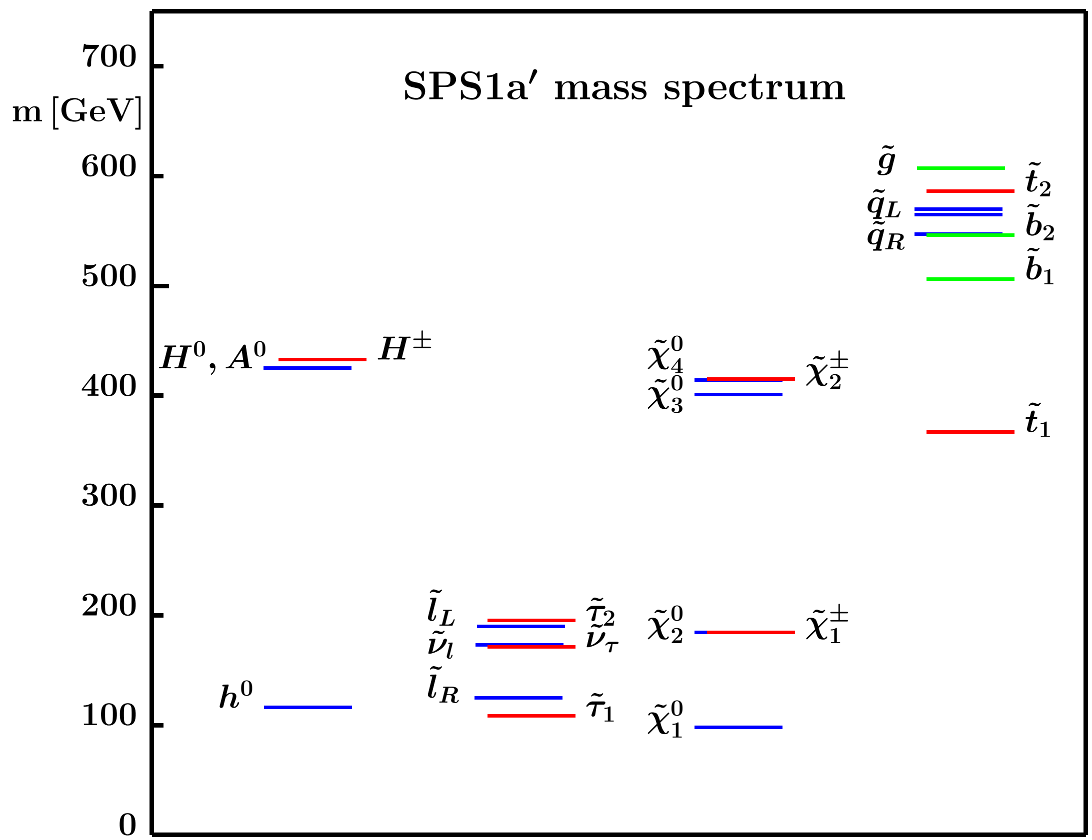
`http://spa.desy.de/spa`

P. Zerwas, J. Kalinowski, H.U. Martyn,
W. Hollik, W. Kilian, W. Majerotto, W. Porod, . . .

`hep-ph/0511344, EPJC 46(2006)43`

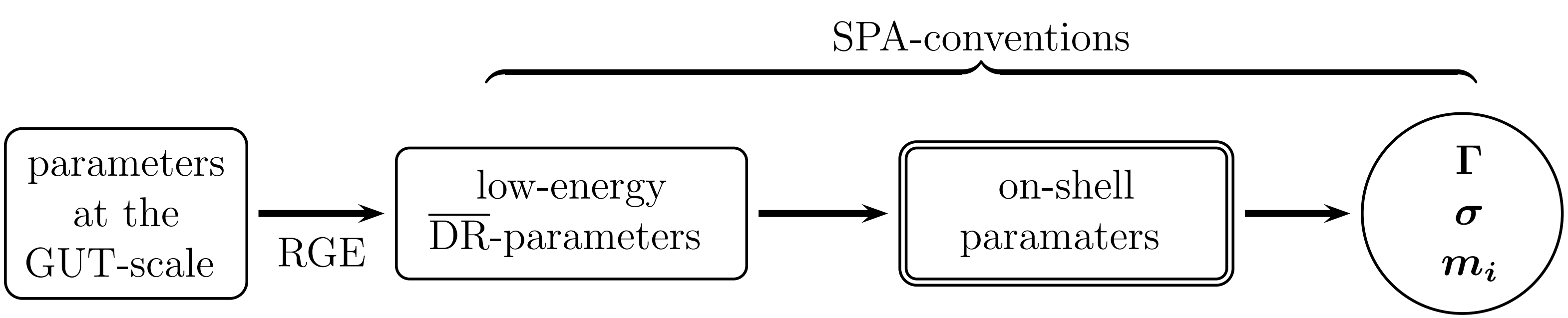
SPS1a′ scenario

$M_{1/2}$	$=$	250 GeV	sign(μ)	$=$	$+1$
M_0	$=$	70 GeV	$\tan\beta(\tilde{M})$	$=$	10
A_0	$=$	-300 GeV			



SPA CONVENTION

- The masses of the SUSY particles and Higgs bosons are defined as pole masses.
- All SUSY Lagrangian parameters, mass parameters and couplings, including $\tan\beta$, are given in the $\overline{\mathrm{DR}}$ scheme and defined at the scale $\tilde{M} = 1$ TeV.
- Gaugino/higgsino and scalar mass matrices, rotation matrices and the corresponding angles are defined in the $\overline{\mathrm{DR}}$ scheme at $\tilde{M}$, except for the Higgs system in which the mixing matrix is defined in the on-shell scheme, the momentum scale chosen as the light Higgs mass.
- The Standard Model input parameters of the gauge sector are chosen as G_F, α, M_Z and $\alpha_s^{\overline{\mathrm{MS}}}(M_Z)$. All lepton masses are defined on-shell. The t quark mass is defined on-shell; the b, c quark masses are introduced in $\overline{\mathrm{MS}}$ at the scale of the masses themselves while taken at a renormalization scale of 2 GeV for the light u, d, s quarks.
- Decay widths/branching ratios and production cross sections are calculated for the set of parameters specified above.



