Exploring the QCD phase diagram on the lattice

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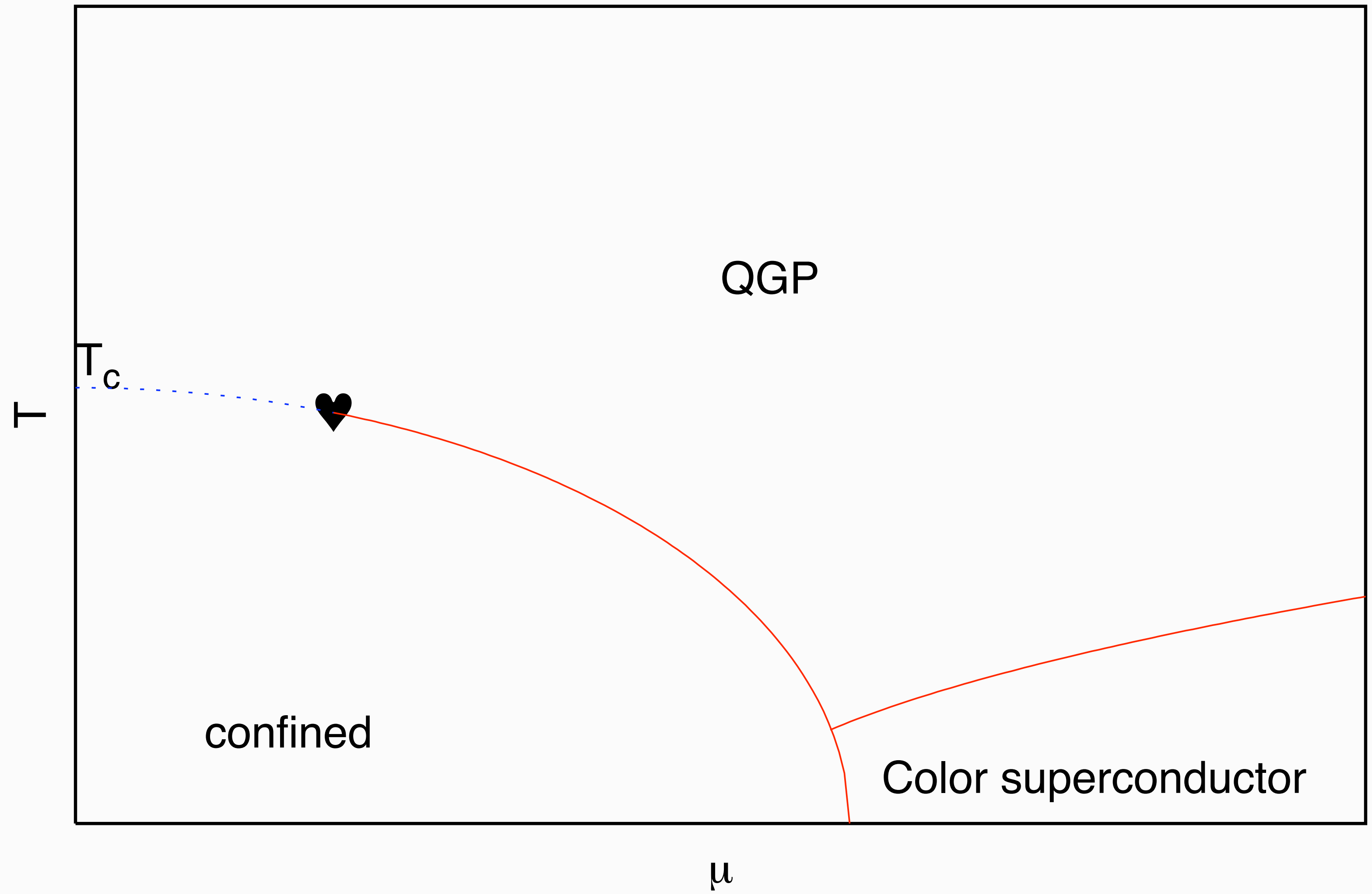
- Overview of lattice techniques for finite temperature and density
- Summary of results: the confusion before clarity

Reviews: Eur.Phys.J.ST 152 (2007) 29; PoS LAT05: 016 (2006)

Original work with Ph. de Forcrand (ETH/CERN): PoS LAT08:208; JHEP 0811:012

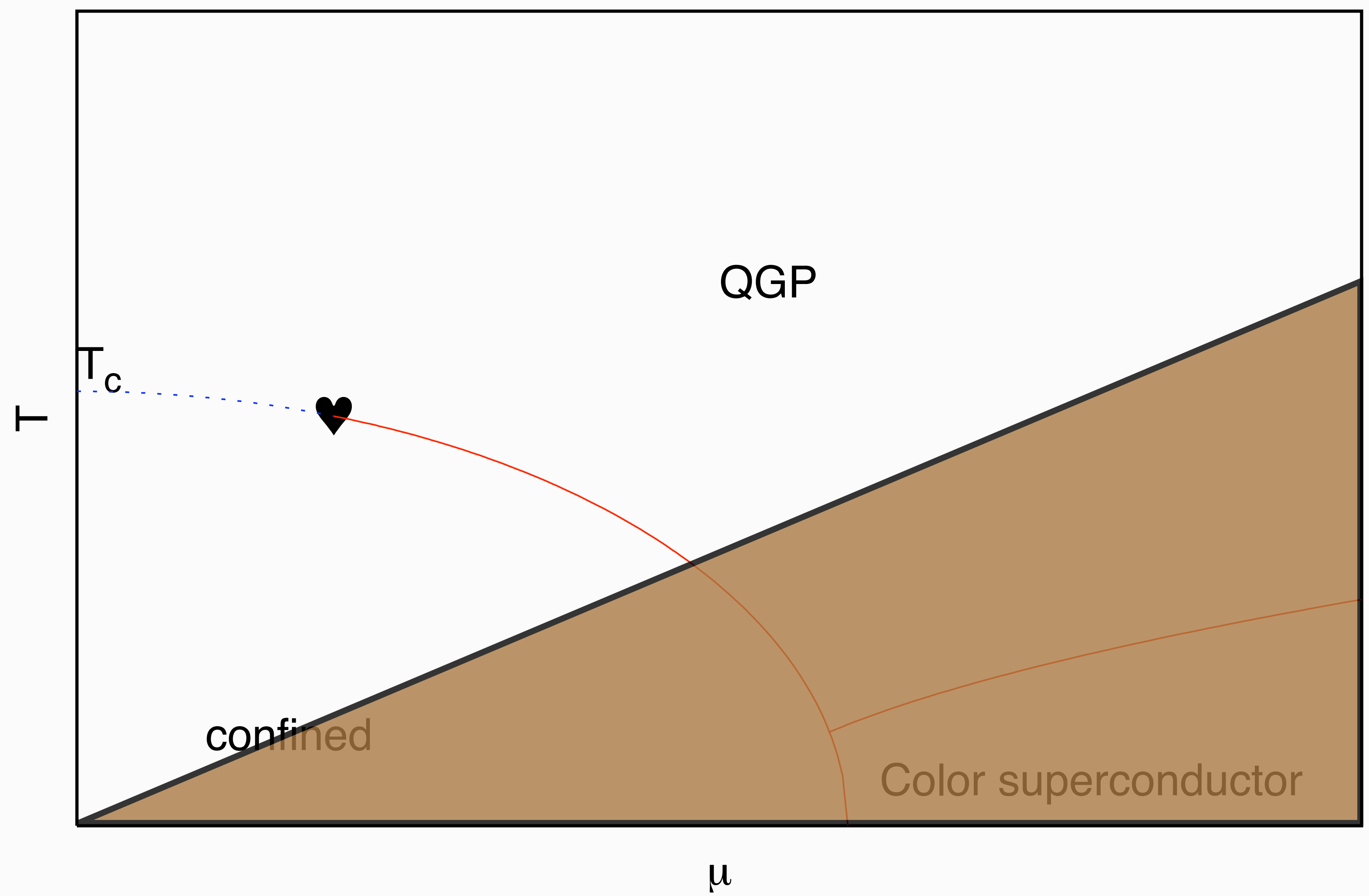
The conjectured QCD phase diagram



- No first principles calculations before 2001: sign problem

Expectation based on models: NJL, NJL+Polyakov loop, linear sigma models, random matrix models, ...

The conjectured QCD phase diagram



- 2001-2008: sign problem not cured, circumvented by approximate methods, need $\mu \lesssim T_c \quad (\mu = \mu_B/3)$

- Upper region: density effects on eq. of state, screening masses, phase diagram

The ‘sign problem’ is a phase problem

$$Z = \int DU \, [\det M(\mu)]^f \mathrm{e}^{-S_g[U]}$$

importance sampling requires
positive weights

Dirac operator: $\quad D\!\!\!/\,(\mu)^\dagger = \gamma_5 D\!\!\!/\,(-\mu^*)\gamma_5$

$\Rightarrow \det(M)$ complex for SU(3), $\mu \neq 0$

$\Rightarrow$ real positive for SU(2), $\mu = i\mu_i$

$\Rightarrow$ real positive for $\mu_u = -\mu_d$

N.B.: all expectation values real, imaginary parts cancel,
but importance sampling config. by config. impossible!

Cut-off effects: $\quad T = \dfrac{1}{aN_t}$

Continuum limit: $\quad N_t \rightarrow \infty, a \rightarrow 0$

Phase diagram: mostly $\quad N_t = 4 : a \sim 0.3 \text{ fm}, \quad N_t = 6 : a \sim 0.2 \text{ fm}$

I. Reweighting

Glasgow Method

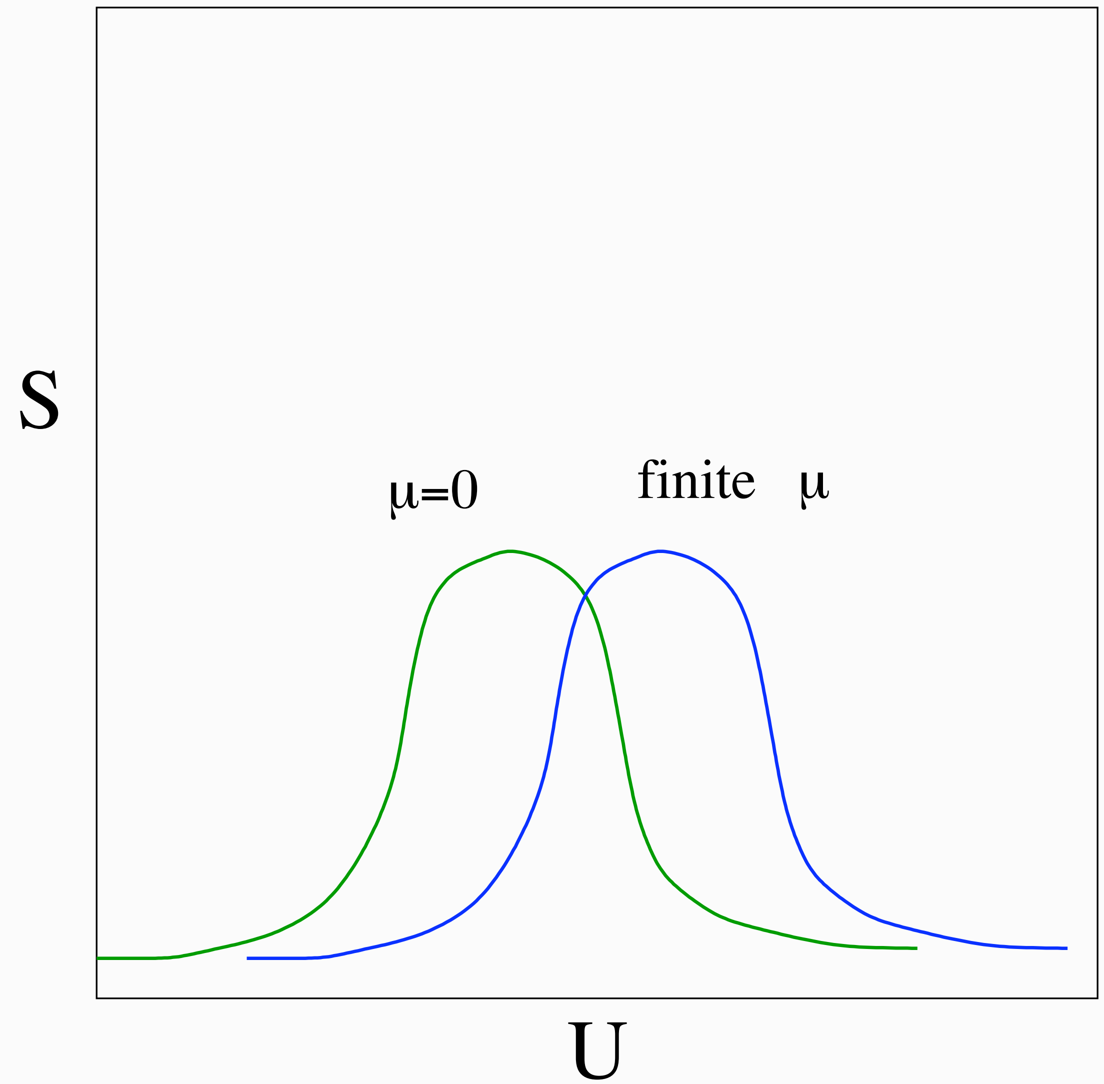
$$Z = \int DU \, [\det M(0)]^f \left[\frac{\det M(\mu)}{\det M(0)}\right]^f e^{-S_g[U]} = \left\langle \frac{\det M(\mu)}{\det M(0)} \right\rangle_{\mu=0} \sim e^{-const.V}$$

use for MC — calculate

Problems:

-MC and target distributions different,
overlap problem

-exponentially suppressed by volume,
need statistics ~ exp(V)



Generalization to two parameters (T, μ): increase overlap along critical line

Fodor, Katz 01

II. Taylor expansion

Measure derivatives w.r.t. chemical potential, zero density calculation

CP-symmetry: partition function even under $\mu \longrightarrow -\mu$

μ-independent O:

$$\langle O\rangle(\mu) = \langle O\rangle(0) + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

- have to compute coeffs. one by one
- convergence?
- higher orders increasingly complex to calculate accurately: delicate cancellations

MILC; Bielefeld/Swansea; Gavai, Gupta....

III.Imaginary chemical potential + analytic continuation

de Forcrand, O.P.
D'Elia, Lombardo
Azcoiti et al.....

