Excited hadrons in 2-flavor LQCD

Georg Engel

Inst. f. Physik, FB Theoretische Physik
Universität Graz

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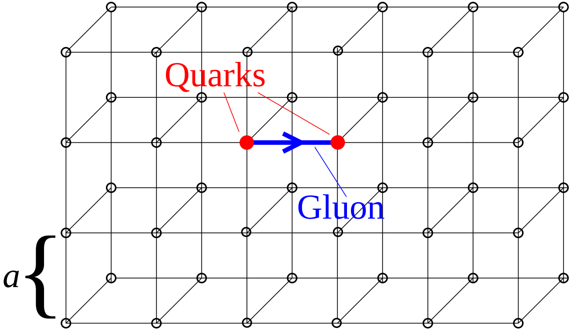
Collaborators:
C. Gattringer, M. Limmer, C. B. Lang,
D. Mohler, A. Schäfer



- Lattice: gauge-covariant regularization.
- Critical point in the continuum limit defines the renormalized QFT.
- Different discretizations agree in the continuum limit.
- Ab-initio calculation possible.
- Discretization effects can be reduced by improvement.

- Masses are extracted from hadron correlators:

  $\langle O(t)\bar{O}(0)\rangle = \sum_n A_n e^{-tE_n}$

- E.g. a meson correlator is given by:

$$\begin{aligned}\langle O(t)\bar{O}(0)\rangle &= \frac{1}{Z}\int \mathcal{D}[U]\mathcal{D}^{n_f}\left[\bar{\psi},\psi\right]e^{-S_G(U)}e^{-S_F(\psi,\bar{\psi},U)}O(t)\bar{O}(0)\\ &= \frac{1}{Z}\int \mathcal{D}[U]e^{-S_G(U)}[det(D)]^{n_f}\,tr\left[D^{-1}(0,t)\Gamma D^{-1}(t,0)\Gamma\right]\end{aligned}$$

- Hadron correlators are computed using Monte Carlo techniques: Importance sampling for the gauge action and the fermion determinant.

- Three limites performed a posteriori by numerical extrapolation:
  - $a \to 0$
  - $V \to \infty$
  - $m_\pi \to m_{\pi,phys}$ or $m_\pi \to 0$

- Chirally improved fermions (D_{CI}), $n_f = 2$ light quarks
- Fermion action includes a level of stout smearing.
- Lüscher-Weisz gauge action
- Hybrid Monte Carlo simulation

- Three ensembles for $16^3 \times 32$:

set	β_{LW}	m_0	configs	a[fm]	m_π[MeV]	m_{AWI}[MeV]
A	4.70	-0.050	100	0.151(2)	525(7)	43.0(4)
B	4.65	-0.060	200	0.150(1)	470(4)	35.1(2)
C	4.58	-0.077	200	0.144(1)	322(5)	15.0(4)

- Coarse lattices possible due to improved action.
- For details of the simulation: Gattringer et al., PRD79(2009)054501

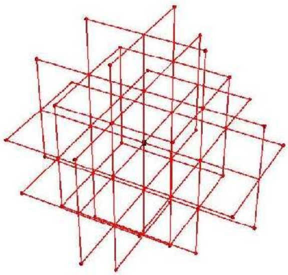
- General ansatz for bilinear fermion action ($\bar{\psi}_n D_{nm} \psi_m$):

$$D_{nm} = \sum_{\alpha=1}^{16} \Gamma_\alpha \sum_{p \in \mathcal{P}^{\alpha}_{m,n}} c^{\alpha}_{p} \prod_{l \in p} U_l \, \delta_{n,m+p}$$

(Gattringer, PRD63(2001)114501)

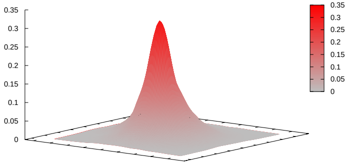
- Plug it in the Ginsparg-Wilson-equation, which describes chiral symmetry on the lattice:

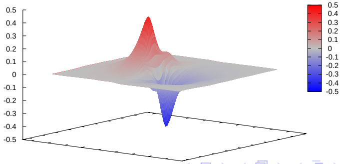
$$\boxed{\mathbf{D}\,\gamma_{\mathbf{5}} + \gamma_{\mathbf{5}}\,\mathbf{D} = \mathbf{a}\,\mathbf{D}\,\gamma_{\mathbf{5}}\,\mathbf{D}}$$



- Truncate the length of the contributions (to, e.g., 4) and compare the coefficients.