$\overline{\mathrm{DR}}$ masses → pole masses (SPS1a′)

	m		δm		m_{phys}		m		δm		m_{phys}
• $m_{\tilde\chi_1^+}$	$= 181.026$	$+$	3.178	$=$	184.204	• $m_{\tilde\chi_1^0}$	$= 100.706$	$+$	(-2.958)	$=$	97.748
• $m_{\tilde\chi_2^+}$	$= 423.420$	$+$	(-2.181)	$=$	421.239	$m_{\tilde\chi_2^0}$	$= 181.404$	$+$	3.022	$=$	184.425
						$m_{\tilde\chi_3^0}$	$= 408.579$	$+$	(-1.626)	$=$	406.952
• $m_{\tilde g}$	$= 572.330$	$+$	40.524	$=$	612.854	$m_{\tilde\chi_4^0}$	$= 422.991$	$+$	(-3.310)	$=$	419.681
• $m_{\tilde\nu_1}$	$= 169.890$	$+$	2.804	$=$	172.695	• $m_{\tilde u_1^1}$	$= 506.424$	$+$	39.255	$=$	545.680
• $m_{\tilde e_1^1}$	$= 123.574$	$+$	1.878	$=$	125.452	• $m_{\tilde u_1^2}$	$= 524.275$	$+$	39.157	$=$	563.433
$m_{\tilde e_1^2}$	$= 186.905$	$+$	3.082	$=$	189.986	• $m_{\tilde d_1^1}$	$= 506.097$	$+$	39.409	$=$	545.506
						$m_{\tilde d_1^2}$	$= 530.033$	$+$	38.826	$=$	568.859
• $m_{\tilde\nu_2}$	$= 169.884$	$+$	2.804	$=$	172.688	• $m_{\tilde u_2^1}$	$= 506.410$	$+$	39.254	$=$	545.664
• $m_{\tilde e_2^1}$	$= 123.510$	$+$	1.877	$=$	125.387	• $m_{\tilde u_2^2}$	$= 524.285$	$+$	39.158	$=$	563.444
$m_{\tilde e_2^2}$	$= 186.929$	$+$	3.080	$=$	190.009	• $m_{\tilde d_2^1}$	$= 506.092$	$+$	39.408	$=$	545.500
						$m_{\tilde d_2^2}$	$= 530.034$	$+$	38.825	$=$	568.859
• $m_{\tilde\nu_3}$	$= 168.001$	$+$	2.629	$=$	170.630	• $m_{\tilde u_3^1}$	$= 333.171$	$+$	35.334	$=$	368.504
• $m_{\tilde e_3^1}$	$= 106.080$	$+$	1.595	$=$	107.674	• $m_{\tilde u_3^2}$	$= 549.649$	$+$	34.223	$=$	583.872
$m_{\tilde e_3^2}$	$= 192.418$	$+$	2.786	$=$	195.203	$m_{\tilde d_3^1}$	$= 470.247$	$+$	34.711	$=$	504.958
						• $m_{\tilde d_3^2}$	$= 506.244$	$+$	38.129	$=$	544.374

$\boxed{\text{“QED corrections”}}$

Full calculation inevitable

- separation of diagrams with virtual photons not UV-finite
- soft-photon bremsstrahlung necessary for getting an IR-finite result
- hard bremsstrahlung needed for realistic treatments

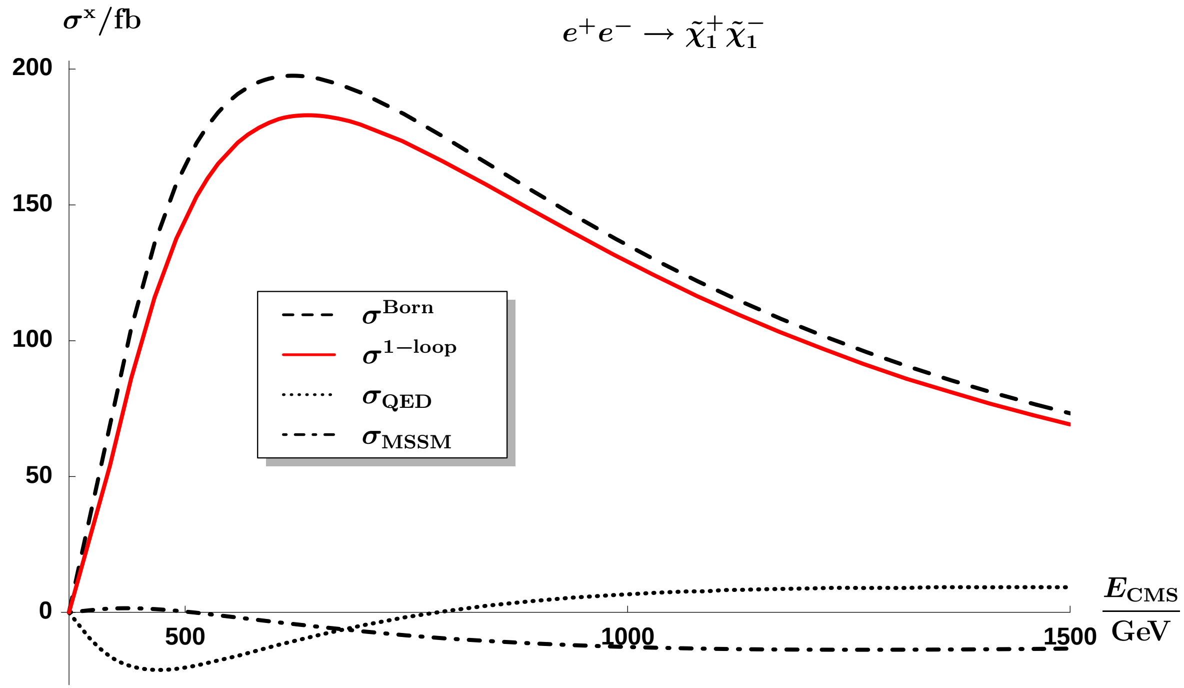
Reasonable separation ($L_e = \log \frac{s}{m_e^2}$, $\Delta E = E_{\gamma\,\mathrm{soft}}^{\mathrm{max}}$):