Fermion determinant positive, no sign problem!

calculate at imag. chem. potential, fit to truncated polynomial

$$\langle O\rangle(\mu_i) = \sum_n^N c_n \left(\frac{\mu_i}{\pi T}\right)^{2n} \Rightarrow \mu_i \longrightarrow i\mu_i$$

- vary two parameters in MC ensemble
- explicit check for convergence in imag. direction
- problems: restricted to $\mu_B \lesssim 500$ MeV, convergence properties different in real direction (T, μ)

IV. Canonical ensemble

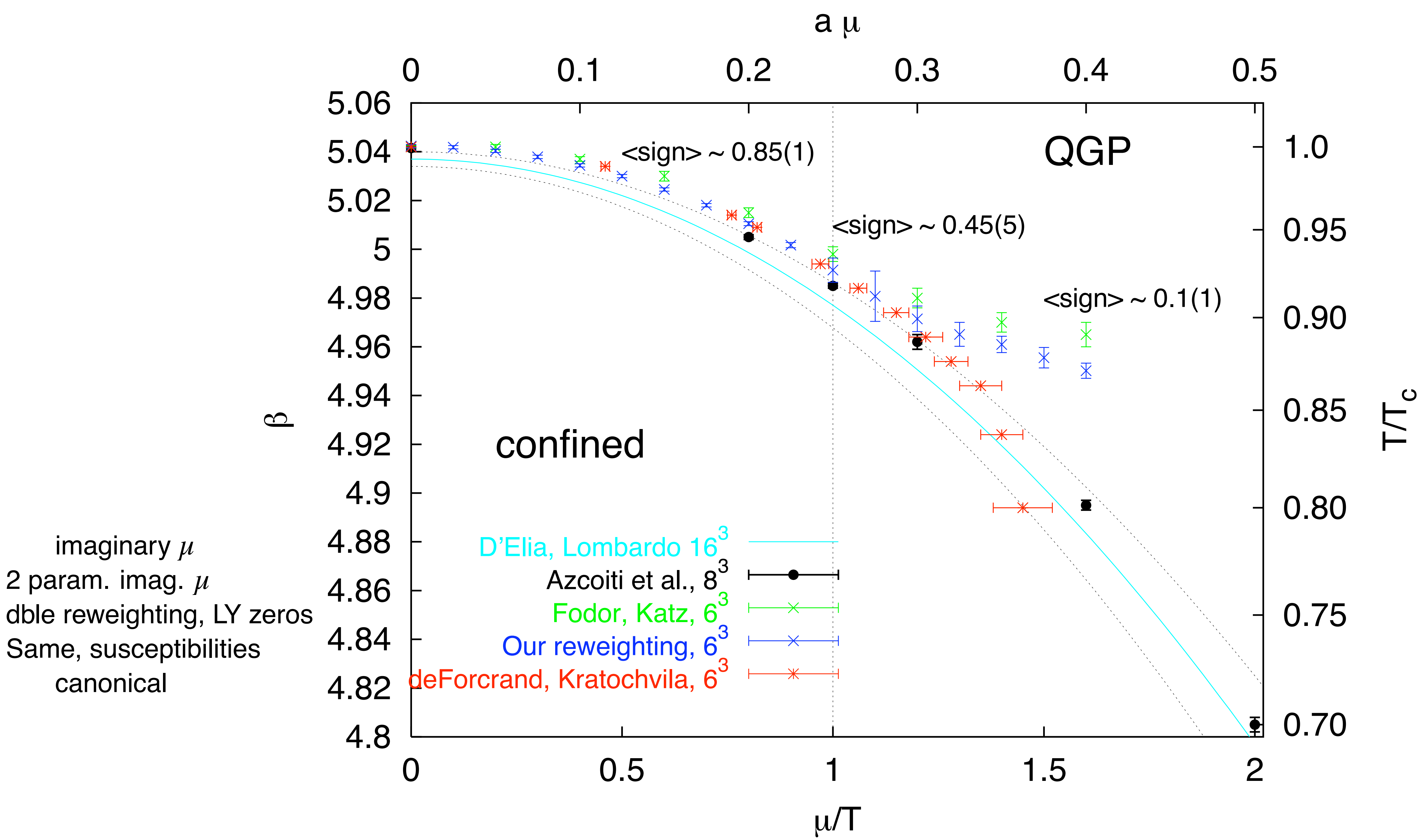
Alford, Kapustin, Wilczek
de Forcrand, Kratochvila
Alexandru et al....

- calculate $Z(B)$ by Fourier trafo from $Z(i\mu_i)$
- restricted to small volumes so far

Comparing approaches: the critical line

de Forcrand, Kratochvila LAT 05

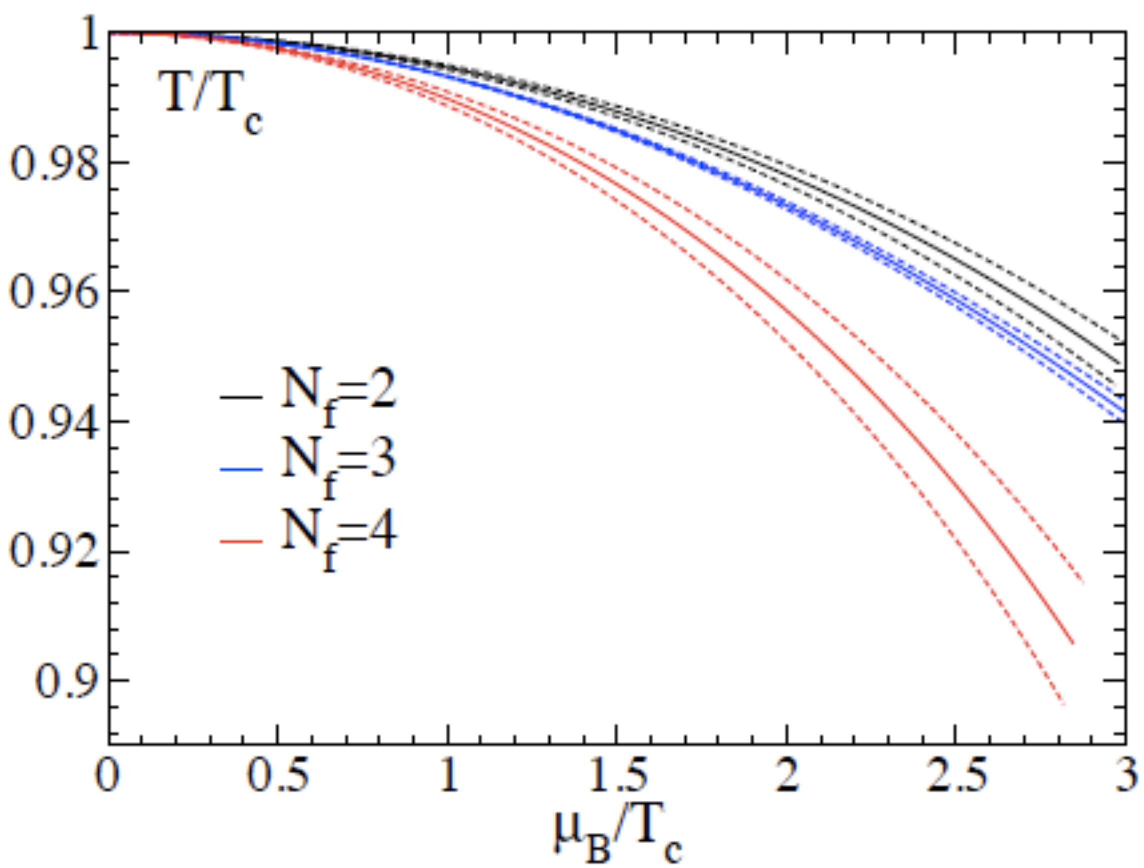
$N_t = 4, N_f = 4$; same actions (unimproved staggered), same mass



Agreement for $\mu/T \lesssim 1$

The (pseudo-) critical temperature

de Forcrand, O.P. 03; d'Elia Lombardo 03



$$\frac{T_c(\mu)}{T_c(\mu=0)} = 1 - c(N_f, m_q)\left(\frac{\mu}{\pi T}\right)^2 + ..$$

$c \approx 0.500(34), 0.602(9), 0.93(10)$

for light $N_f = 2, 3, 4$

cf. Toublan:($c \propto N_f/N_c$)

smaller than freeze-out curve $t_2 \approx$ **2.05** Cleymans et al.

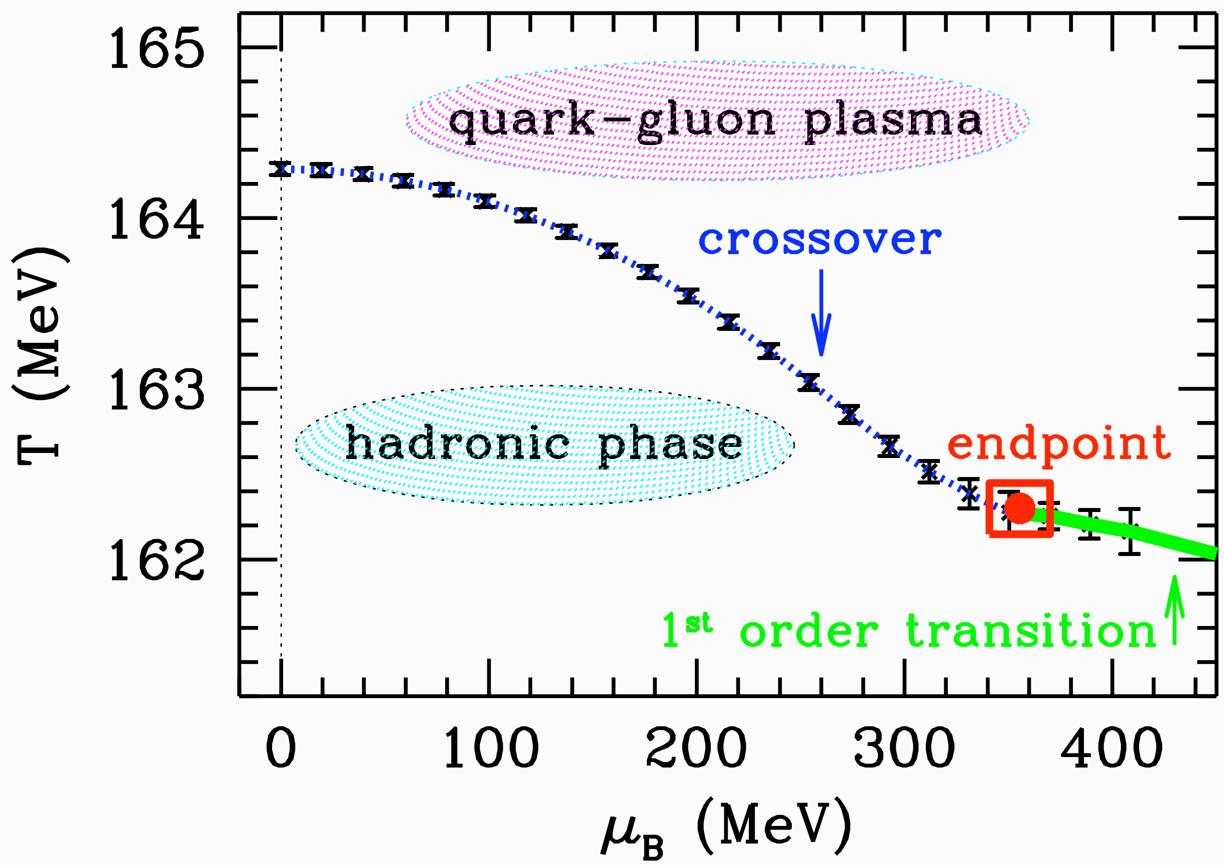
but coarse lattices; slightly larger values on finer lattices Wuppertal 07

physical masses in progress, cf. Endrodi

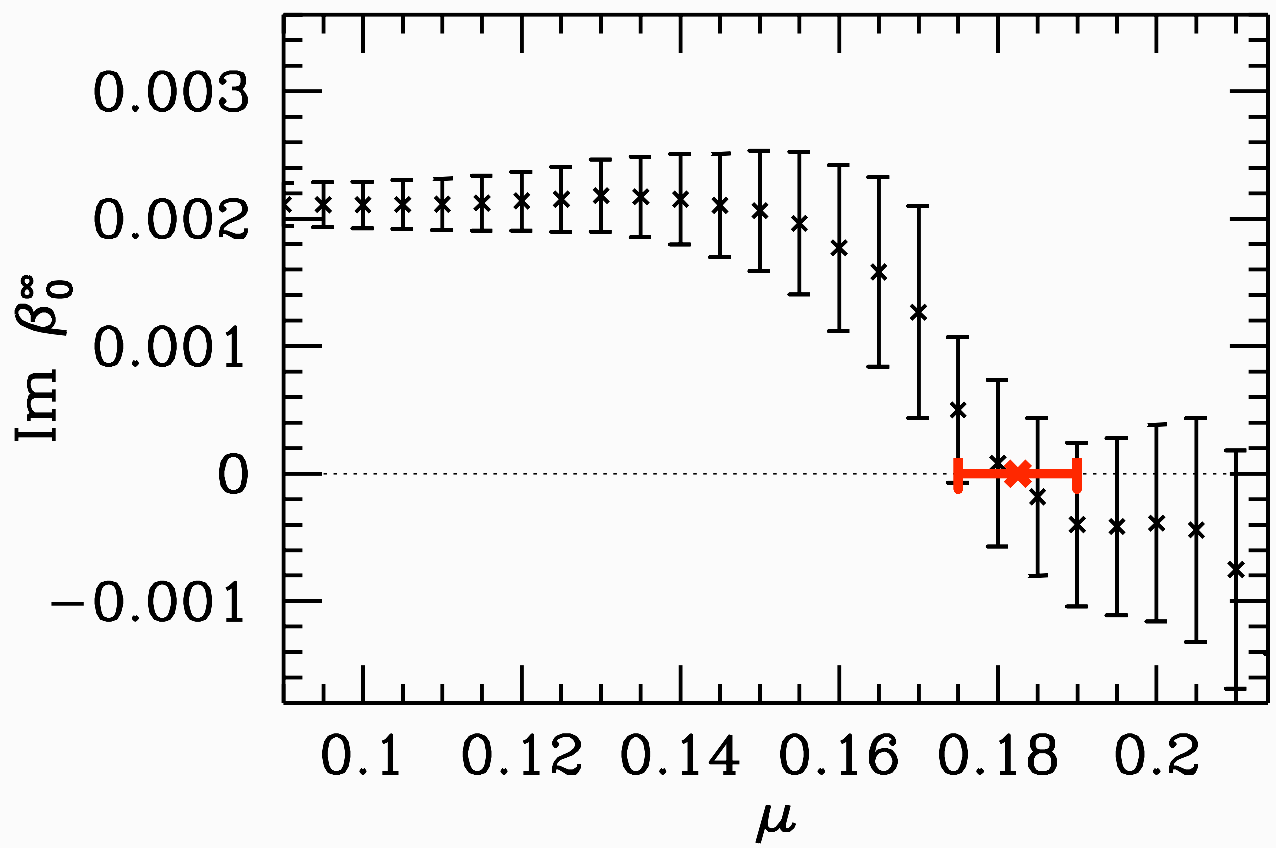
Critical point from reweighting

Fodor,Katz JHEP 04

$N_t = 4, N_f = 2+1$ physical quark masses, unimproved staggered fermions



Lee-Yang zero:



$$(\mu_E^q, T_E) = \ (120(13), 162(2))\ \text{MeV}$$

abrupt change: physics or problem of the method?

