Vacuum configurations for renormalisable noncommutative scalar field theory

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Plan

- Scalar field theory with harmonic term on the Moyal space
- Equation of motion
- Vacuum configurations
- Linear sigma model
- Conclusions and perspectives

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Moyal space

Deformation of Euclidean $\mathbb{R}^D$: $\mathcal{M}_\theta = (\mathcal{S}(\mathbb{R}^D), \star_\theta)$

$$(f \star_\theta g)(x) = \frac{1}{\pi^D |\det \Theta|} \int d^D y d^D z \, f(x+y) g(x+z) e^{-2iy\Theta^{-1}z}$$

$$\Theta = \begin{pmatrix} 0 & -\theta & & & 0 \\ \theta & 0 & & & \\ & & \ddots & & \\ & & & 0 & -\theta \\ 0 & & & \theta & 0 \end{pmatrix}$$

Properties: $[x_\mu, x_\nu]_\star = i\Theta_{\mu\nu}$ $\qquad (\widetilde{x}_\mu = 2\Theta^{-1}_{\mu\nu} x_\nu)$

$$\int d^D x \, (f \star g)(x) = \int d^D x \, f(x) g(x)$$

$$\partial_\mu \phi = -\frac{i}{2}[\widetilde{x}_\mu, \phi]_\star \qquad \widetilde{x}_\mu \phi = \frac{1}{2}\{\widetilde{x}_\mu, \phi\}_\star$$

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Matrix base (2 dimensions)

$(f_{mn}(x))_{m,n\in\mathbb{N}}$, eigenfunctions of the 2D harmonic oscillator: base of $\mathcal{M}_\theta$

$$f_{mn}(x) = 2(-1)^m \sqrt{\frac{m!}{n!}} e^{i(n-m)\varphi} \left(\frac{2r^2}{\theta}\right)^{\frac{n-m}{2}} L_m^{n-m}\left(\frac{2r^2}{\theta}\right) e^{-\frac{r^2}{\theta}}$$

Properties: $f_{mn}^\dagger(x) = f_{nm}(x)$

$$(f_{mn} \star f_{kl})(x) = \delta_{nk} f_{ml}(x)$$

if $g(x) = \sum_{m,n=0}^\infty g_{mn} f_{mn}(x)$, then

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Scalar field theory with harmonic term

Renormalisable theory (for $\Omega \neq 0$,hep-th/0401128):

$$S[\phi] = \int d^D x \Big(\frac{1}{2} \partial_\mu \phi \star \partial_\mu \phi + \frac{\Omega^2}{2} (\widetilde{x}_\mu \phi) \star (\widetilde{x}_\mu \phi) + \frac{M^2}{2} \phi \star \phi + \lambda\, \phi \star \phi \star \phi \star \phi \Big)$$

Spontaneous symmetry breaking: $M^2 \to -\mu^2$

Special value $\Omega = 1$:

- invariant under Langmann-Szabo duality (hep-th/0202039) $(p_\mu \leftrightarrow \widetilde{x}_\mu)$
- stable under renormalisation group (hep-th/0612251)

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Equation of motion

For $\Omega = 1$, the equation of motion:

$$-\partial^2 v(x) + \widetilde{x}^2 v(x) - \mu^2 v(x) + 4\lambda \, (v \star v \star v)(x) = 0$$

$$\frac{1}{2}\{\widetilde{x}^2, v\}_\star - \mu^2 v + 4\lambda \; v \star v \star v = 0$$

if $v(x) = \sum_{m,n=0}^{\infty} v_{mn} f_{mn}(x)$, then $\quad ((\widetilde{x}^2)_{mn} = \frac{4}{\theta}(2m+1)\delta_{mn})$

$$\left(\frac{4}{\theta}(m+n+1) - \mu^2\right) v_{mn} + 4\lambda \sum_{k,l=0}^{\infty} v_{mk} v_{kl} v_{ln} = 0$$