$$\sigma(1-\mathrm{loop}) = \sigma_{\mathrm{QED}} + \sigma_{\mathrm{MSSM}}\,,$$

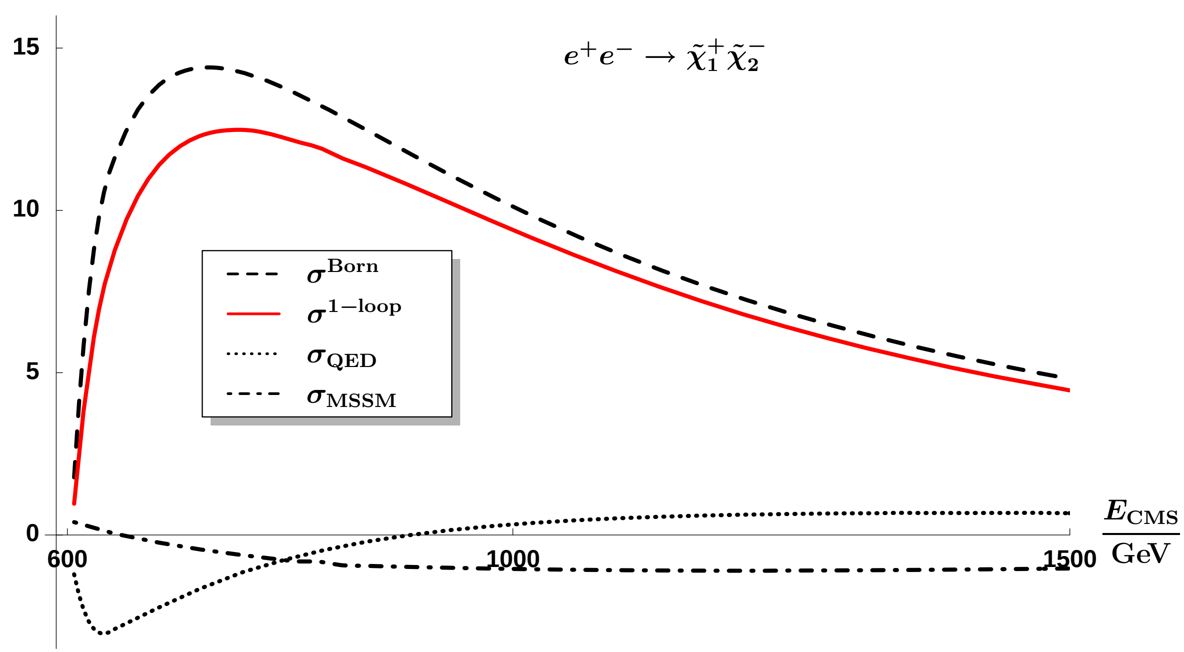
$$\sigma_{\mathrm{QED}} = \sigma^{\mathrm{hard}} + \frac{\alpha}{\pi}\left[(L_e-1)\log\frac{4\,\Delta E^{\,2}}{s} + \frac{3}{2}\,L_e\right]\sigma_0$$

$$\sigma_{\mathrm{MSSM}} = \sigma^{\mathrm{v+s}} - \frac{\alpha}{\pi}\left[(L_e-1)\log\frac{4\,\Delta E^{\,2}}{s} + \frac{3}{2}\,L_e\right]\sigma_0$$

