Anomalies and Transport

Karl Landsteiner
Instituto de Física Teórica, UAM-CSIC

in collaboration with
I. Amado, E. Megías, L. Melgar, F. Peña-Benitez,
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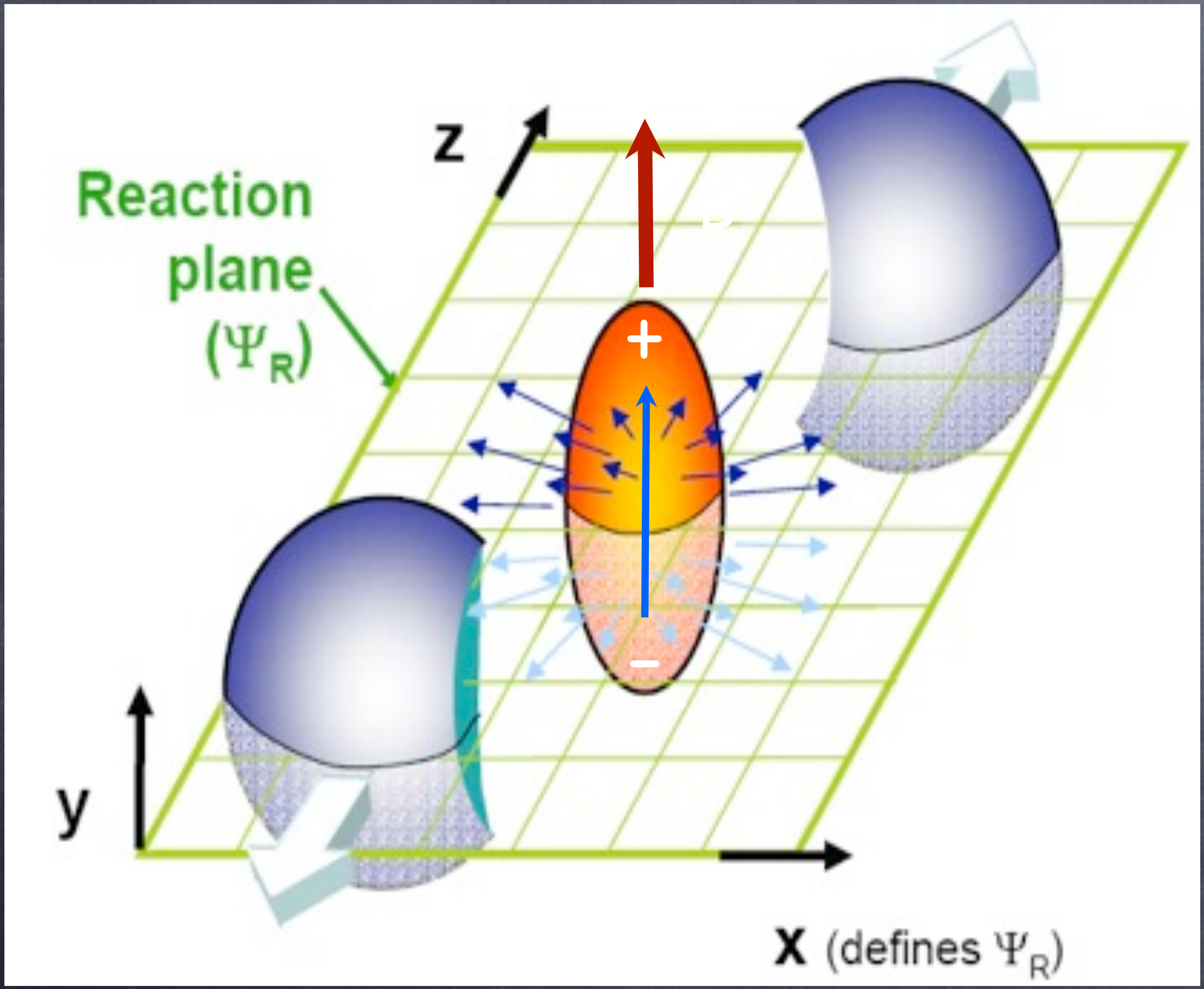
Outline