Splittorff 05;
Han, Stephanov 08

Critical point from Taylor expansion

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

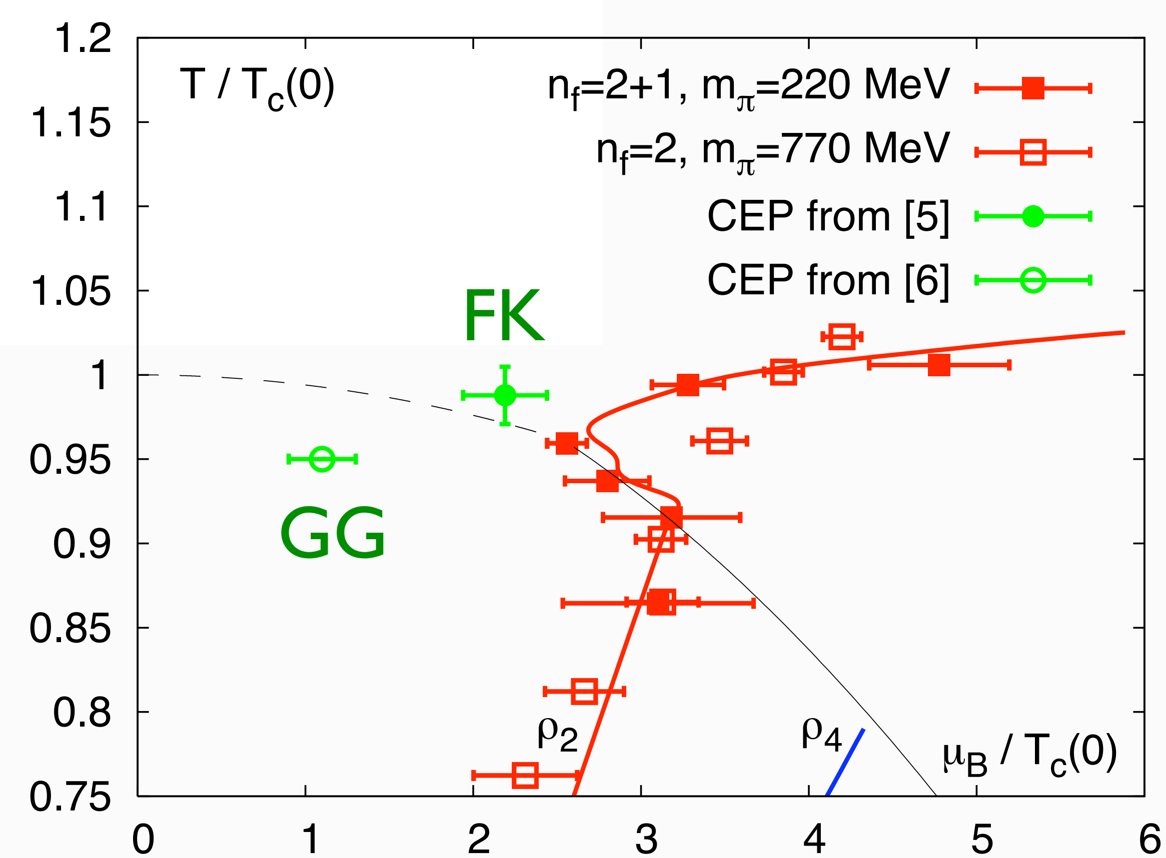
Singularity $(\mu_E, T_E) \Rightarrow$ $\boxed{\frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}}$ Karsch et al.

- Need $n \longrightarrow \infty$, not $n = 1$ or 2; $\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound
- Other definitions just as good, eg. $\lim_{n\to\infty} \left|\frac{c_0}{c_{2n}}\right|^{\frac{1}{2n}}$

Attn, different definitions used in following:

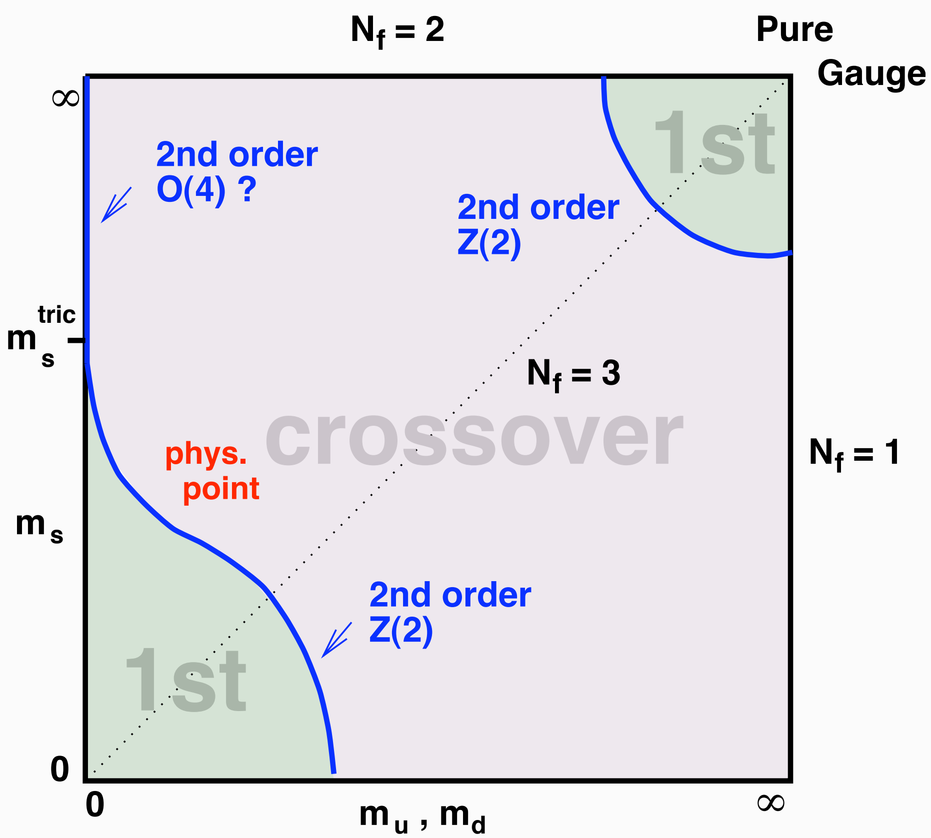
Bielefeld-RBC-collab 08: $N_t = 4, N_f = 2+1$

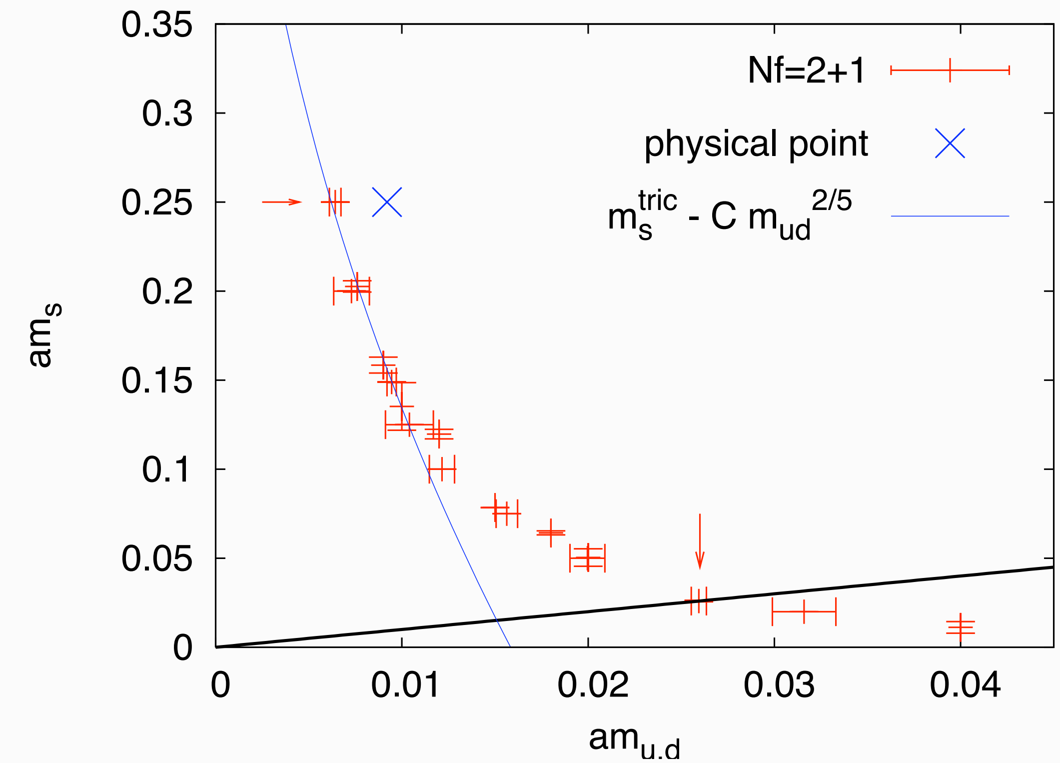
improved staggered



Gavai/Gupta: $N_f = 2, m/T_c = 0.1$, standard KS, critical point at $\mu_B^c/T = 1.1 \pm 0.2$

More general: p.t. as function of quark masses, zero density



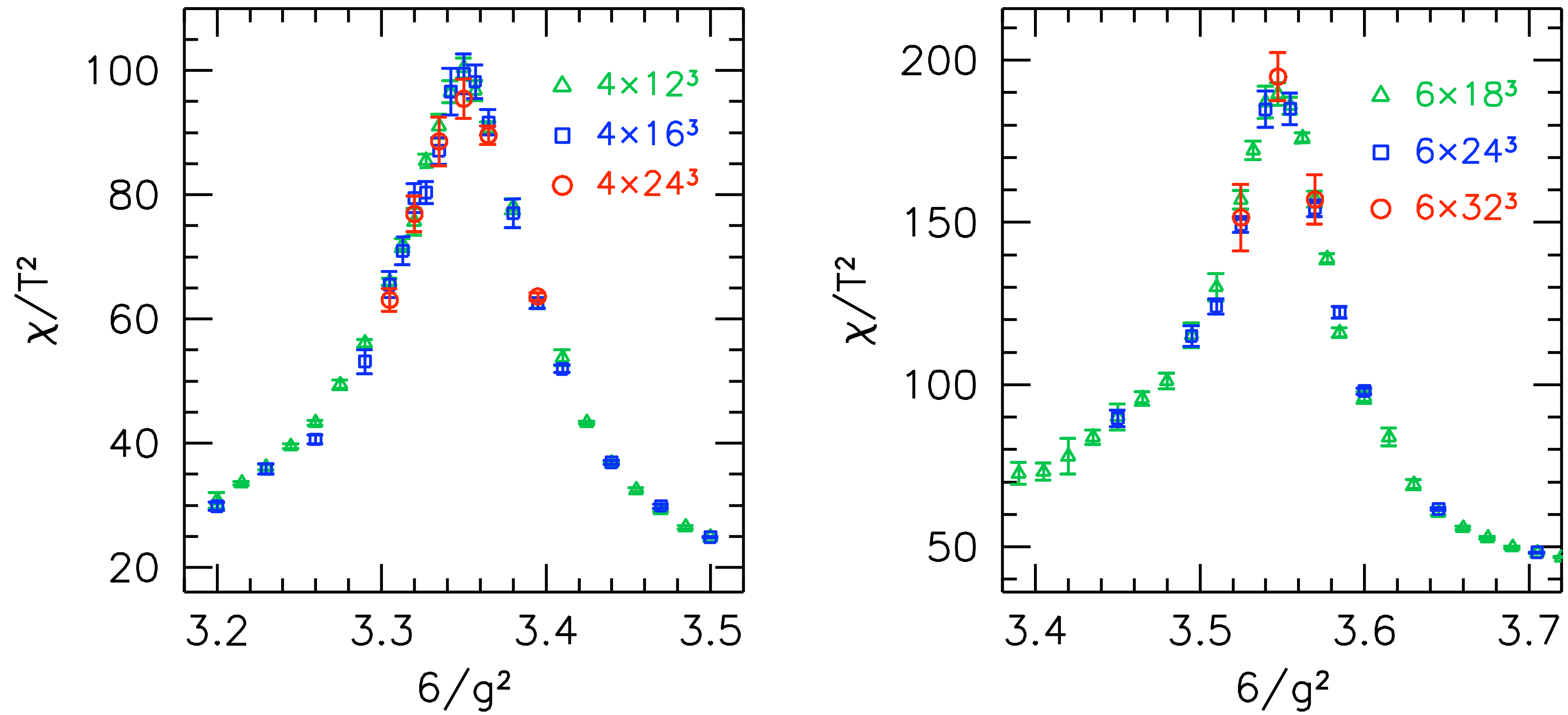


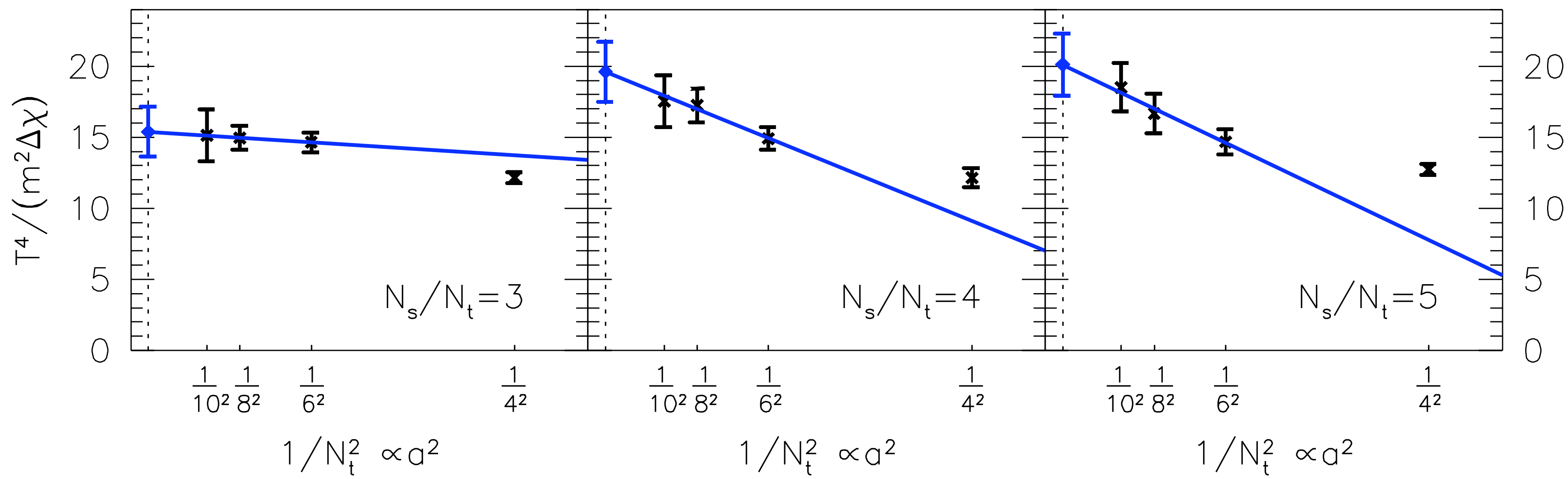
- $N_f = 2, m = 0$: $N_t = 4$, still not settled ax. U(1) anomaly — DiGiacomo et al. 05; Kogut, Sinclair 06; Chandrasekharan, Mehta 07
- phys. point: crossover in continuum — Aoki et al. 06
- chiral critical line: two points on
  - $N_t = 4$ — de Forcrand, O.P. 07
  - $N_t = 6$ — de Forcrand, O.P. 07; Endrodi et al. 07

The nature of the phase transition at the physical point

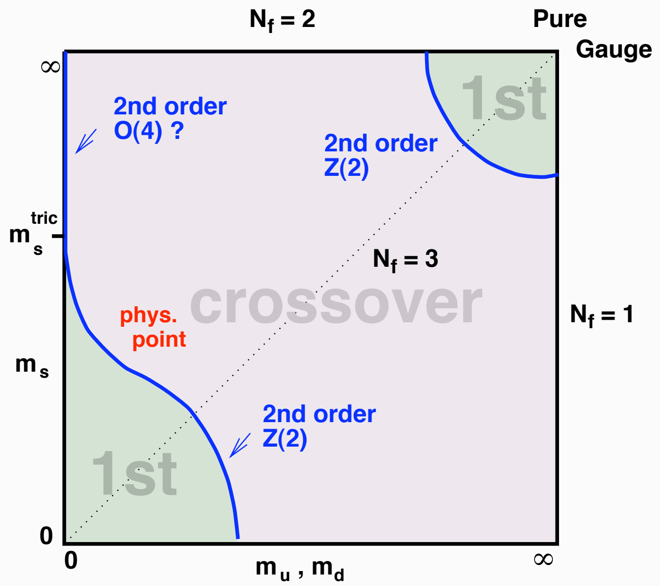
Fodor et al. 06

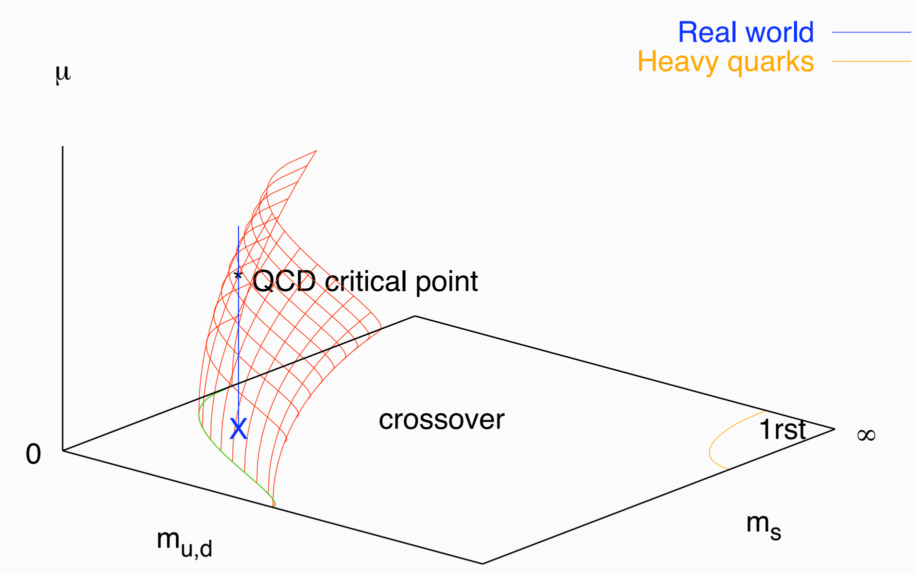
...in the staggered approximation...in the continuum...is a crossover!