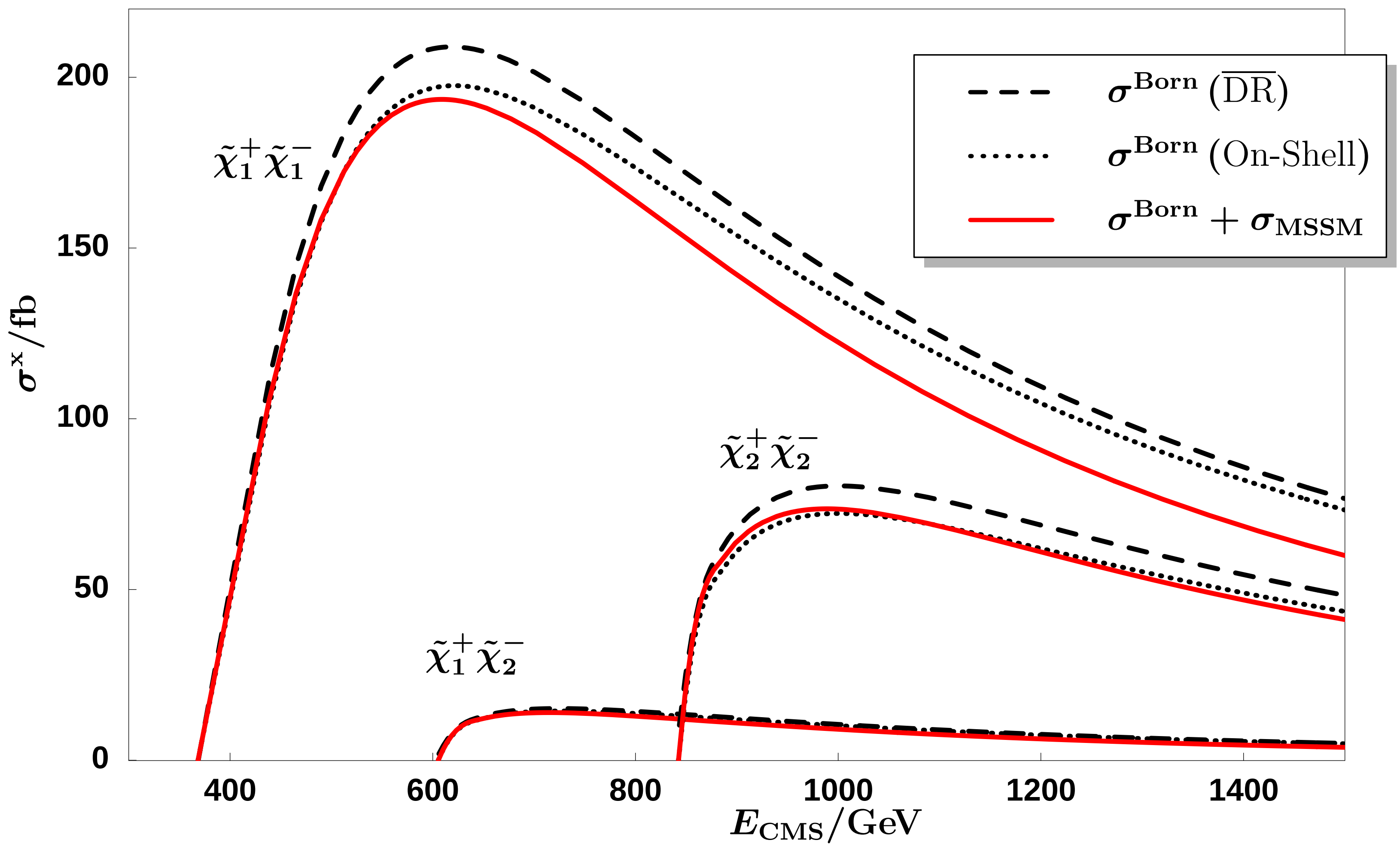
- gauge invariant
- σ_{MSSM} free of large soft and collinear photon terms



σ^{x}/fb
$e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$
200
150
100
50
0
500
1000
1500
$\frac{E_{\mathrm{CMS}}}{\mathrm{GeV}}$
σ^{Born}
$\sigma^{1-\mathrm{loop}}$
σ_{QED}
σ_{MSSM}