- The Chiral Magnetic Effect
- Kubo formulas I
- Kubo formulas II
- Hydrodynamics
- Strong coupling (Holography)
- Observable gravitational anomaly?
- Summary

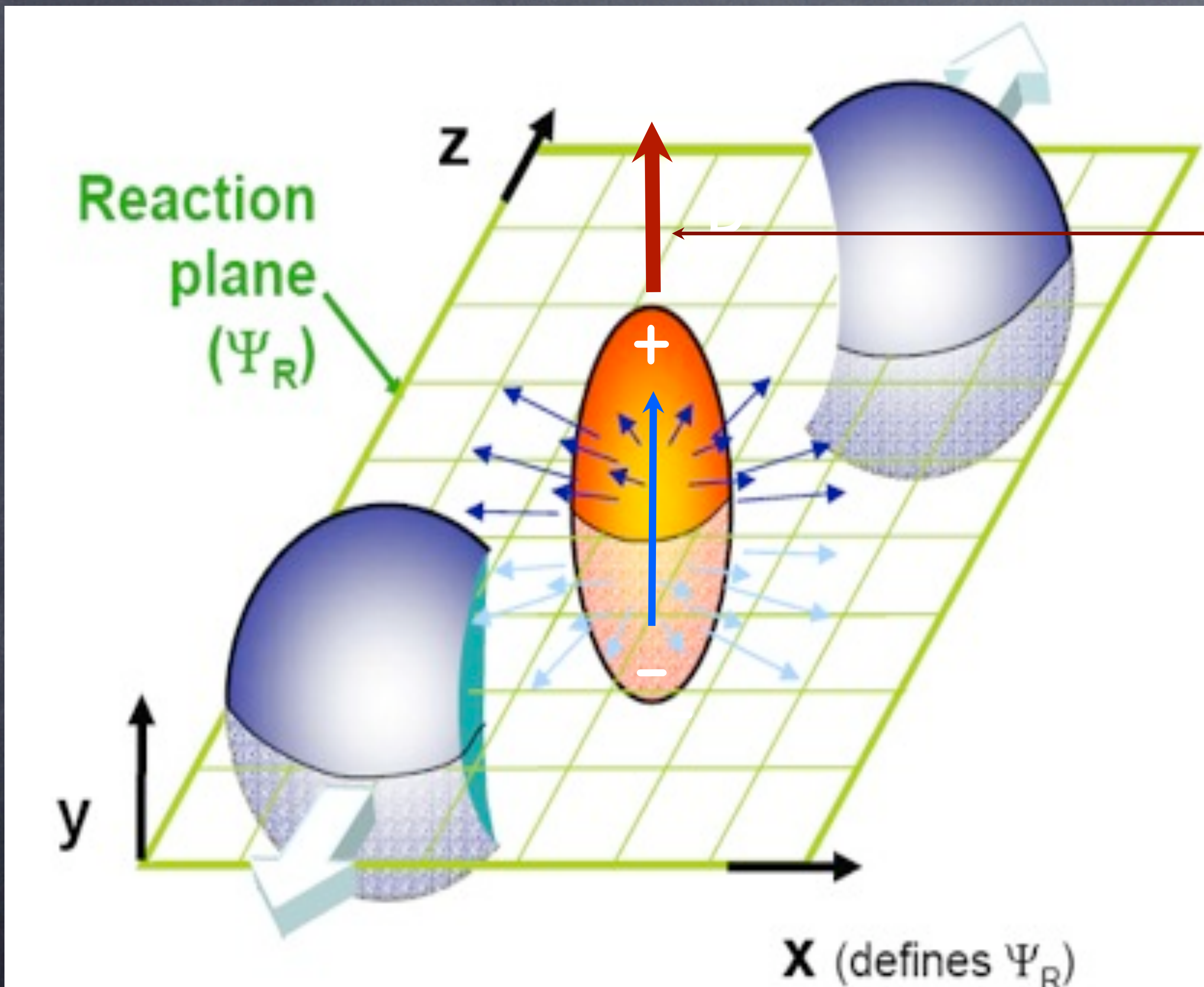
The Chiral Magnetic Effect



[Kharzeev, McLarren, Warringa]