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Symmetry of the solution

Vacuum invariant under rotations: $v(x) = g(x^2)$

$$v_{mn} = \frac{1}{2\pi\theta}\int d^2x\; g(x^2) f_{nm}(x)$$

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The vacuum is of the form:

$$v_{mn} = a_m \delta_{mn}, \quad a_m \in \mathbb{R}$$

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$$\left(\frac{4}{\theta}(m+n+1)-\mu^2\right)v_{mn}+4\lambda\sum_{k,l=0}^{\infty}v_{mk}v_{kl}v_{ln}=0$$

$$v_{mn}=a_m\delta_{mn}$$

$$\forall m\in\mathbb{N},\quad \left(\frac{4}{\theta}(2m+1)-\mu^2+4\lambda\, a_m^2\right)a_m=0$$

Solutions: $a_m=0$ or $a_m^2=\frac{2}{\lambda\theta}\left(\frac{\mu^2\theta}{8}-\frac{1}{2}-m\right)\geq 0$

$$p=\left\lfloor\frac{\mu^2\theta}{8}-\frac{1}{2}\right\rfloor \quad \rightarrow 2^p \text{ solutions}$$

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Propagator

Spontaneous symmetry breaking: $\phi(x) \to v(x) + \phi(x)$

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$$+4\lambda\, v \star v \star \phi \star \phi + 2\lambda\, v \star \phi \star v \star \phi$$
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The propagator is given by

$$S_{quadr} = 2\pi\theta \sum_{m,n=0}^{\infty} \Big(\frac{2}{\theta}(m+n+1) - \frac{\mu^2}{2} + 2\lambda \sum_{k=0}^{p} a_k^2 (\delta_{mk} + \delta_{nk})$$
$$+2\lambda \sum_{k,l=0}^{p} a_k a_l \delta_{mk} \delta_{nl} \Big) \phi_{mn} \phi_{nm}$$

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Spontaneous symmetry breaking

Vacuum condition: $\forall m \in \{0,..p\},\ a_m^2 > 0$ $\qquad (p = \lfloor \frac{\mu^2\theta}{8} - \frac{1}{2} \rfloor)$

- $-\mu^2 > -\frac{4}{\theta}$ $(p < 0)$:
  $v(x) = 0$: global minimum of the action
- $\frac{\mu^2\theta}{8} - \frac{1}{2} \in \mathbb{N}$:
  Local zero mode in the propagator
- $-\mu^2 < -\frac{4}{\theta}$ $(p \geq 0)$ and $\frac{\mu^2\theta}{8} - \frac{1}{2} \notin \mathbb{N}$:

$$v(x) = \sum_{k=0}^{p} \sqrt{\frac{2}{\lambda\theta}\left(\frac{\mu^2\theta}{8} - \frac{1}{2} - m\right)} f_{mm}(x)$$

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$$v(x) = \sum_{k=0}^{p} \sqrt{\frac{2}{\lambda\theta}\left(\frac{\mu^2\theta}{8} - \frac{1}{2} - m\right)} f_{mm}(x)$$

  is a local minimum of the action

Spontaneous symmetry breaking

Vacuum condition: $\forall m \in \{0, ..p\}$, $a_m^2 > 0$ $\quad (p = \lfloor \frac{\mu^2\theta}{8} - \frac{1}{2} \rfloor)$

- $-\mu^2 > -\frac{4}{\theta}$ $(p < 0)$:
  $v(x) = 0$: global minimum of the action
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Linear sigma model(1)

$$S = \int d^4x \Big(\frac{1}{2}(\partial_\mu \Phi_i) \star (\partial_\mu \Phi_i) + \frac{1}{2}\widetilde{x}^2 \Phi_i \Phi_i - \frac{\mu^2}{2}\Phi_i\Phi_i + \lambda \Phi_i \star \Phi_i \star \Phi_j \star \Phi_j \Big)$$

Vacuum: $< \Phi >= (0, \ldots, 0, v(x))$
fluctuation: $\delta\Phi = (\pi_1, \ldots, \pi_{N-1}, \sigma(x))$
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Condition for vanishing π-mass term:

$$\int d^4x\Big(\frac{1}{2}\widetilde{x}^2\pi_i\pi_i - \frac{\mu^2}{2}\pi_i\pi_i + 2\lambda v\star v\star\pi_i\star\pi_i\Big)$$
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