- Extended quark sources show better overlap with physical states.
- Allow for a larger basis in the variational method.
- We use 3 types:
  - Gaussian narrow (ψ_n): 0.27fm
  - Gaussian wide (ψ_w): 0.55fm
  - Derivative (ψ_{∂_i}): applied on wide source
- Gauge-covariant construction by Jacobi-smearing (using link-variables as gauge-transporter).
  Gusken et al, PLB227(1989)266; Best et al, PRD56(1997)2743; Gattringer et al, PRD78(2008)034501





- Interpolating fields are constructed according to the **quantum numbers of a physical state**.
- Realized by using particular combinations of smeared quarks and gamma-structures.
- Examples for Pion:

$$\overline{u}_n \,\gamma_5\, d_n ,\;\; \overline{u}_n \,\gamma_5\, d_w ,\;\; \overline{u}_w \,\gamma_5\, d_w ,$$
$$\overline{u}_n \,\gamma_t\gamma_5\, d_n ,\;\; \overline{u}_n \,\gamma_t\gamma_5\, d_w ,\;\; \overline{u}_w \,\gamma_t\gamma_5\, d_w ,$$
$$\overline{u}_{\partial_i} \,\gamma_i\gamma_5\, d_n ,\;\; \overline{u}_{\partial_i} \,\gamma_i\gamma_5\, d_w ,$$
$$\overline{u}_{\partial_i} \,\gamma_i\gamma_t\gamma_5\, d_n ,\;\; \overline{u}_{\partial_i} \,\gamma_i\gamma_t\gamma_5\, d_w ,$$
$$\overline{u}_{\partial_i} \,\gamma_5\, d_{\partial_i} ,\;\; \overline{u}_{\partial_i} \,\gamma_t\gamma_5\, d_{\partial_i}$$

- Examples for Nucleon:

$$N^{(i)} = \epsilon_{abc}\, \Gamma_1^{(i)}\, u_a \left(u_b^T\, \Gamma_2^{(i)}\, d_c - d_b^T\, \Gamma_2^{(i)}\, u_c \right)$$

with appropriate gamma-structures Γ_1 and Γ_2, and u/d being gaussian smeared quarks.

- **Variational Method**: In each channel, compute all cross-correlations, obtaining the matrix:

$$C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger|0\rangle \; = \sum_n \langle 0|O_i|n\rangle\langle n|O_j^\dagger|0\rangle$$

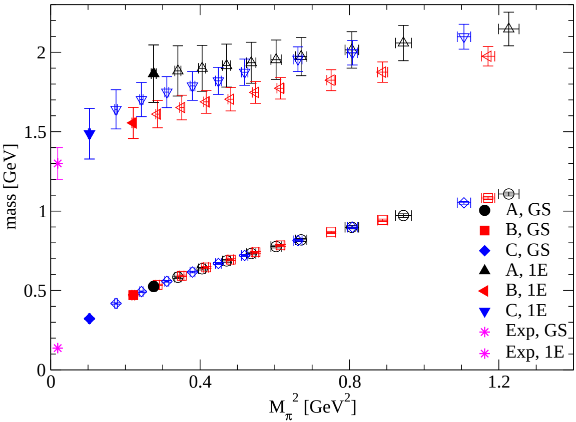
- Solve the generalized eigenvalue problem:

$$C(t)\,\vec{v}_k = \lambda_k(t)\,C(t_0)\,\vec{v}_k$$

yielding

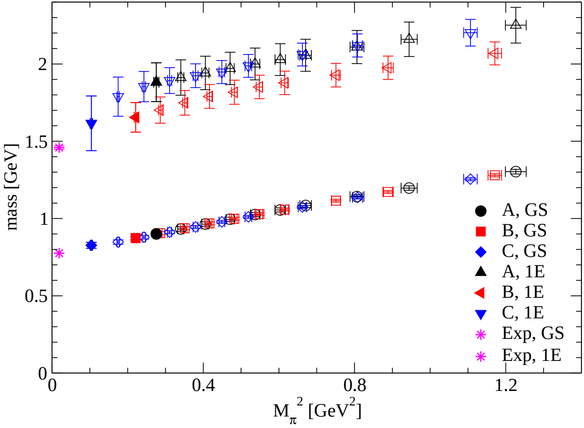
$$\lambda_k(t,t_0) \propto e^{-t\,m_k}\left(1 + \mathcal{O}(e^{-t\,\Delta m_k})\right)$$

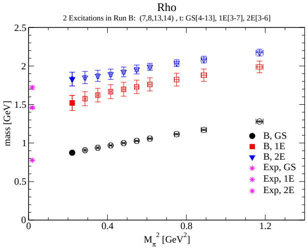
- Each **eigenvalue** is related only to a **single mass** at large time separations.
- Corresponding eigenvectors are “fingerprints” the states.
- Extraction of **excited states** possible.
- A good basis of different interpolators is crucial.
- Michael NPB259(1985)58; Lüscher/Wolff NPB339(1990)222

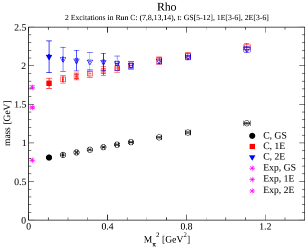


Ground state shown vs. itself. A signal of the first excited state seen. 3 ensembles shown, full (empty) symbols denote dynamical (partially quenched) results.