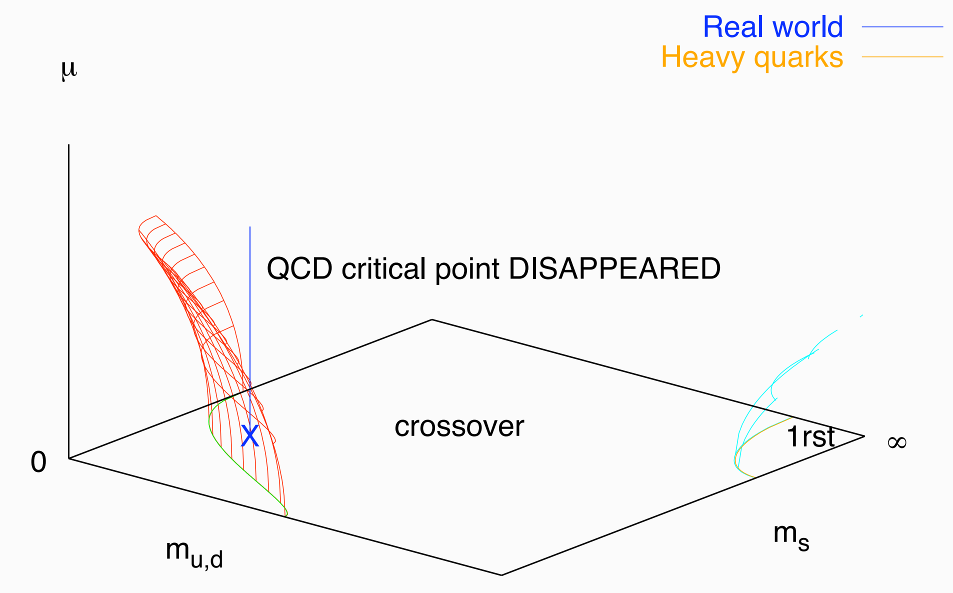




Finite density: chiral critical line ⟶ critical surface

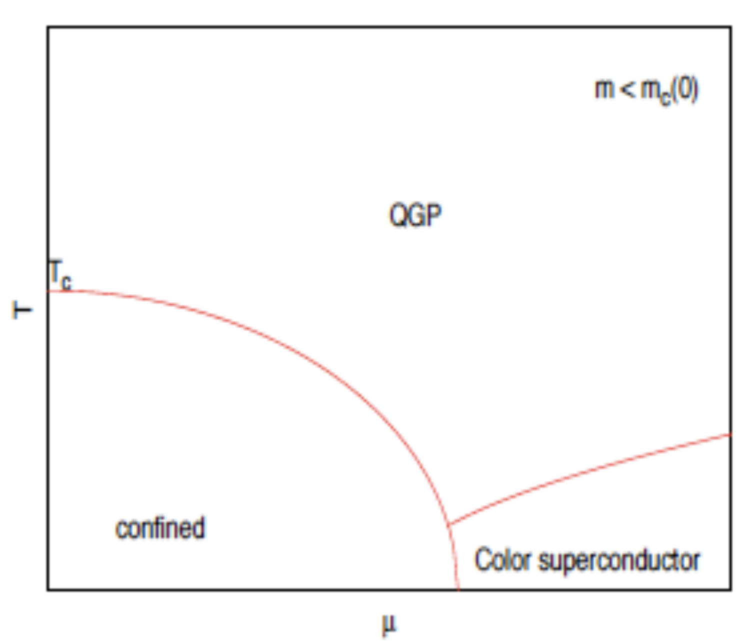


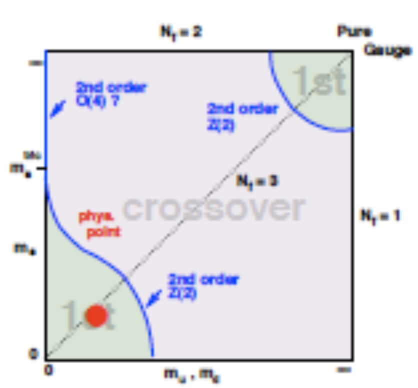


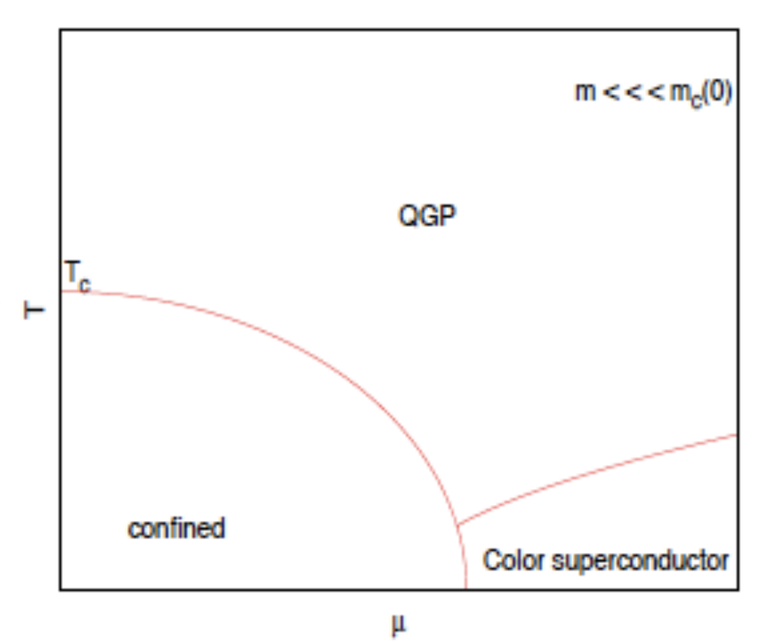


$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

$c_1 > 0$ $\qquad\qquad$ $c_1 < 0$







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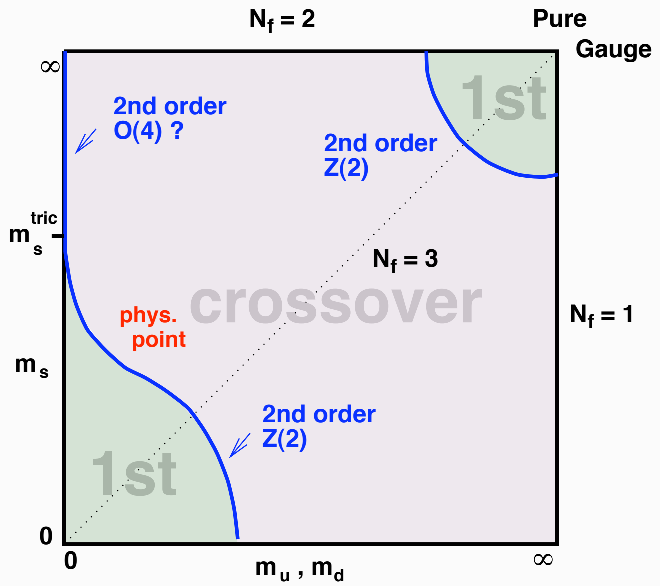
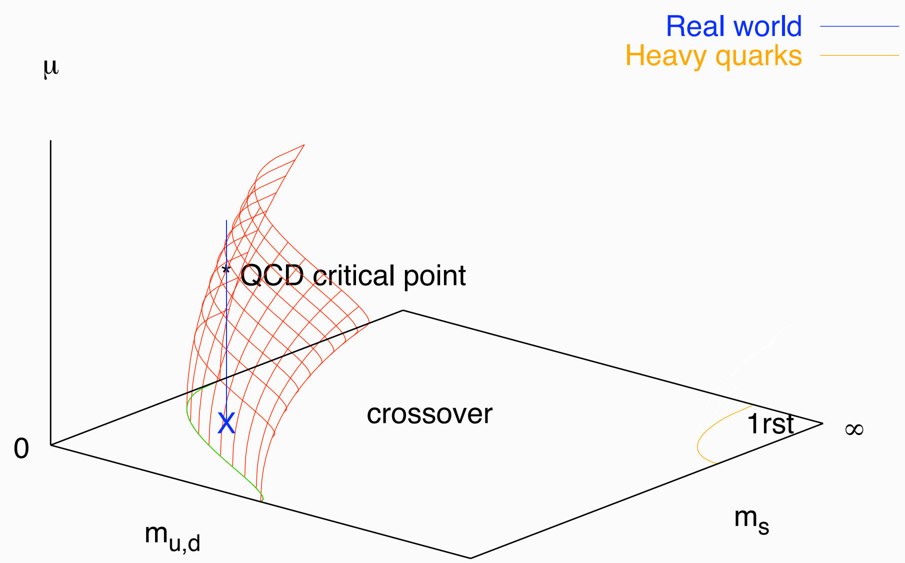
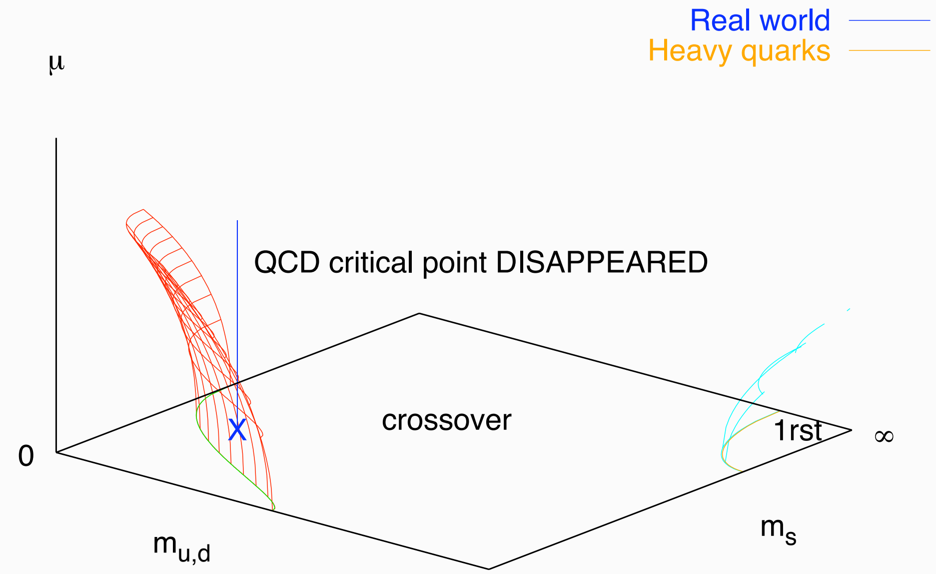
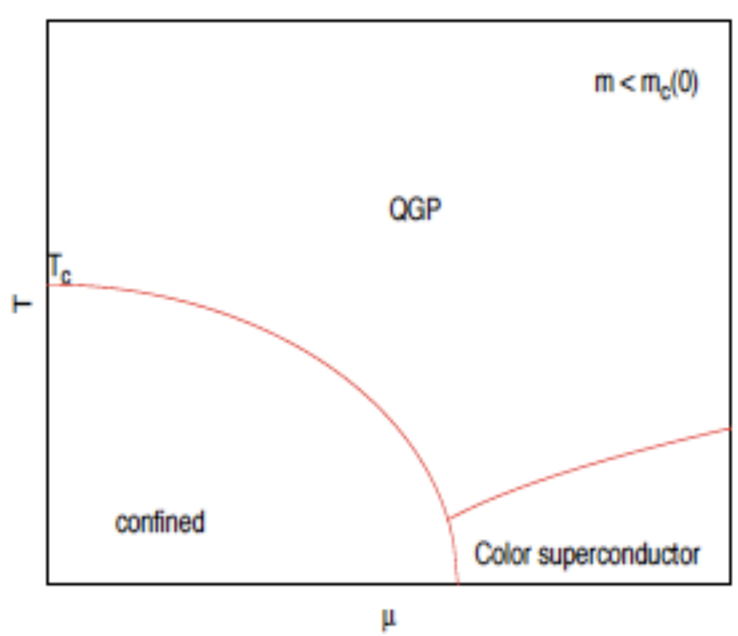


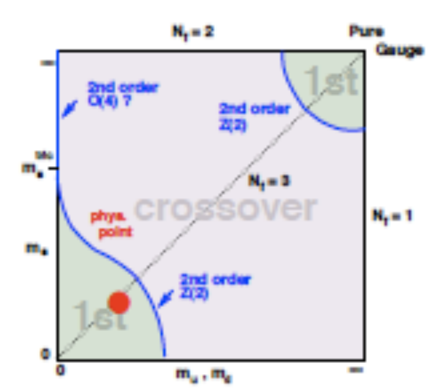


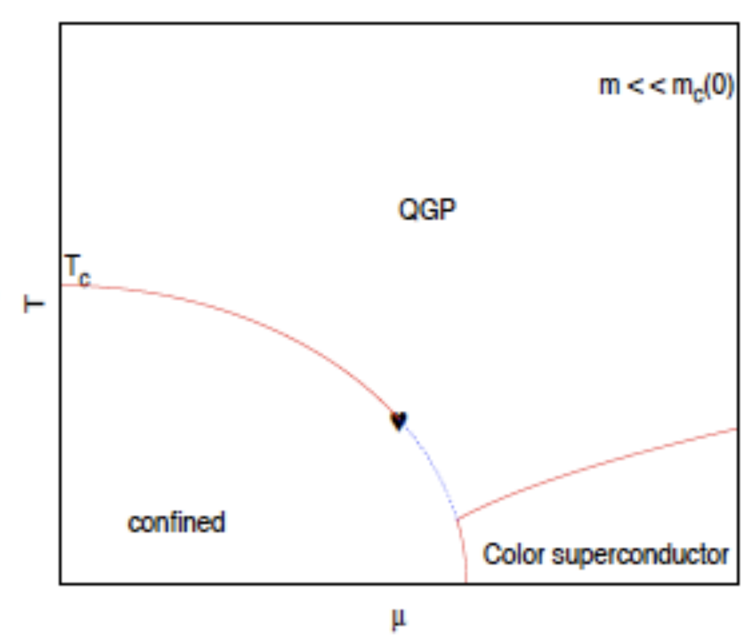


$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} \mathbf{c_k} \left(\frac{\mu}{\pi T}\right)^{2k}$$