[parity violating currents: Vilenkin '80, Giovannini, Shaposhnikov '98, Alekseev, Chaianov, Frohlich '98]

The Chiral Magnetic Effect



[Kharzeev, McLarren, Warringa]

Magnetic Field

[parity violating currents: Vilenkin ’80, Giovannini, Shaposhnikov ’98, Alekseev, Chaianov, Frohlich ’98]

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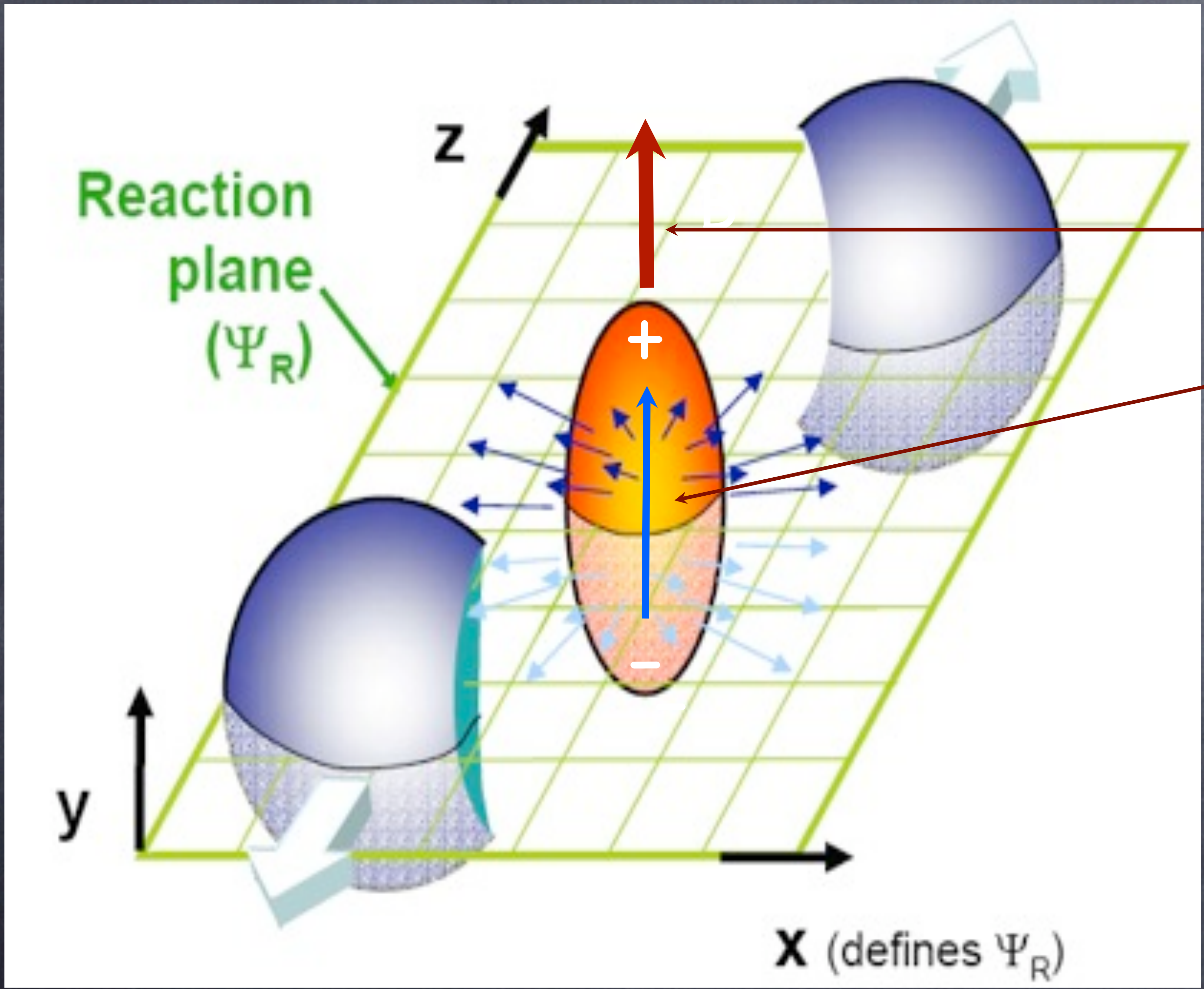