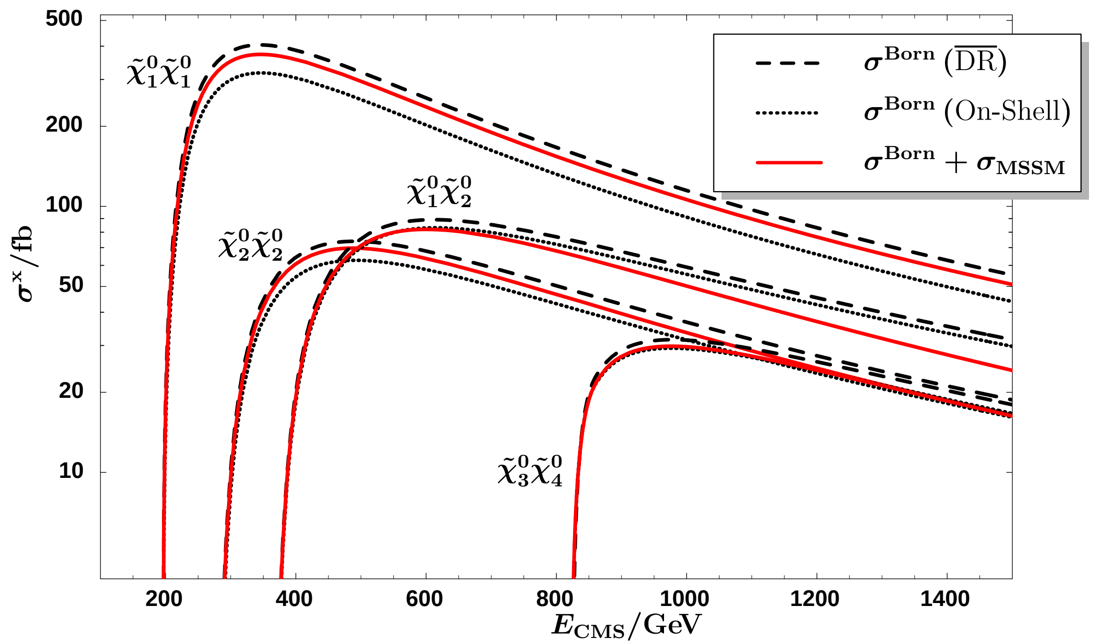


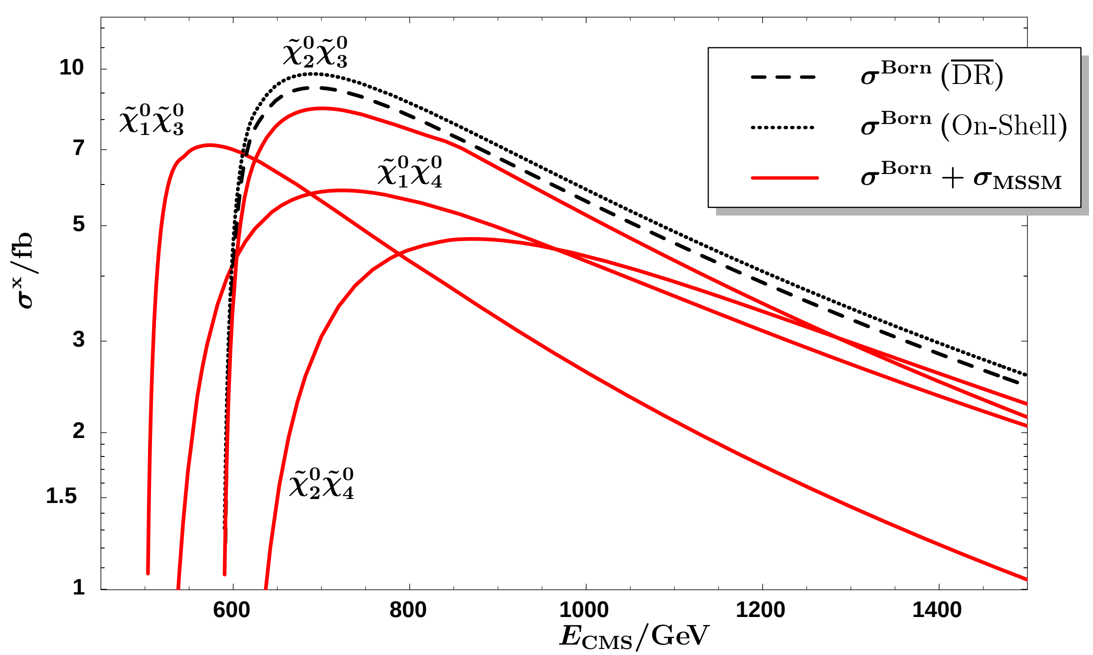
$e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$
15
10
5
0
600
1000
1500
$\frac{E_{\mathrm{CMS}}}{\mathrm{GeV}}$
σ^{Born}
$\sigma^{1-\mathrm{loop}}$
σ_{QED}
σ_{MSSM}



$\sigma^{\mathrm{Born}}\,(\overline{\mathrm{DR}})$
$\sigma^{\mathrm{Born}}\,(\mathrm{On\text{-}Shell})$
$\sigma^{\mathrm{Born}} + \sigma_{\mathrm{MSSM}}$
$\tilde{\chi}_1^+\tilde{\chi}_1^-$
$\tilde{\chi}_2^+\tilde{\chi}_2^-$
$\tilde{\chi}_1^+\tilde{\chi}_2^-$
$\sigma^{\mathrm{x}}/\mathrm{fb}$
$E_{\mathrm{CMS}}/\mathrm{GeV}$
0
50
100
150
200
400
600
800
1000
1200
1400