Results: Rho (1^{--})

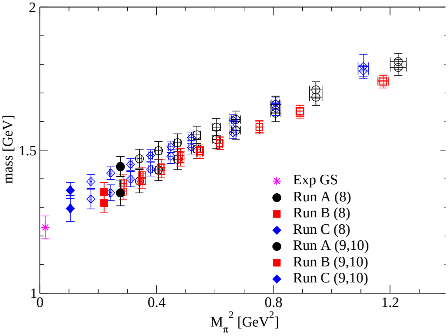




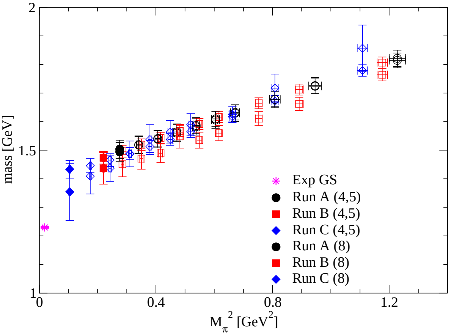


Ground state with very small error bar. First excited state signal found.

2nd excitation $\rho(1720)$ signal is seen in ensemble B, but bad signal in others.

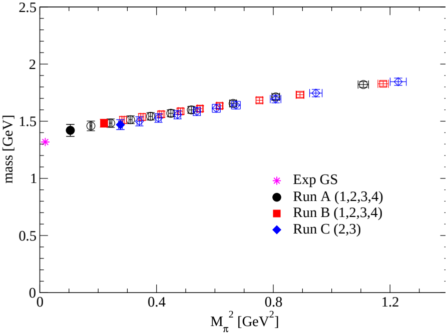


(a) a_1 ($\mathbf{1}^{++}$)

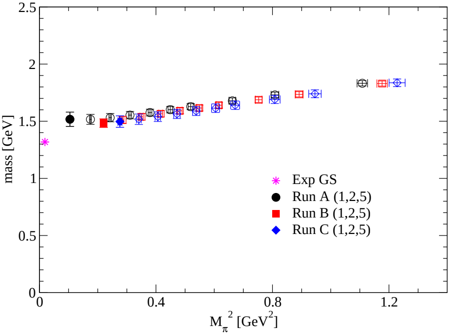


(b) b_1 ($\mathbf{1}^{+-}$)

In both cases systematic uncertainty of choosing the interpolators is expected to shrink when using higher statistics. Derivative interpolators are found crucial for a good signal.



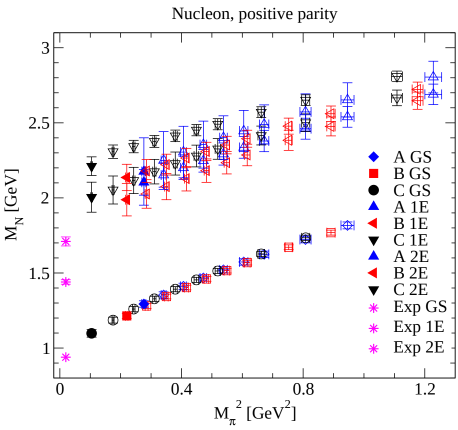
(a) Representation $\mathbf{T}_2$

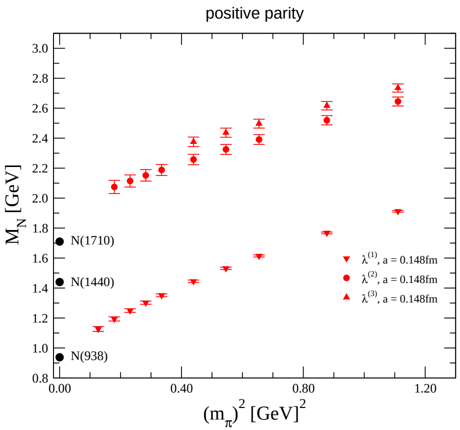


(b) Representation **E**

Reasonable agreement between different lattice spin representations found. Derivative interpolators and variational method are both found crucial for a good signal.