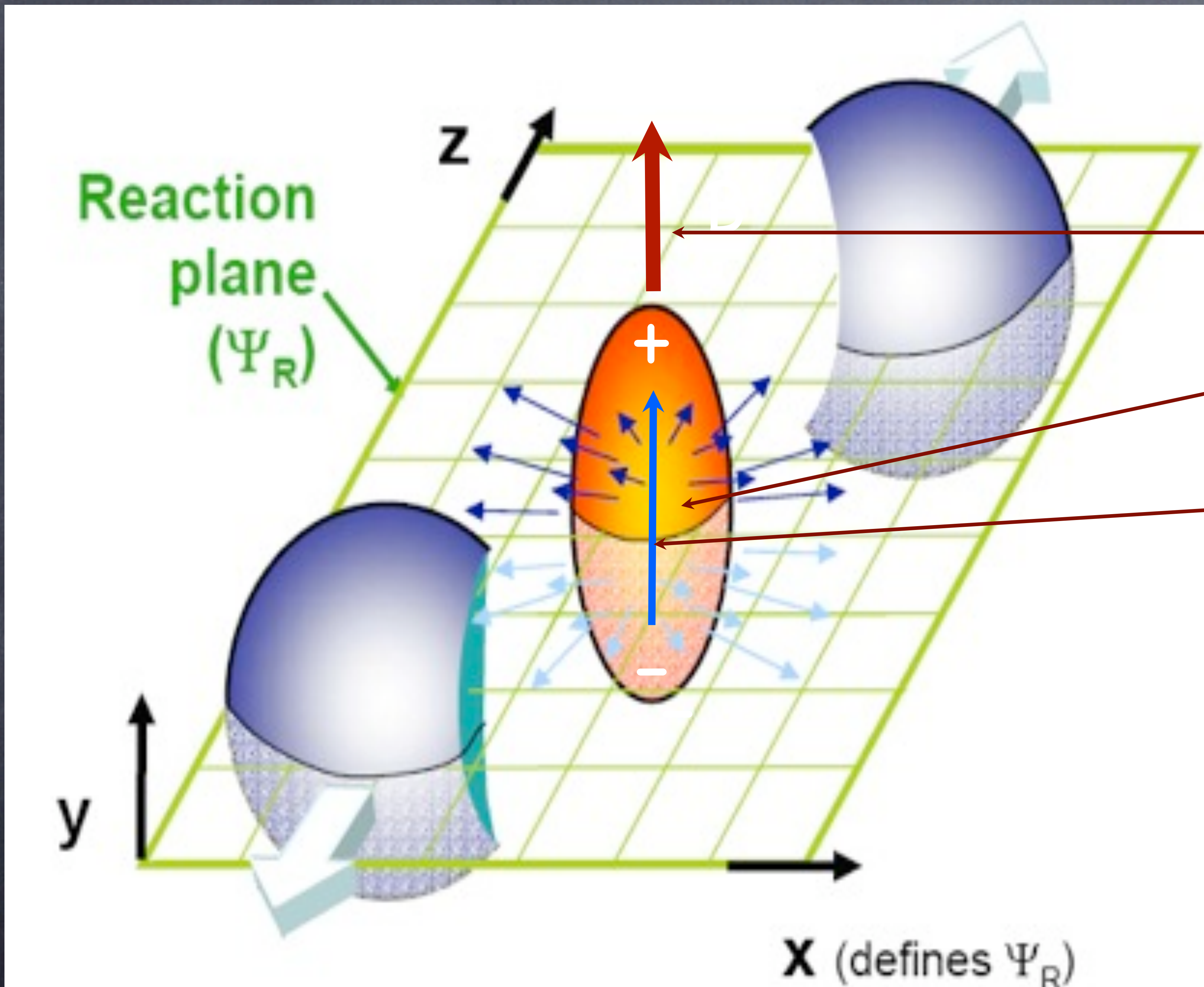
[Kharzeev, McLarren, Warringa]