$e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$



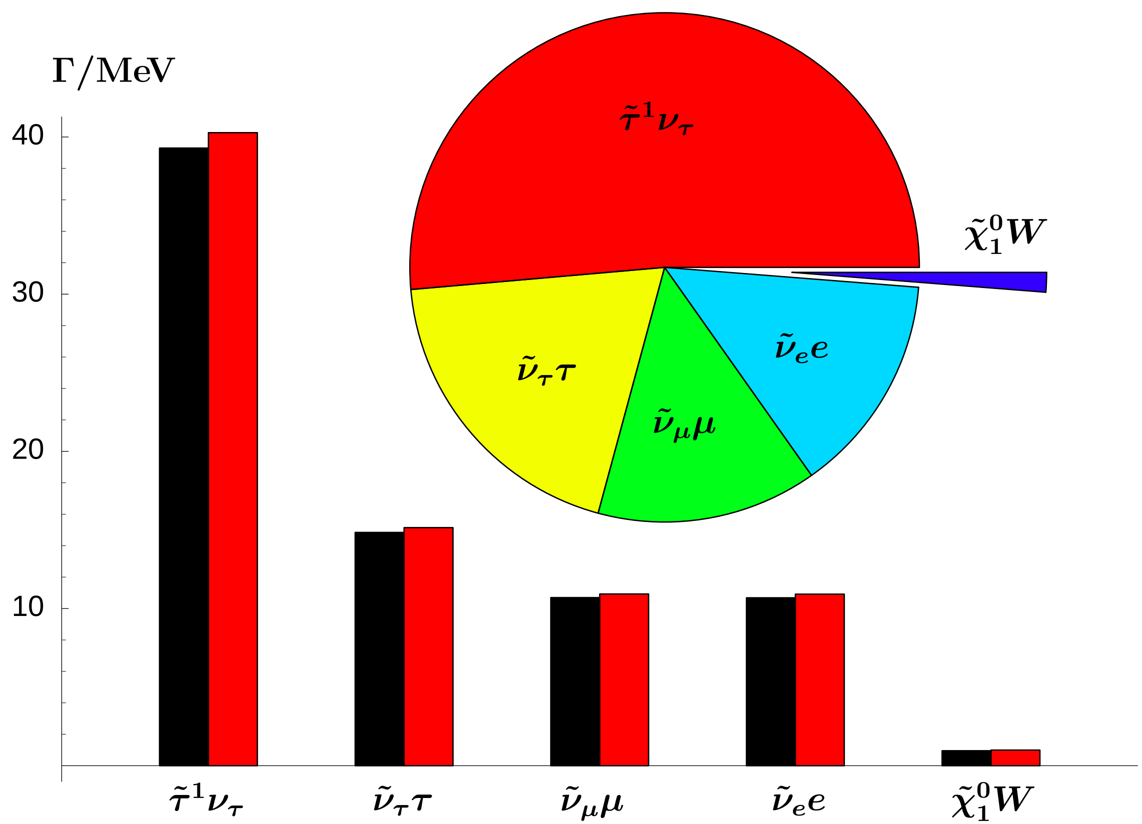


SUSY particle decay rates

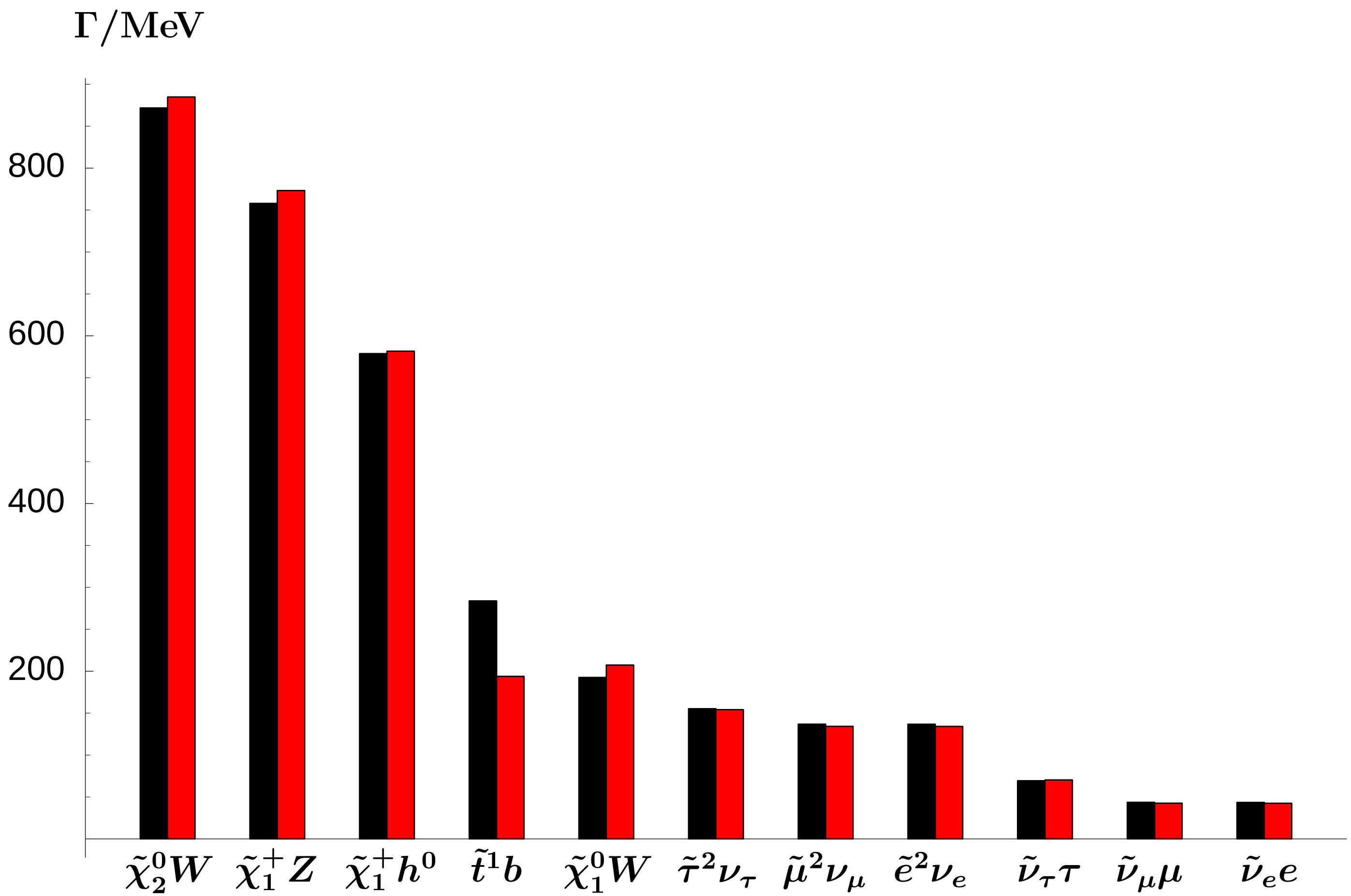
2-particle decays of $\tilde{\chi}^{\pm}_{1,2}$ and $\tilde{\chi}^{0}_{2,3,4}$

tree level (black) and 1-loop (red) *[Fritzsche, WH]*

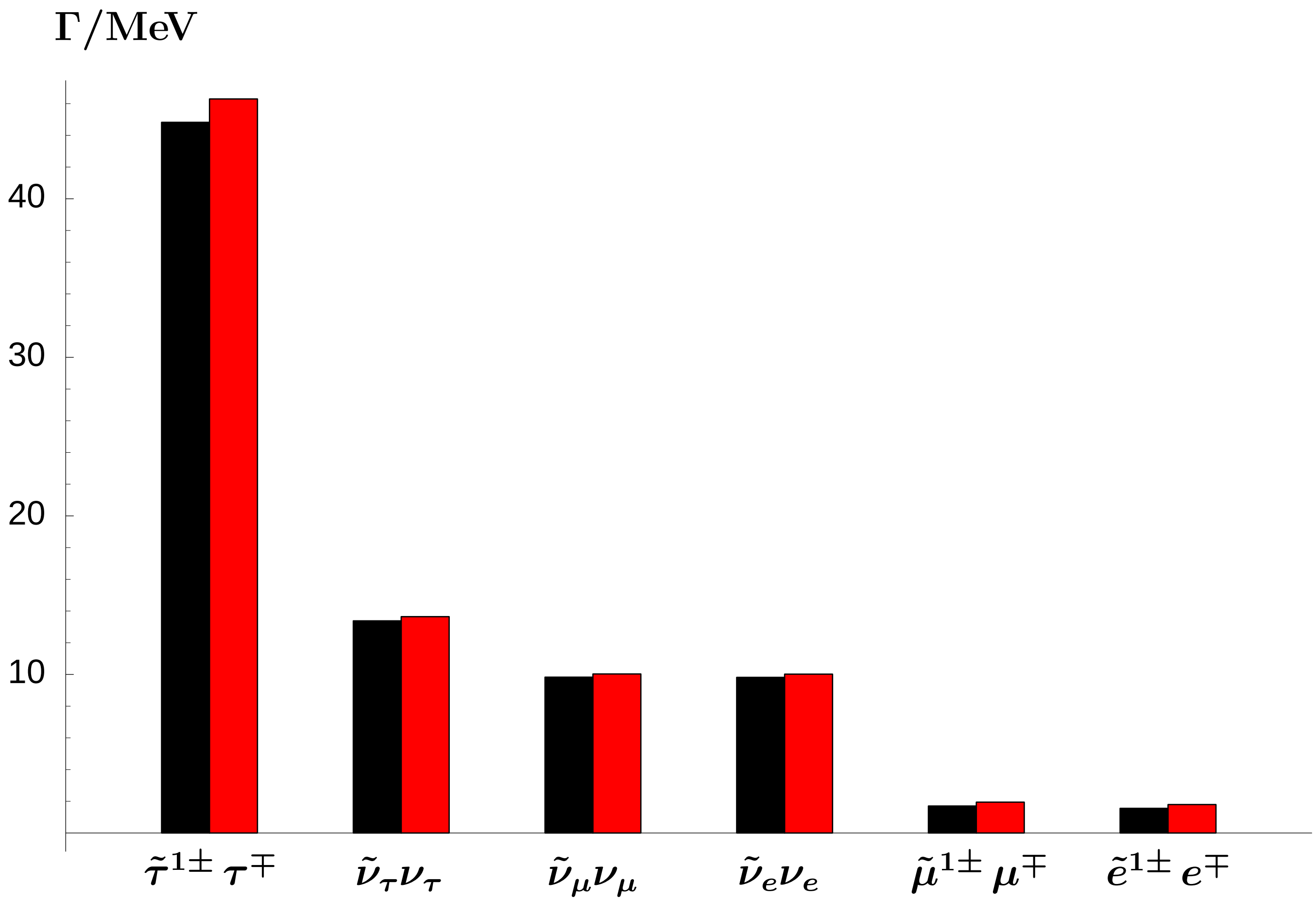
$\tilde{\chi}_1^-$ decay modes (SPS1a′)