$c_1 > 0$ $c_1 < 0$







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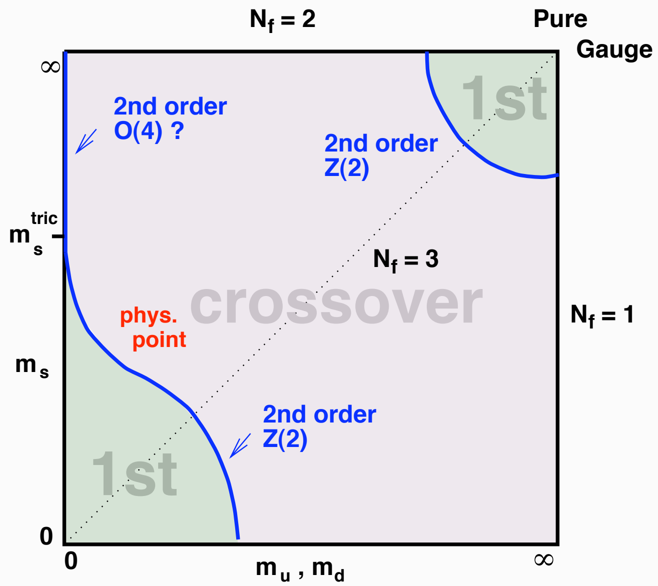

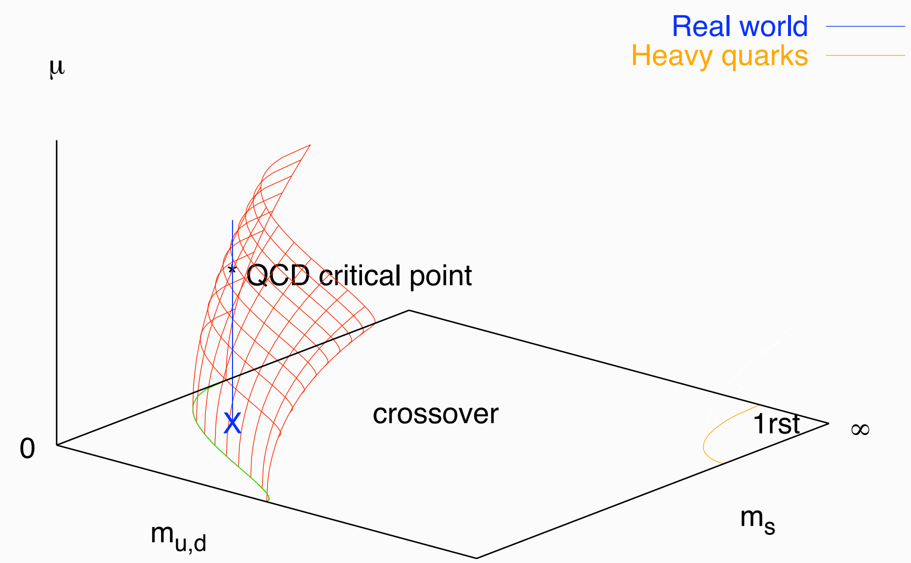

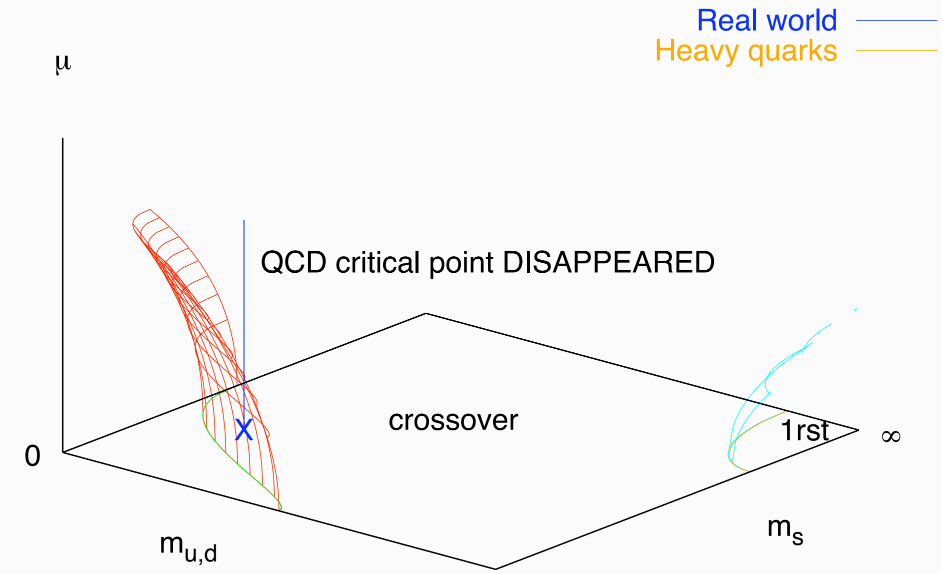

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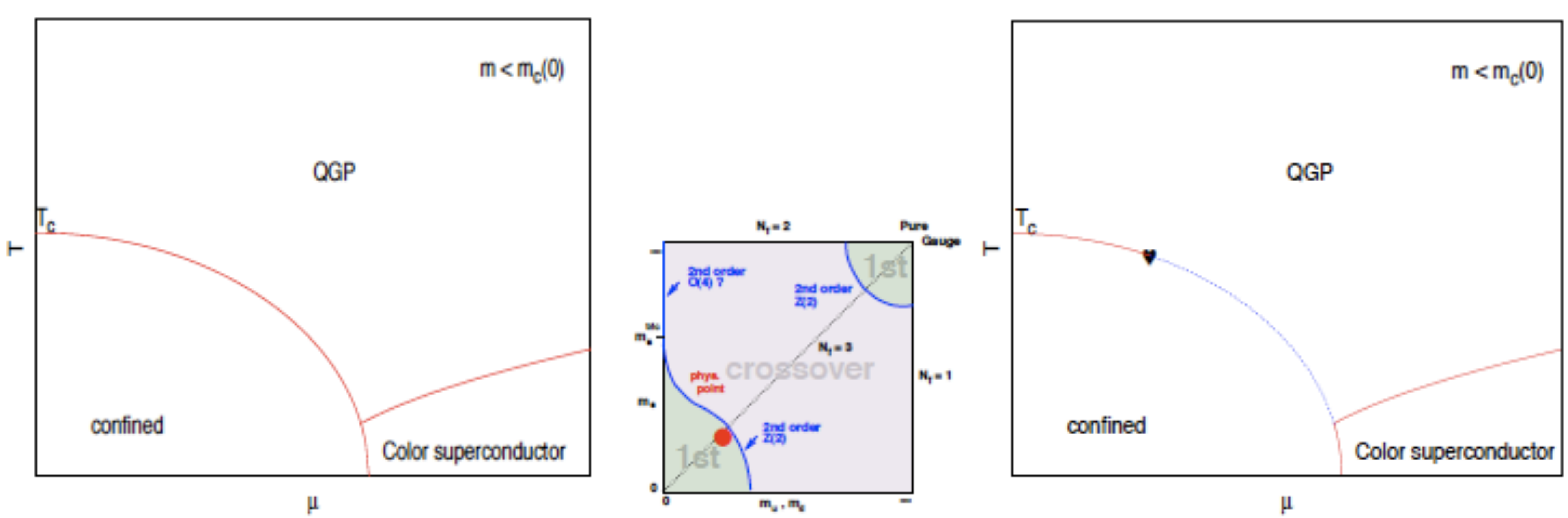

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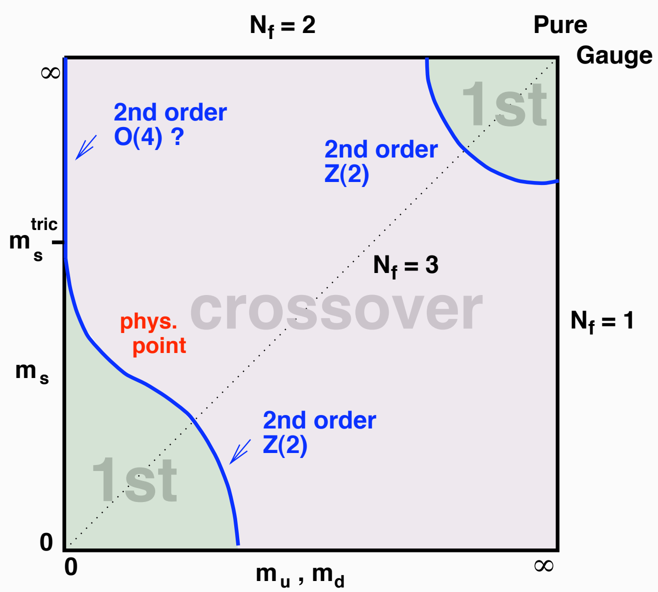

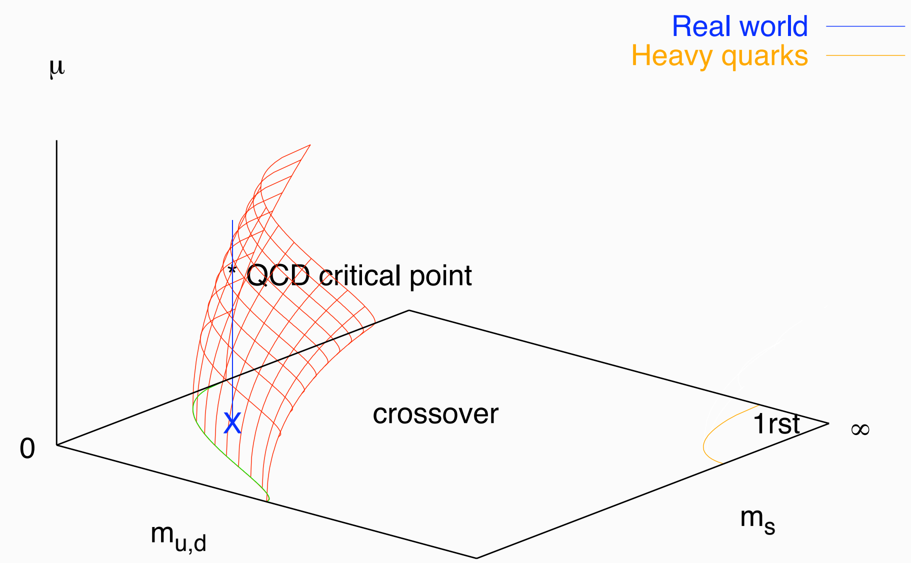

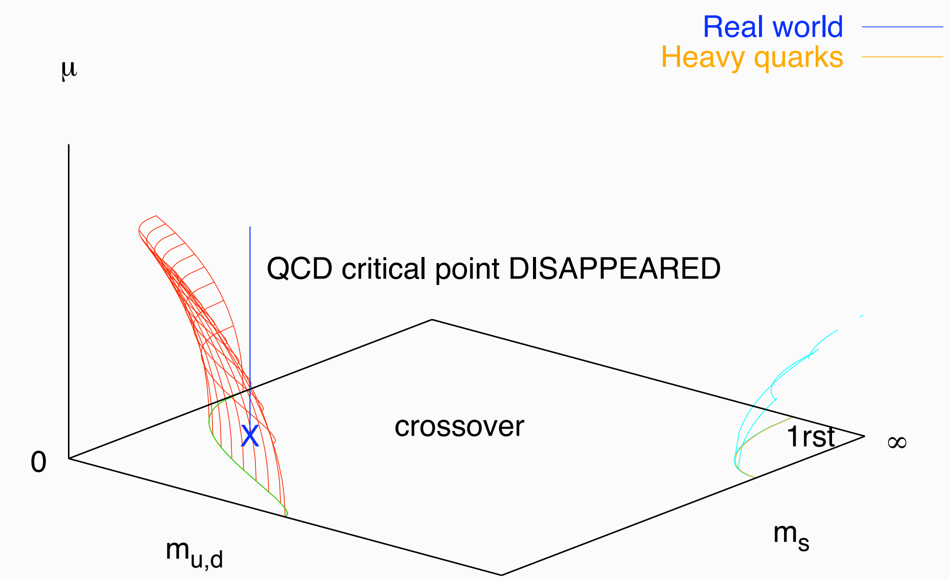

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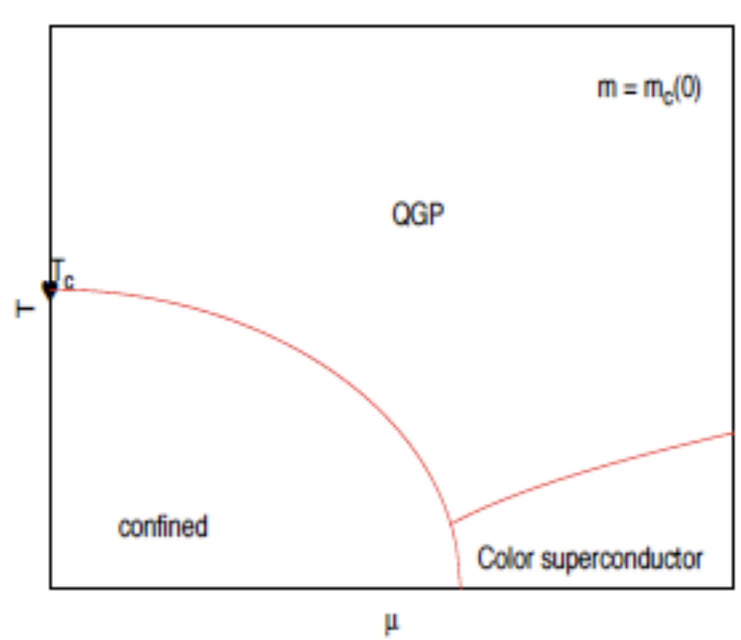

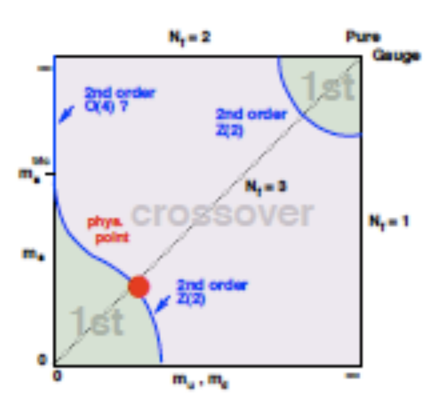

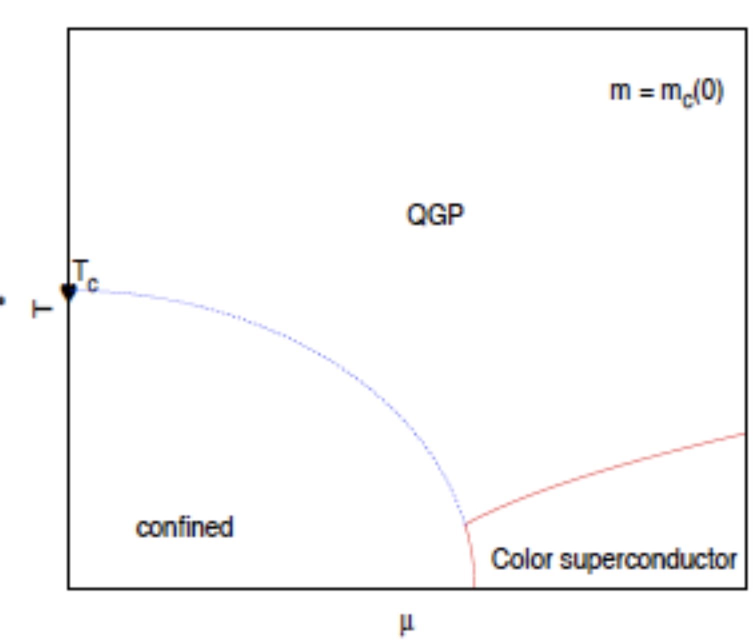

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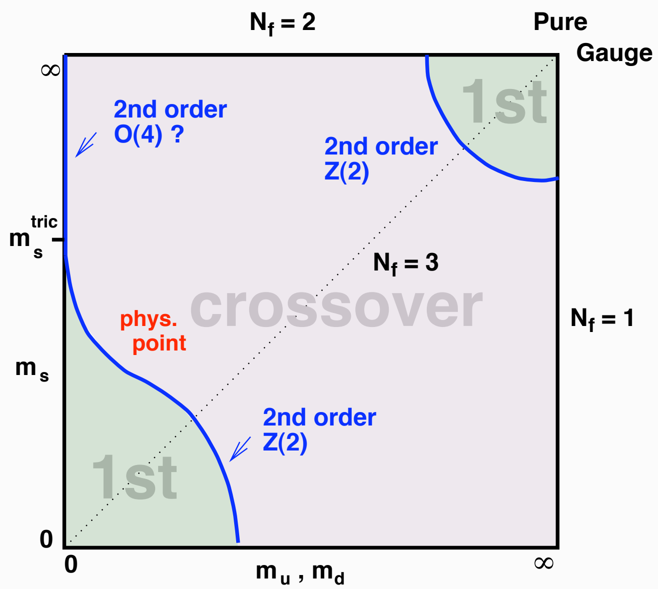