Results: Nucleon pos. parity **1/2(1/2$^+$)**

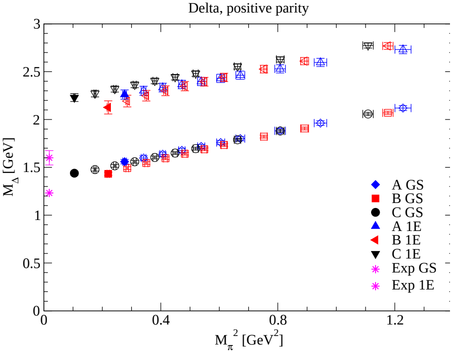


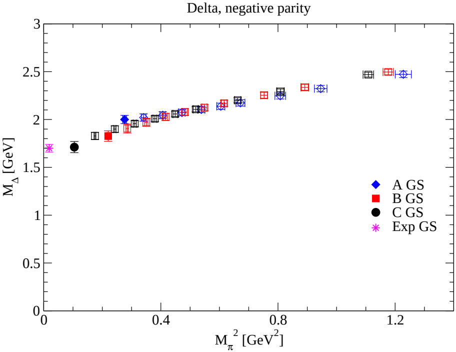


Two excitations, but too high up.

Roper: For confirmation probably larger volume and smaller quark masses needed.

Quenched results (Burch et al., PRD 74 (2006) 014504)

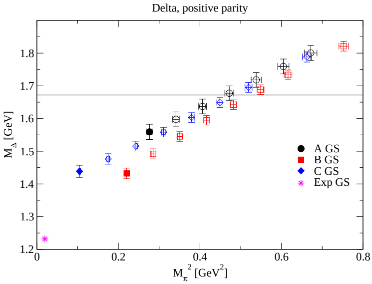




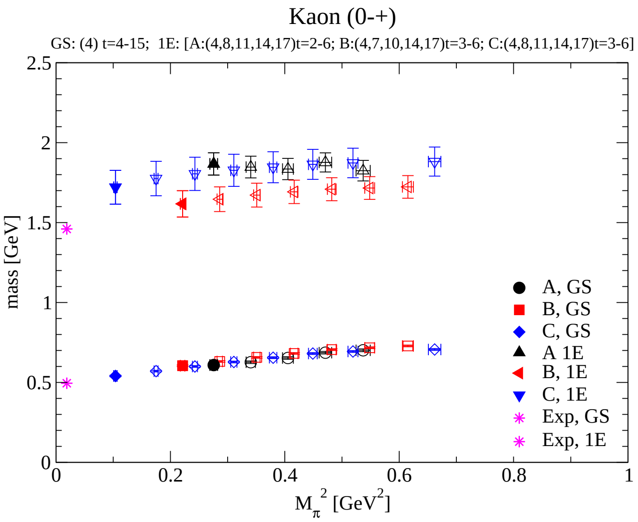
Ground and first excited state clearly seen, but too high, maybe due to finite volume effects.

Ground state comes out nicely.

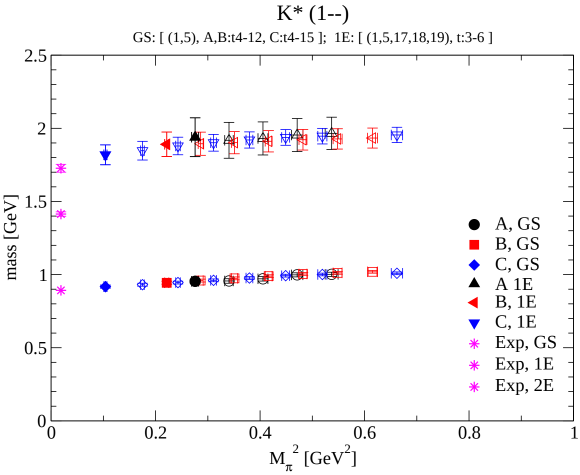
- Two mass degenerate light sea quarks
- $\rightarrow$ strange hadrons calculated using partially quenched quarks
- Valence quark mass parameter fixed to strange quark mass by identifying a **partially quenched** Delta with **Omega** (1672).



- Interpolation between existing partially quenched data or recalculation at strange quark mass.
- Possible partially quenched effects in the results.



Accurate ground state, a weak signal of the first excited state seen.



Accurate ground state, first excited state may be a hybrid (L. Burakovsky et al PRD57(1998)2879), second excitation seen.

- Lattice QCD is used for ab-initio calculation of hadron masses.
- Though the coarse lattice, ground states of several hadrons are predicted with high accuracy.
- Excited states of pion, rho, nucleon, delta, kaon and K^* found.
- Agreement between different lattice spin representations in the a_2 ($\mathbf{2}^{++}$) channel verified.
- Further results of strange hadrons will be published soon.
- In future we plan to calculate additional phase points and to perform the chiral extrapolation to the physical pion mass.

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