Magnetic Field

Net chirality

[parity violating currents: Vilenkin '80, Giovannini, Shaposhnikov '98, Alekseev, Chaianov, Frohlich '98]

The Chiral Magnetic Effect



[Kharzeev, McLarren, Warringa]

Magnetic Field

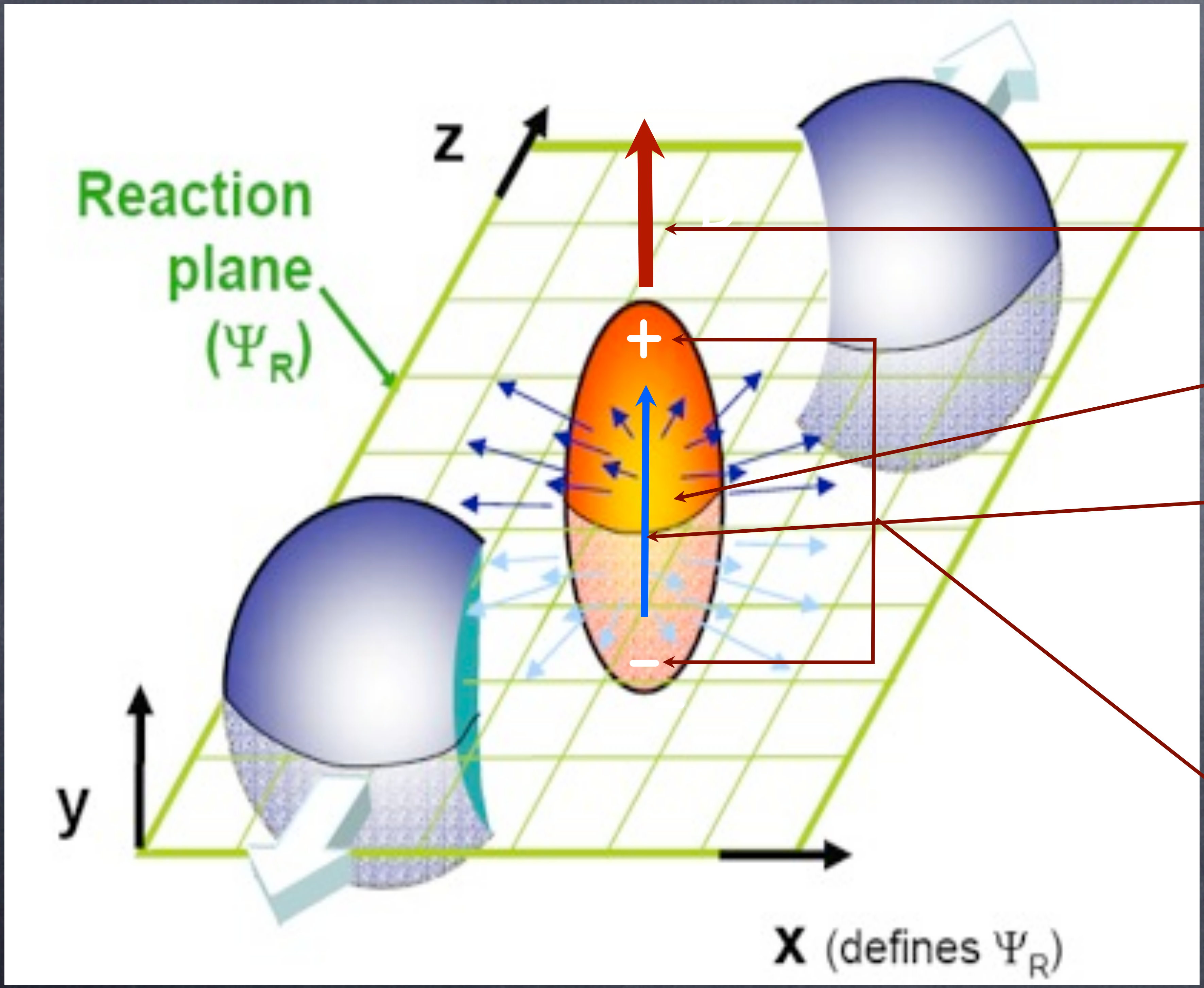
Net chirality

Electric current

$$J^i = \frac{\mu_5}{2\pi^2} B^i$$

[parity violating currents: Vilenkin '80, Giovannini, Shaposhnikov '98, Alekseev, Chaianov, Frohlich '98]

The Chiral Magnetic Effect



[Kharzeev, McLarren, Warringa]

Magnetic Field

Net chirality

Electric current

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P-odd charge separation

[parity violating currents: Vilenkin '80, Giovannini, Shaposhnikov '98, Alekseev, Chaianov, Frohlich '98]

Kubo formulas I

- Chiral magnetic conductivity

$$\vec{J} = \sigma \vec{B}$$

[Kharzeev, Warringa]

$$J_i = \sigma \epsilon_{ijk}(ip_j)A_k$$

- Kubo formula, general symmetry group

$$[T^A, T^B] = i f_C^{AB} T^C$$

$$\sigma^{AB} = \lim_{p_j \to 0} \sum_{i,k} \frac{i}{2p_j} \epsilon_{ijk} \langle J_i^A J_k^B \rangle \Big|_{\omega=0}$$

Kubo formulas I

- chiral fermions

$$J_i^A = \sum_{f,g=1}^{N} (T^A)^g{}_f \bar{\Psi}_g \gamma_i P_+ \Psi^f$$

- chemical potentials and Cartan generators

$$H_A = q_A^f \delta^f{}_g \qquad\qquad \mu^f = \sum_A q_A^f \mu_A$$

- 1-loop calculation

$$\sigma_{AB} = \frac{1}{8\pi^2} \sum_C \mathrm{tr}\left(T^A \{T^B, H^C\}\right) \mu_C = \frac{1}{4\pi^2} d^{ABC} \mu_C$$

[Newman, Son], [Newman]