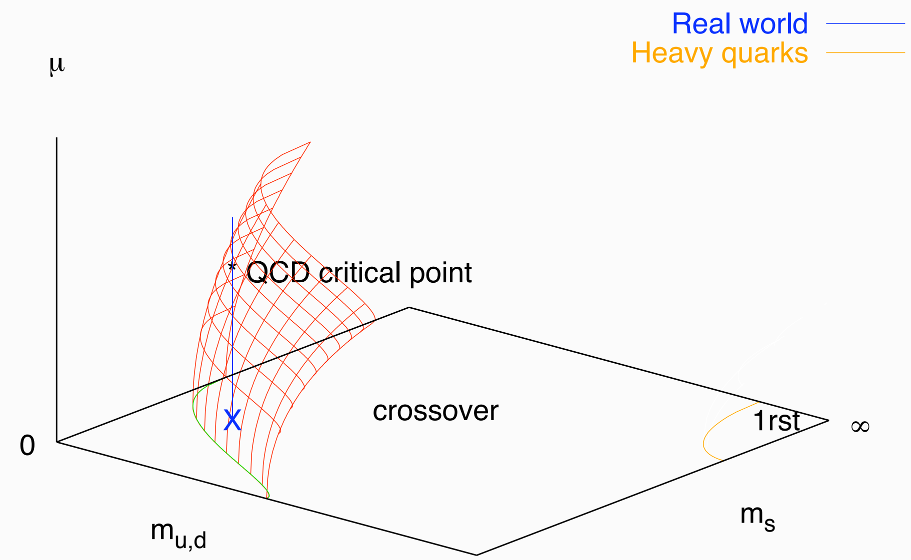

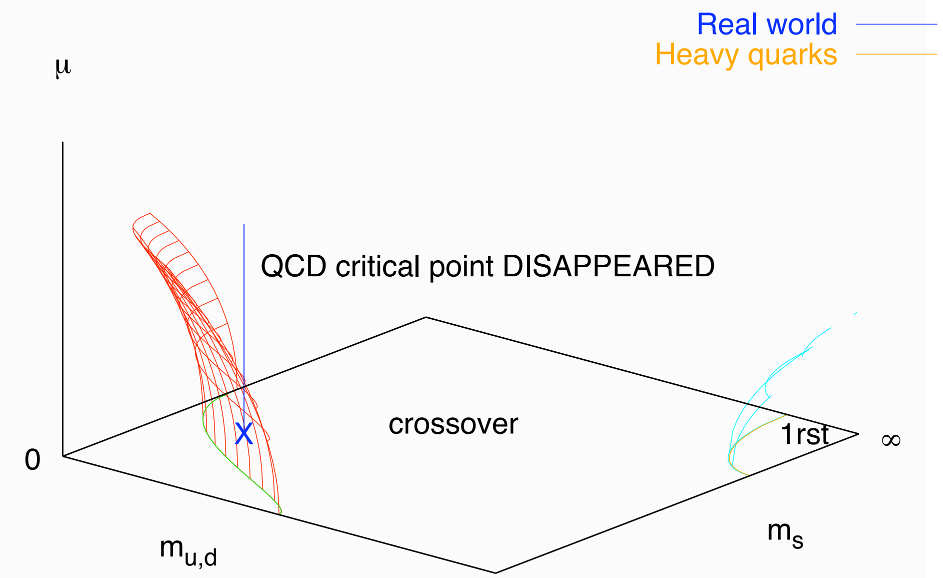

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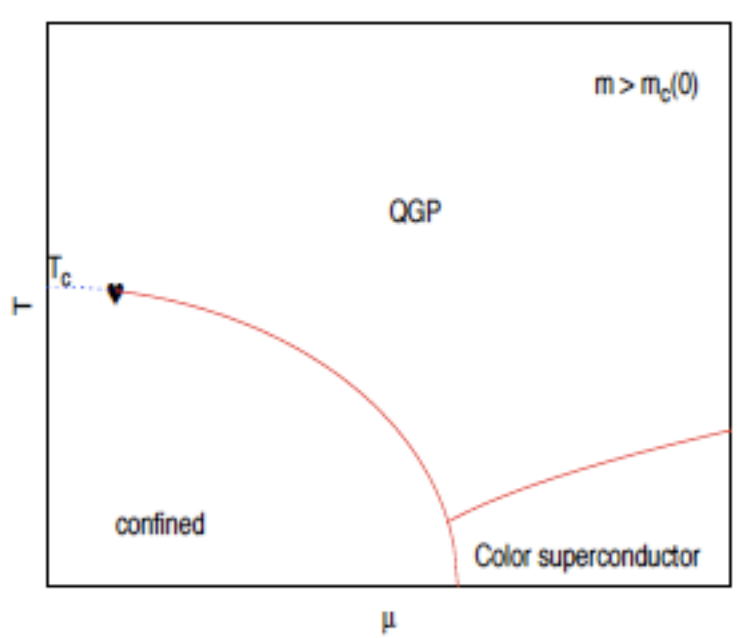
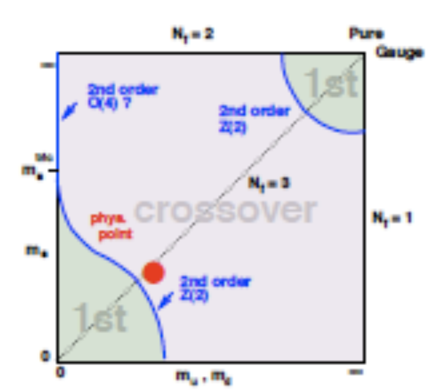
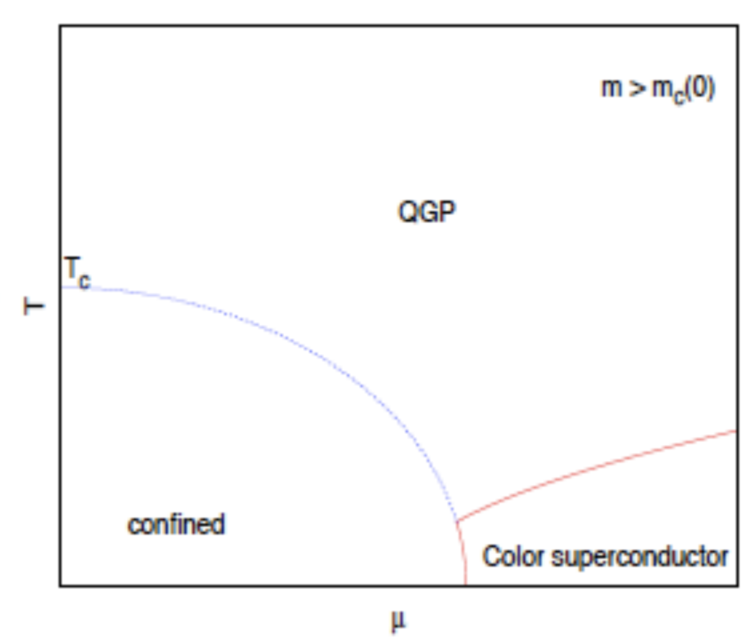


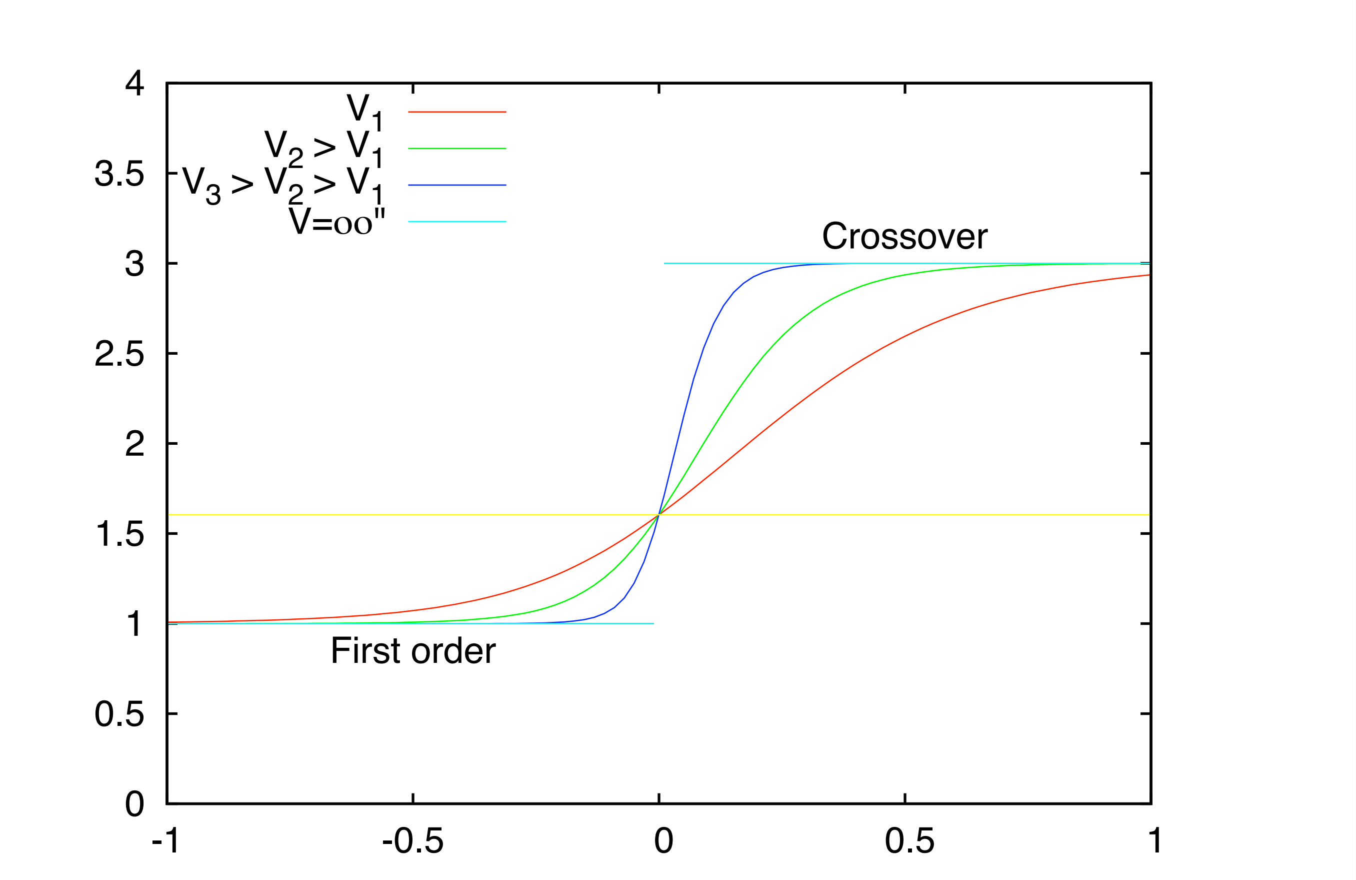


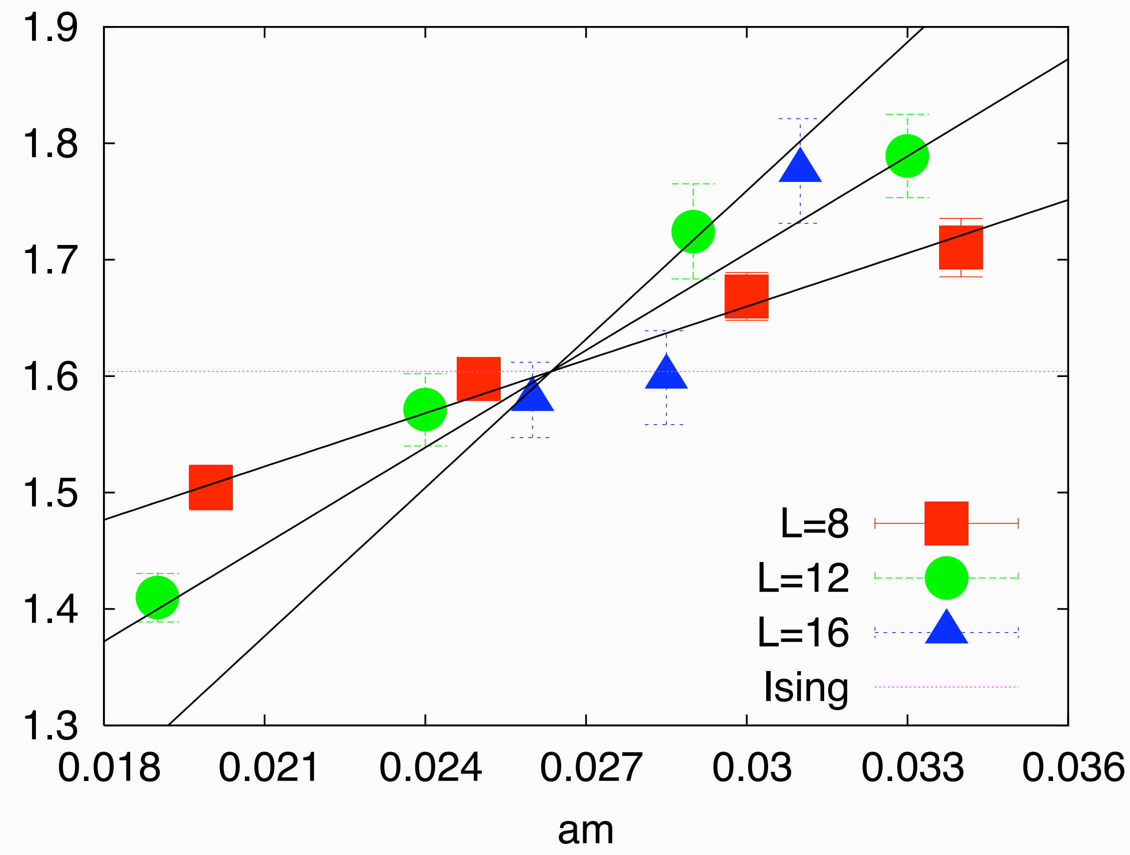


How to identify the critical surface: Binder cumulant

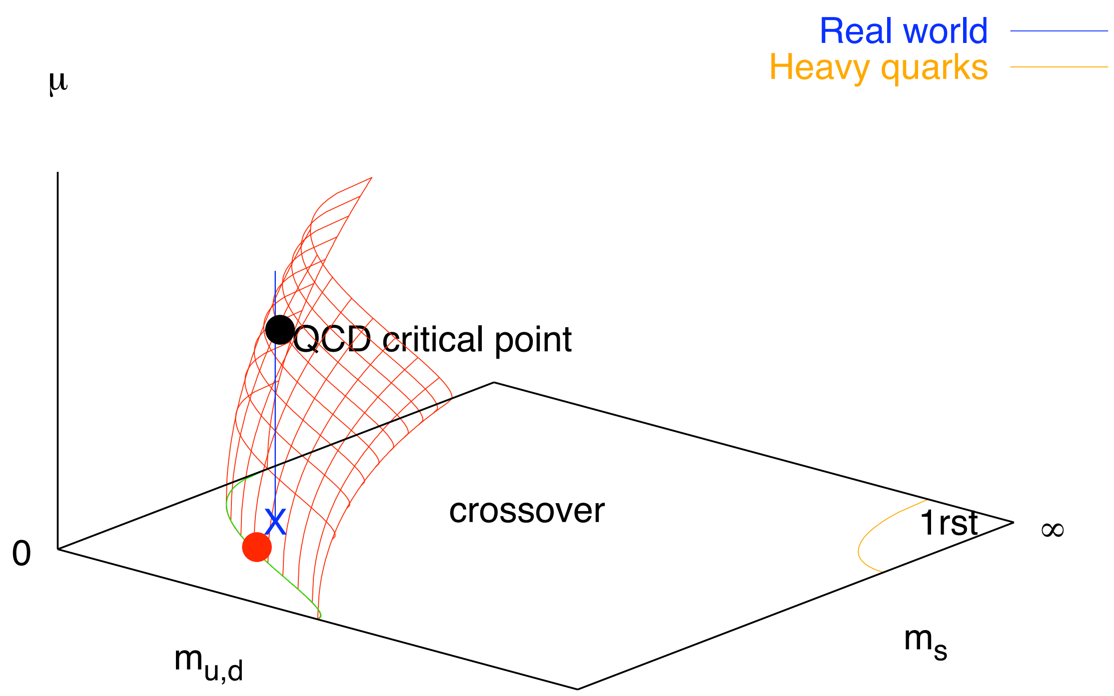
$$B_4(\bar\psi\psi) \equiv \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2} \xrightarrow{V\to\infty} \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first-order} \\ 3 & \text{crossover} \end{cases}$$