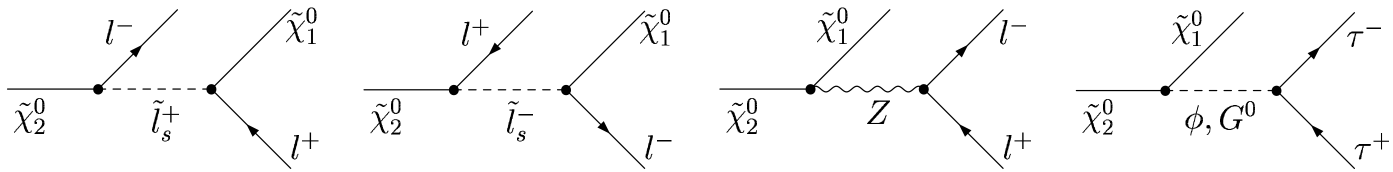
$\tilde{\chi}_2^-$ decay modes

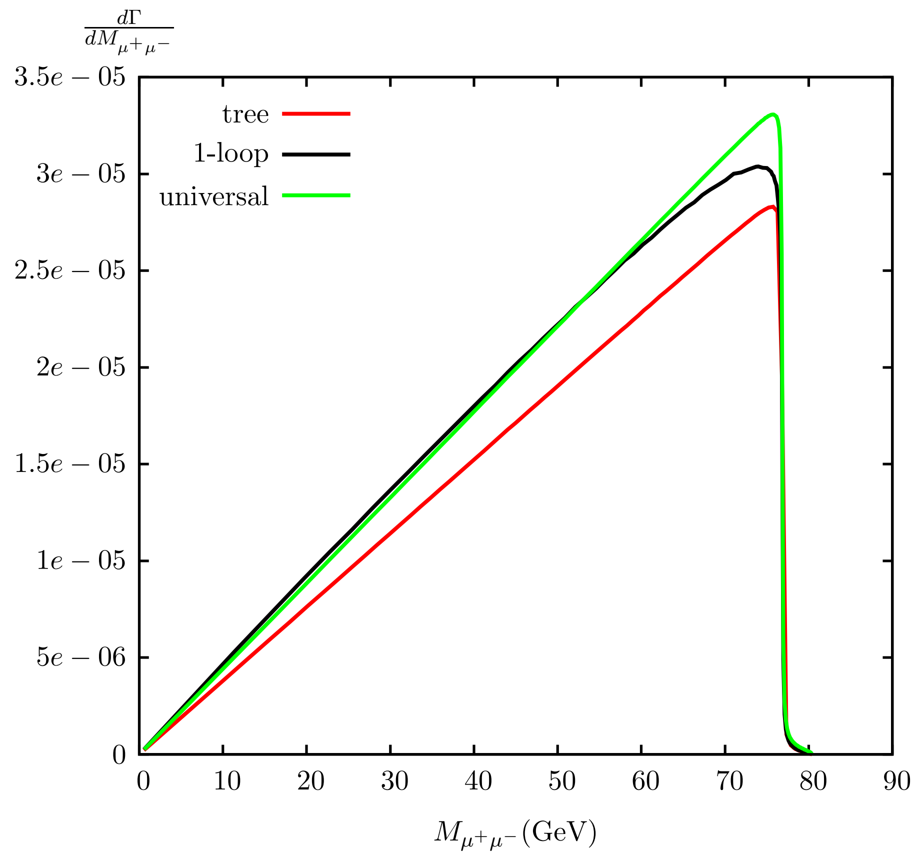


$\tilde{\chi}_2^0$ decay modes



important 3-body decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\, \ell^+ \ell^-$





[Drees, WH, Qingjun Xu]

Reconstructing Lagrangian parameters

based on 82 simulated measurements at LHC and ILC

Parameter	SPS1a$'$value	Fit error [exp]
M_1	103.3	0.1
M_2	193.4	0.1
M_3	568.9	7.8
μ	400.4	1.1
$M_{\tilde{e}_L}$	181.3	0.2
$M_{\tilde{e}_R}$	115.6	0.4
$M_{\tilde{\tau}_L}$	179.5	1.2
$M_{\tilde{u}_L}$	523.2	5.2
$M_{\tilde{u}_R}$	503.9	17.3
$M_{\tilde{t}_L}$	467.7	4.9
$m_{\rm A}$	374.9	0.8
$A_{\rm t}$	-525.6	24.6
$\tan\beta$	10.0	0.3