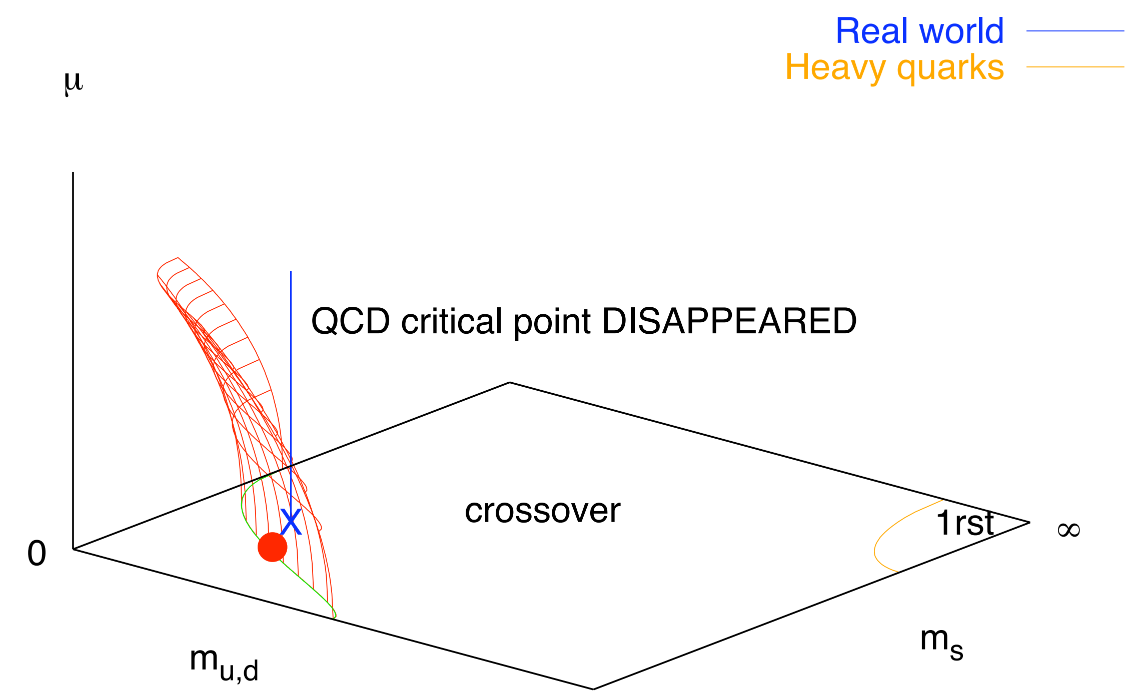
$\mu = 0:$ $$B_4(m,L) = 1.604 + bL^{1/\nu}(m - m_0^c), \quad \nu = 0.63$$





$\mu \neq 0$





Strategy: tune m_q for 2nd-order P.T. at $\mu = 0$, then turn on [imaginary] μ

Does the transition become 1rst-order (left) or crossover (right)?

$$B_4(am, a\mu) = 1.604 + \sum_{k,l=1} b_{kl} \, (am - am_0^c)^k (a\mu)^{2l}$$

$$\frac{d\, am^c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial (a\mu)^2} \Big/ \frac{\partial B_4}{\partial am} = -b_{01}/b_{10}, \quad \text{hard / easy}$$

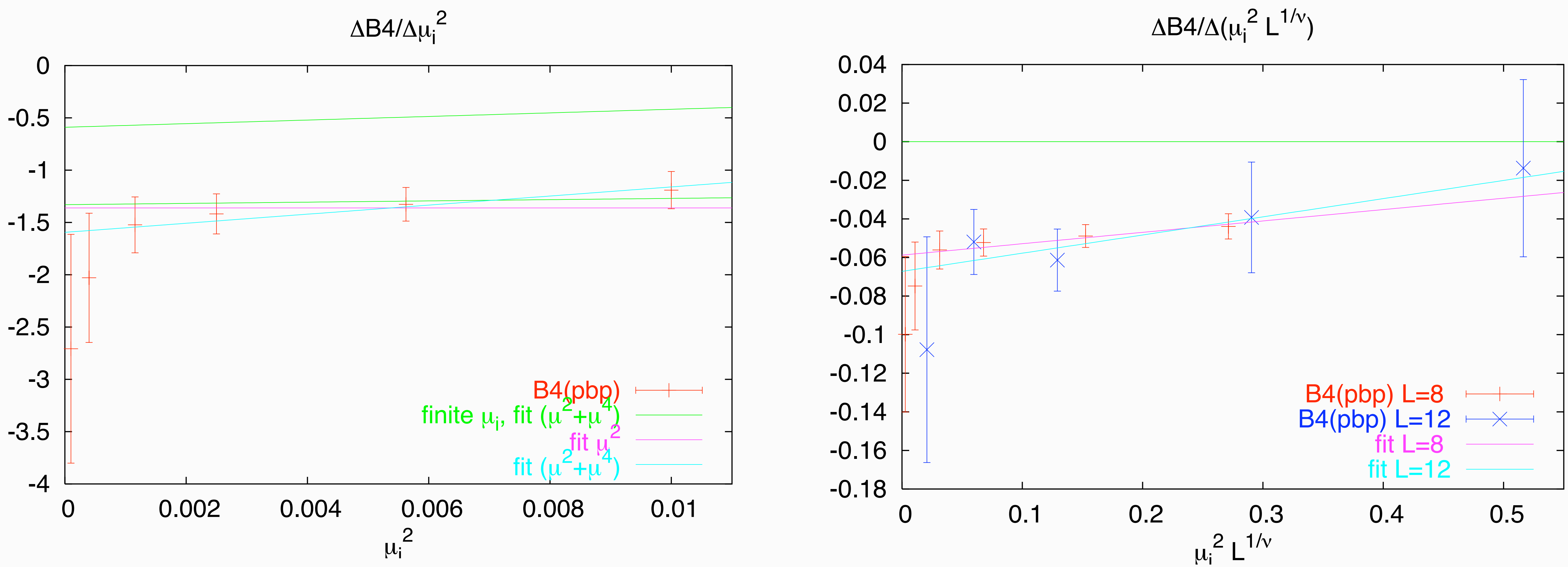
Numerical results for $N_f = 3, N_t = 4$

de Forcrand, O.P. 08

unimproved staggered fermions, RHMC algorithm

I. imag. μ : $8^3 \times 4, 42$ pairs $(am, a\mu_i)$ > 20 million traj., 18 unconstrained dof's in fits

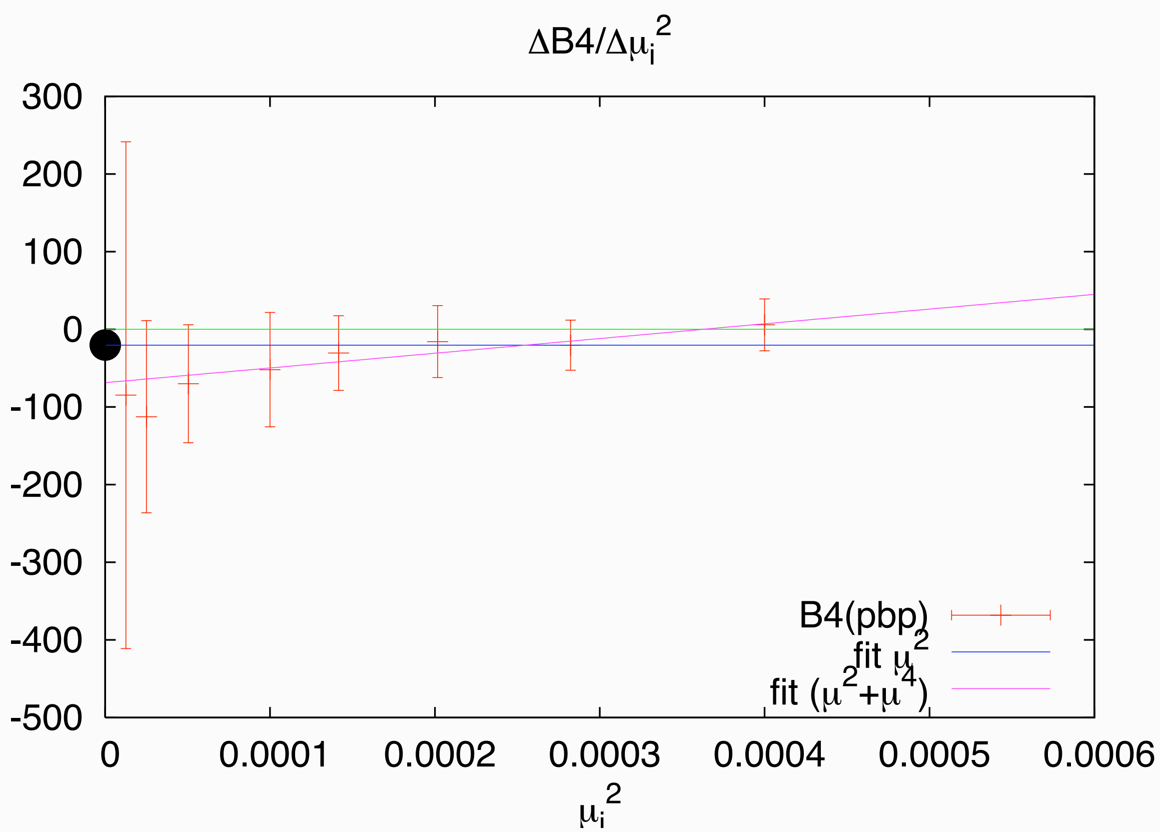
II: deriv. at $\mu = 0$: $8^3, 12^3 \times 4$ $m_\pi L \gtrsim 3, 4.5$ > 5 million, 0.5 million traj.

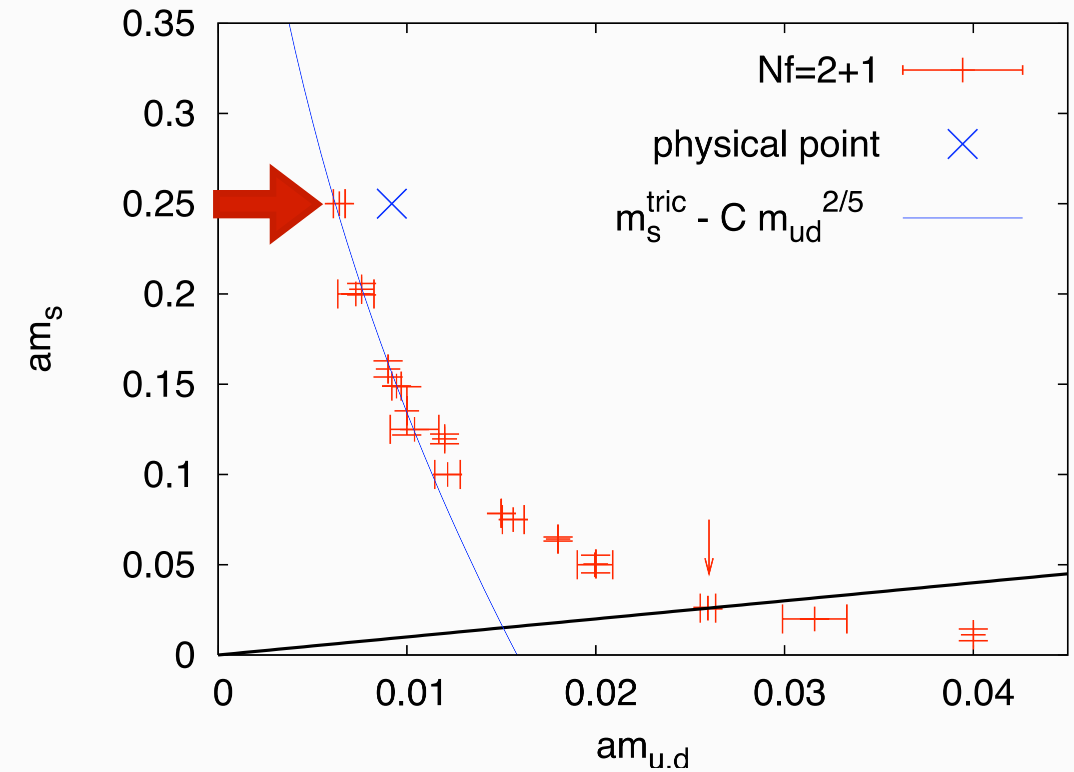


$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 - 47(20)\left(\frac{\mu}{\pi T}\right)^4 - \ldots$$