Accuracy of measurements

	Mass	"LHC"	"LC"	"LHC+LC"
h^0	115.4	0.25	0.05	0.05
H^0	431.1		1.5	1.5
$\tilde{\chi}_1^0$	97.75	4.8	0.05	0.05
$\tilde{\chi}_2^0$	184.4	4.7	1.2	0.08
$\tilde{\chi}_4^0$	419.6	5.1	$3-5$	2.5
$\tilde{\chi}_1^\pm$	184.2		0.55	0.55
$\tilde{e}_R$	125.2	4.8	0.05	0.05
$\tilde{e}_L$	190.1	5.0	0.18	0.18
$\tilde{\tau}_1$	107.4	$5-8$	0.24	0.24
$\tilde{q}_R$	547.7	$7-12$	–	$5-11$
$\tilde{q}_L$	565.7	8.7	–	4.9
$\tilde{t}_1$	368.9		1.9	1.9
$\tilde{b}_1$	506.3	7.5	–	5.7
$\tilde{g}$	607.6	8.0	–	6.5

High Scale Extrapolations

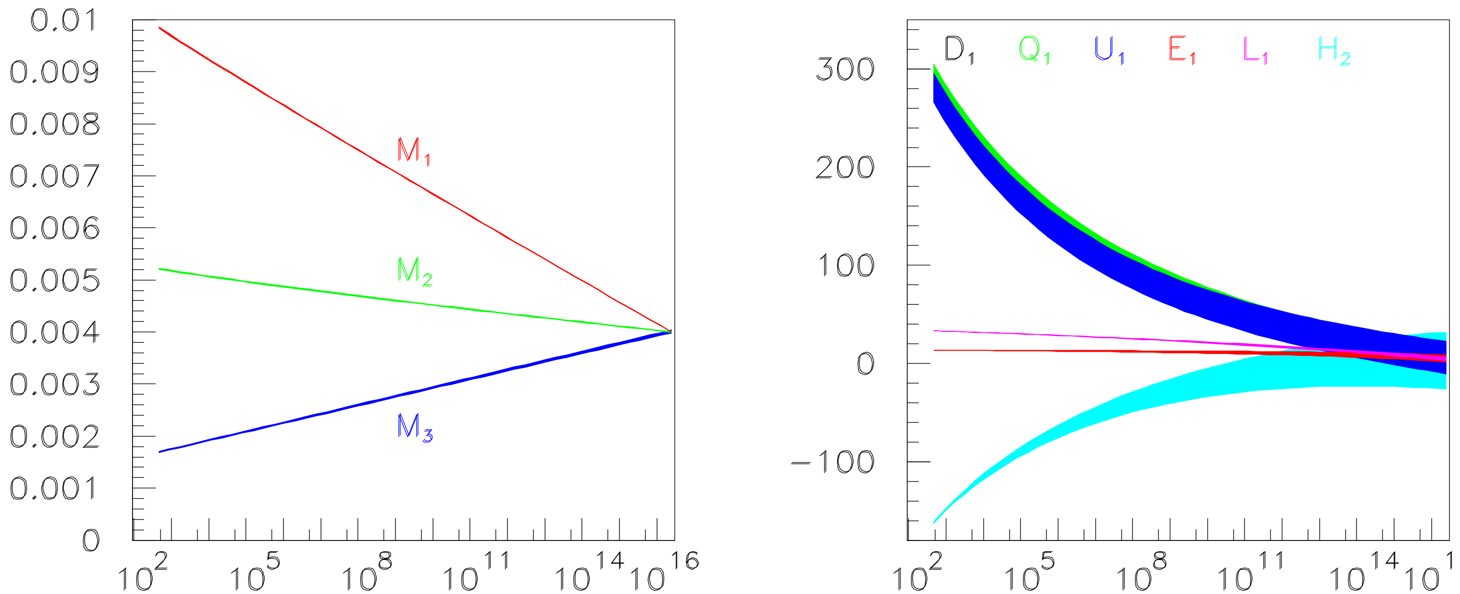


Fig. 1. *Running of the gaugino and scalar mass parameters in SPS1a′ [*`SPheno 2.2.2` *]. Only experimental errors are taken into account; theoretical errors are assumed to be reduced to the same size in the future.*

Conclusions

- precision calculations for SUSY (MSSM) are well advanced
- electroweak precision observables $\rightarrow$ 2-loop level

  global fits of similar quality as in standard model

  indirect sensitivity to SUSY parameters
- m_{h^0} is another precision observable
  – dependent on all SUSY sectors
  – accurate theoretical evaluation ($\delta m_{h^0} \simeq 4$ GeV),
  to be further improved
- progress for loop contributions to SUSY processes

  future precision allows tests of breaking scenarios

Detailed analysis for SPS1a benchmark scenario: potential of LHC (300 fb^{-1}) alone and LHC $+$ LC

	LHC	LHC+LC
$\Delta m_{\tilde{\chi}_1^0}$	4.8	0.05 (input)
$\Delta m_{\tilde{l}_R}$	4.8	0.05 (input)
$\Delta m_{\tilde{\chi}_2^0}$	4.7	0.08
$\Delta m_{\tilde{q}_L}$	8.7	4.9
$\Delta m_{\tilde{q}_R}$	11.8	10.9
$\Delta m_{\tilde{g}}$	8.0	6.4
$\Delta m_{\tilde{b}_1}$	7.5	5.7
$\Delta m_{\tilde{b}_2}$	7.9	6.2
$\Delta m_{\tilde{l}_L}$	5.0	0.2 (input)
$\Delta m_{\tilde{\chi}_4^0}$	5.1	2.23

LHC+LC accuracy limited by LHC jet energy scale resolution

SPS 1a benchmark scenario:

favorable scenario for both LHC and LC

$\Rightarrow$ LC input improves accuracy significantly