Non-degenerate fermion masses, $N_f = 2+1, N_t = 4$

de Forcrand, O.P., preliminary, thanks to CERN IT/ Grid

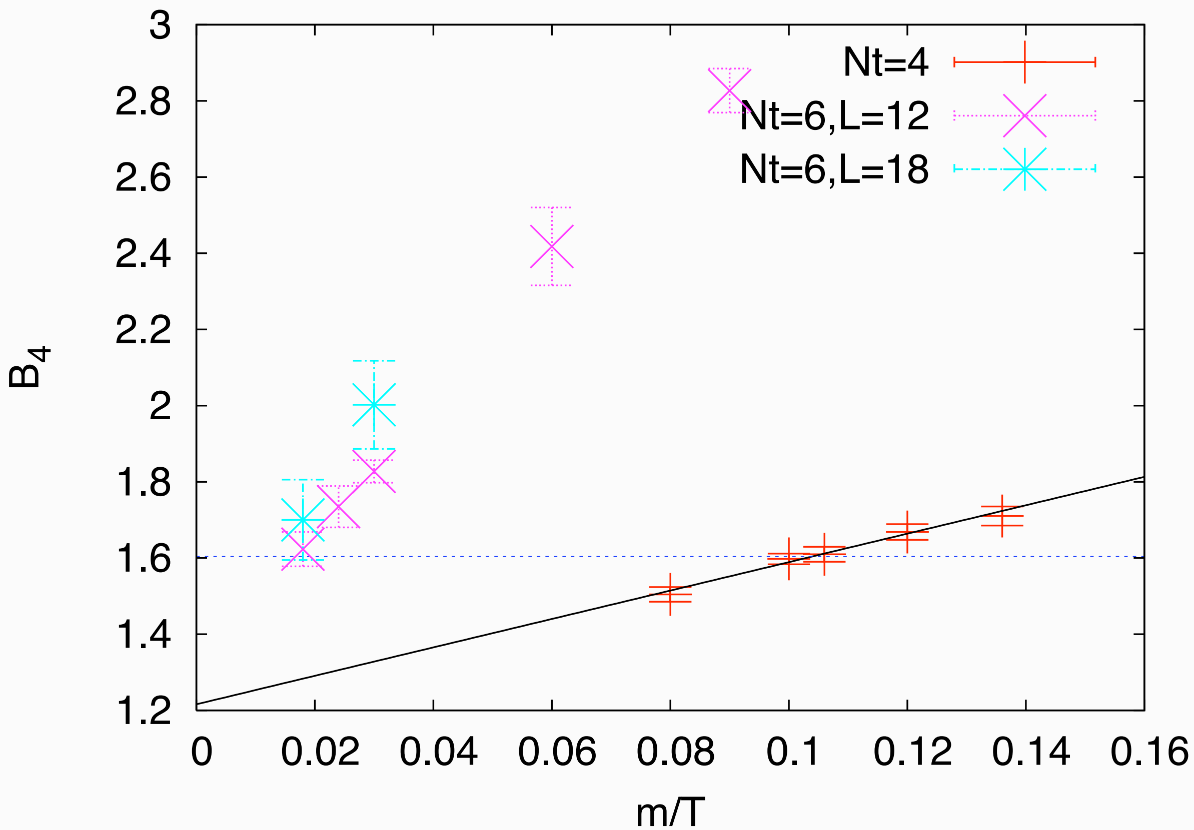




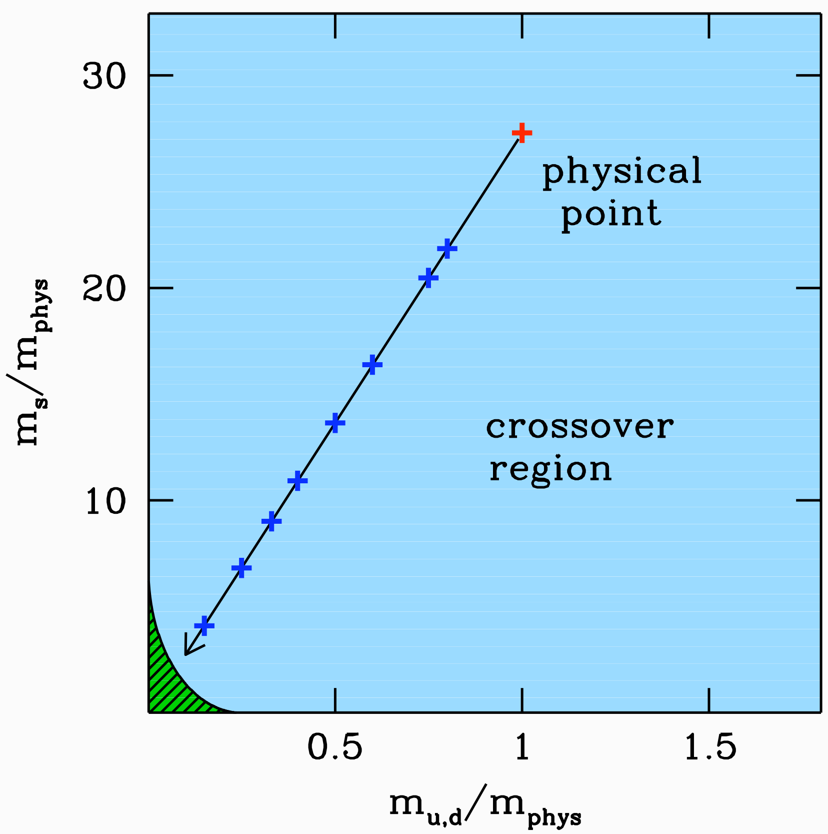
$16^3 \times 4, am_s = 0.25, am_{u,d} = 0.005, m_\pi L \sim 3$ lighter than in nature 700k traj.

$$\frac{m_c(\mu)}{m_c(0)} = 1 - 24(11)\left(\frac{\mu}{\pi T}\right)^2 - \ldots$$

Towards the continuum: $N_f = 3, N_t = 6, \mu = 0$



de Forcrand, Kim, O.P. (LAT07)

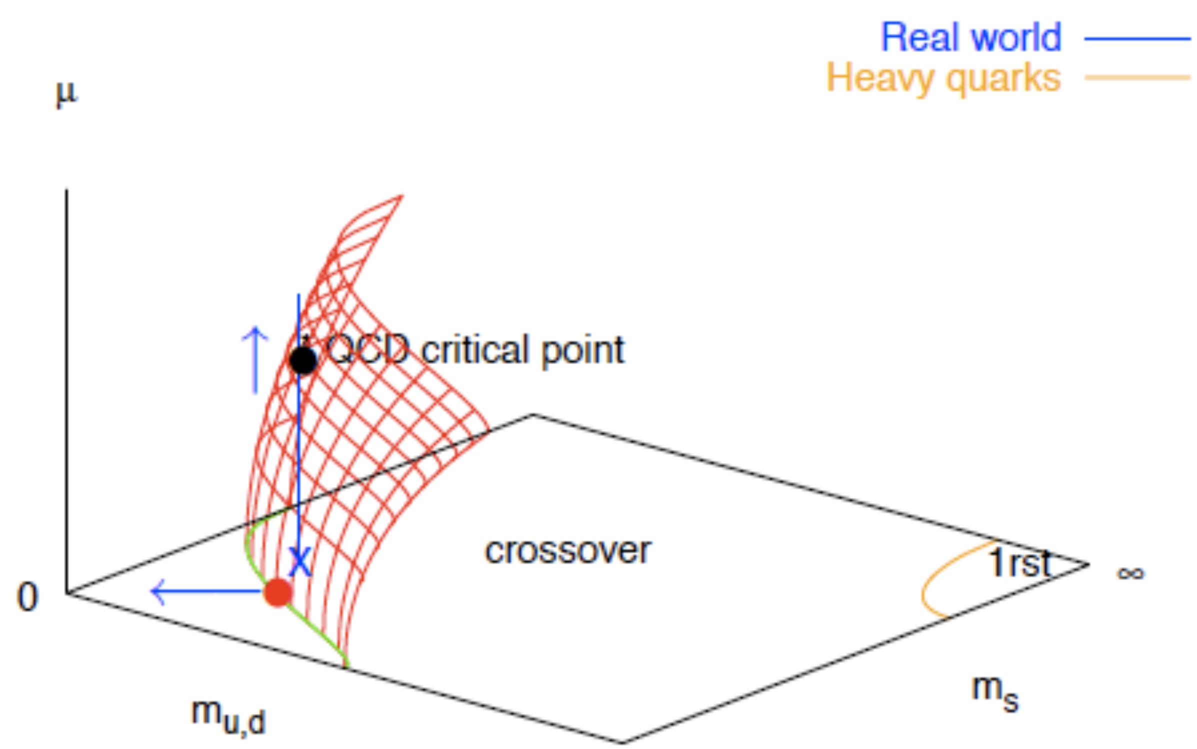


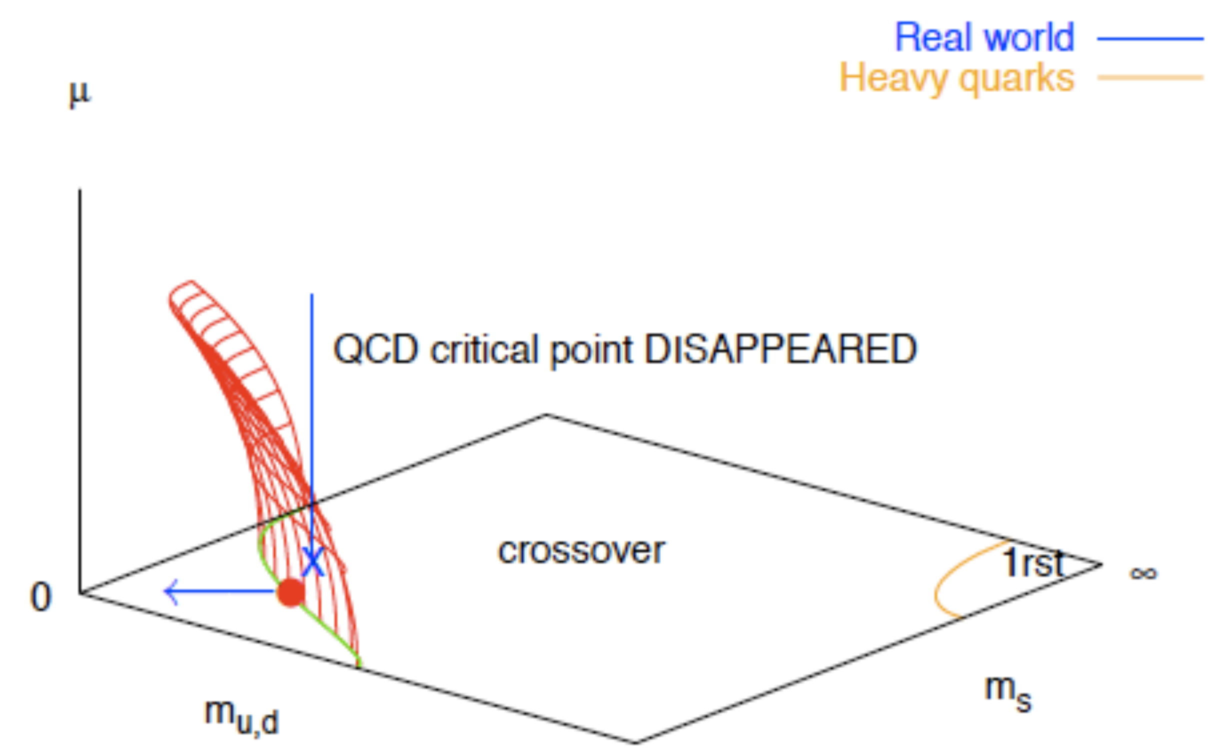
Endrodi et al. (LAT07)

$$\frac{m_\pi^c(N_t = 4)}{m_\pi^c(N_t = 6)} \approx 1.77 \approx \sqrt{3}$$

Distance between physical point and critical line grows as $a \longrightarrow 0$

Prospects for critical point at $\mu_B \lesssim 600 MeV$





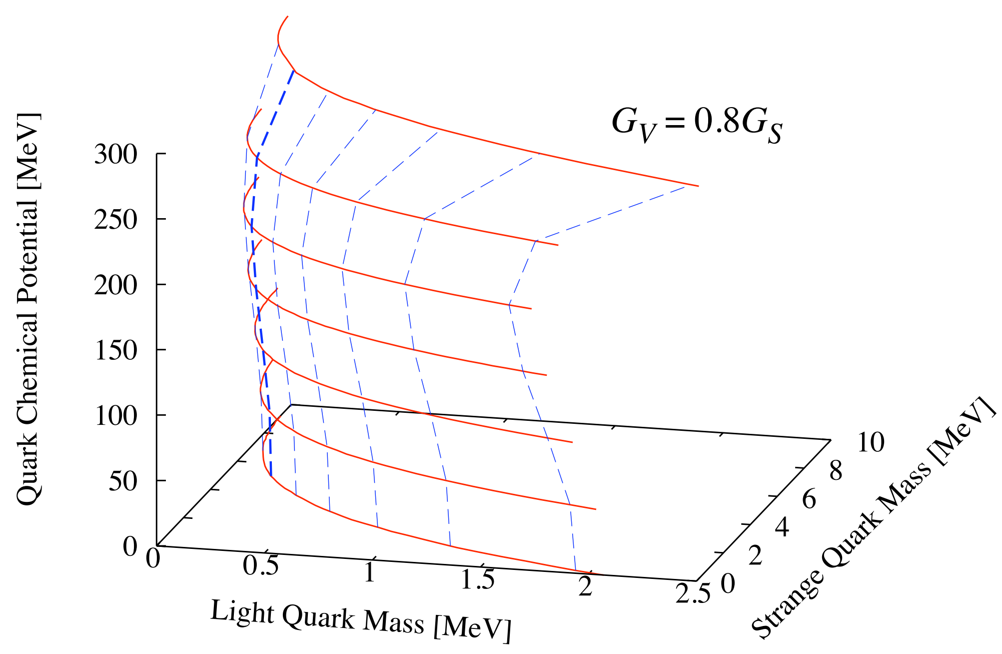
A critical point at "small" μ (ie. $\mu/T \lesssim 1$) would require curvature to

(i) change sign and (ii) become large

as $a \rightarrow 0$

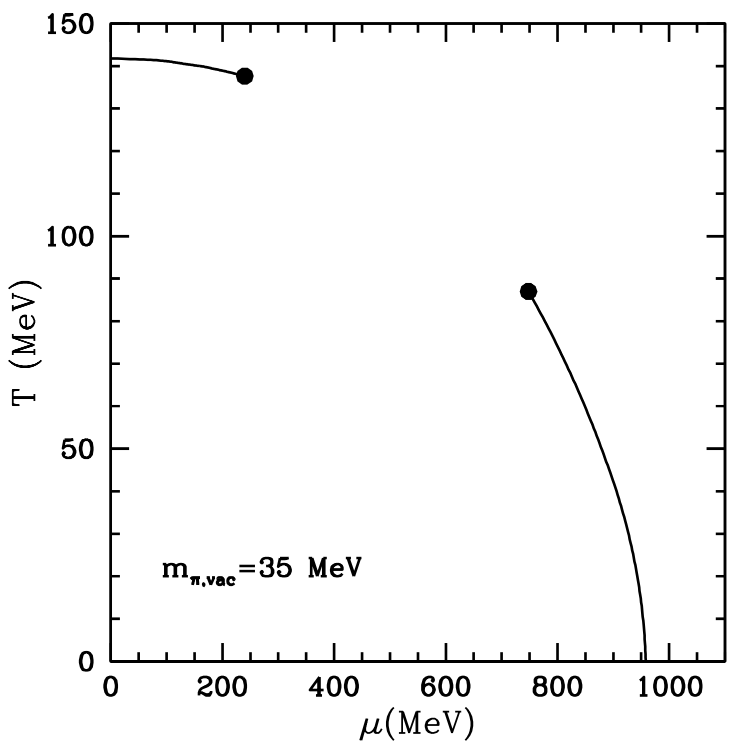
Recent model studies with similar results

K. Fukushima 08



NJL-Polyakov loop model with vector-vector interaction

Bowman, Kapusta 08



Linear sigma model with quarks

Qualitative behaviour as in exotic lattice scenario!

Conclusions

- Working lattice methods available for $\mu \lesssim T_c$
- $T_c(\mu)$, EoS, screening masses under control at finite density
- Critical endpoint extremely quark mass sensitive
- For $N_f = 3, N_t = 4$ no chiral critical point for $\mu \lesssim T_c$
- Beware of cut-off effects: a=0.2-0.3 fm, need finer lattices, other discretizations
- Picture not yet clear, but systematic errors will reduce!