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Anomalycoeff

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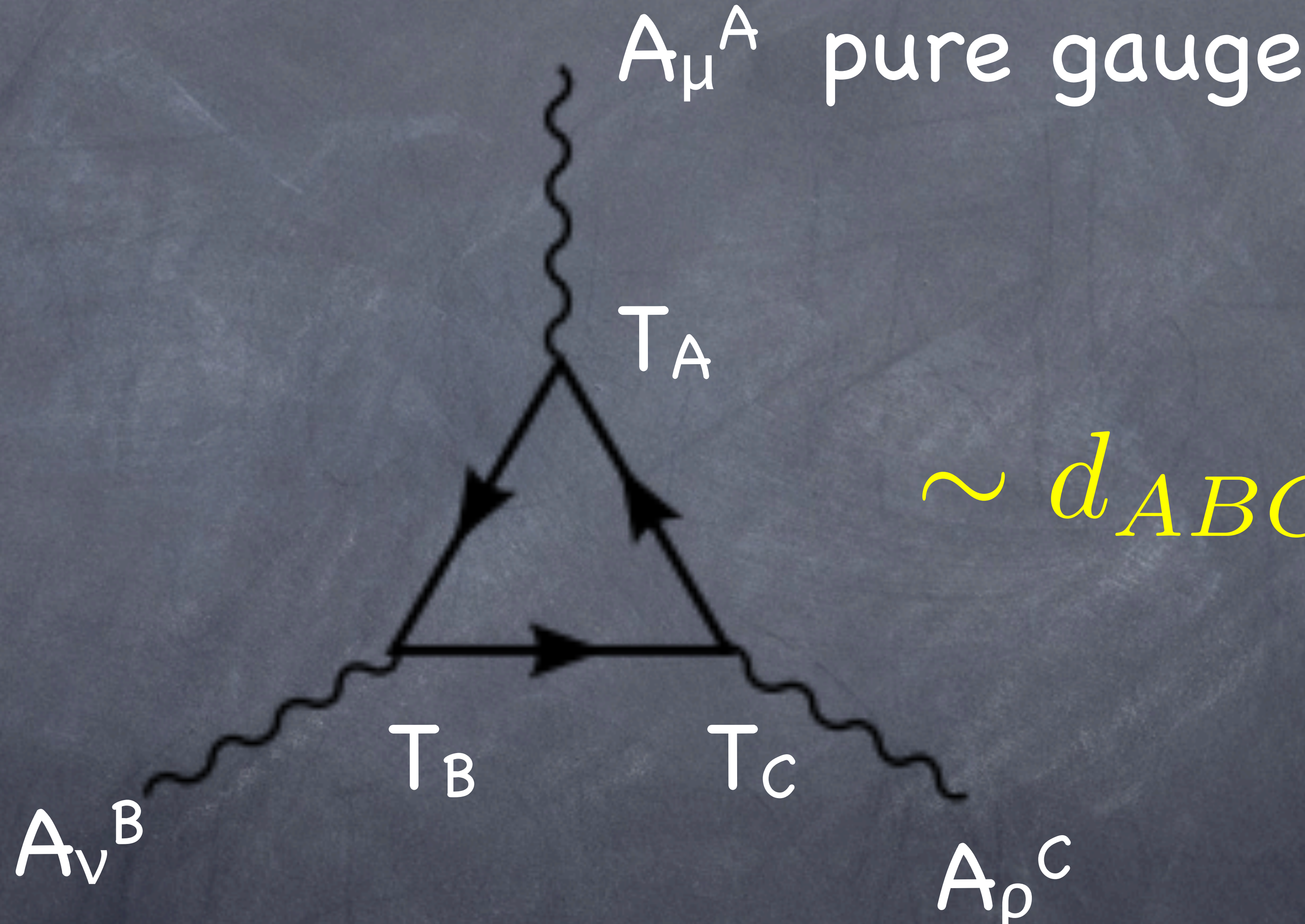
Kubo formulas I



$\sim d_{ABC}$

Kubo formulas II

- finite density: charge transport => energy transport

$$\delta T_{0i} = \mu \delta J_i = \mu \delta\sigma B_i$$

- energy flux sourced by magnetic fields

$$\frac{i}{2p_j} \sum_{i,k} \epsilon_{ijk} \left.\langle T_{0i} J_k \rangle\right|_{\omega=0} = \int \mu d\sigma + \text{const.}$$

- at ω=0 reverse order of operators

$$\sigma_V = \frac{i}{2p_j} \sum_{i,k} \epsilon_{ijk} \left.\langle J_i T_{0k} \rangle\right|_{\omega=0} = \int \mu d\sigma + \text{const.}$$

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conductivity ?

Kubo formulas II

- $T_{\mu\nu}$ sourced by metric

$$ds^2 = -(1-2\Phi)dt^2 + 2\vec{A}_g dt d\vec{x} + (1+2\Phi)d\vec{x}^2$$

- A_g "gravitomagnetic field" -> chiral gravitomagnetic effect

$$\vec{J} = \sigma_V \vec{B}_g$$

- chiral vortical effect: fluid velocities

$$u^\mu = (1,0,0,0) \qquad u_\mu = (-1, \vec{A}_g)$$

$$J^i = \sigma_V \epsilon^{ijk} \partial_j u_k$$

Kubo formulas II

- as before: general symmetry group

$$T^{0i} = \frac{i}{2}\sum_{f=1}^{N} \bar{\Psi}_f(\gamma^0\partial^i + \gamma^i\partial^0)P_+\Psi^f$$

$$\sigma_V^A = \frac{1}{8\pi^2}\sum_{f=1}^{N}(T^A)^f{}_f\left[(\mu^f)^2 + \frac{\pi^2}{3}T^2\right]$$

$$= \frac{1}{8\pi^2}\sum_{B,C} d^{ABC}\mu_B\,\mu_C + \frac{T^2}{24}\mathrm{tr}\left(T^A\right)$$

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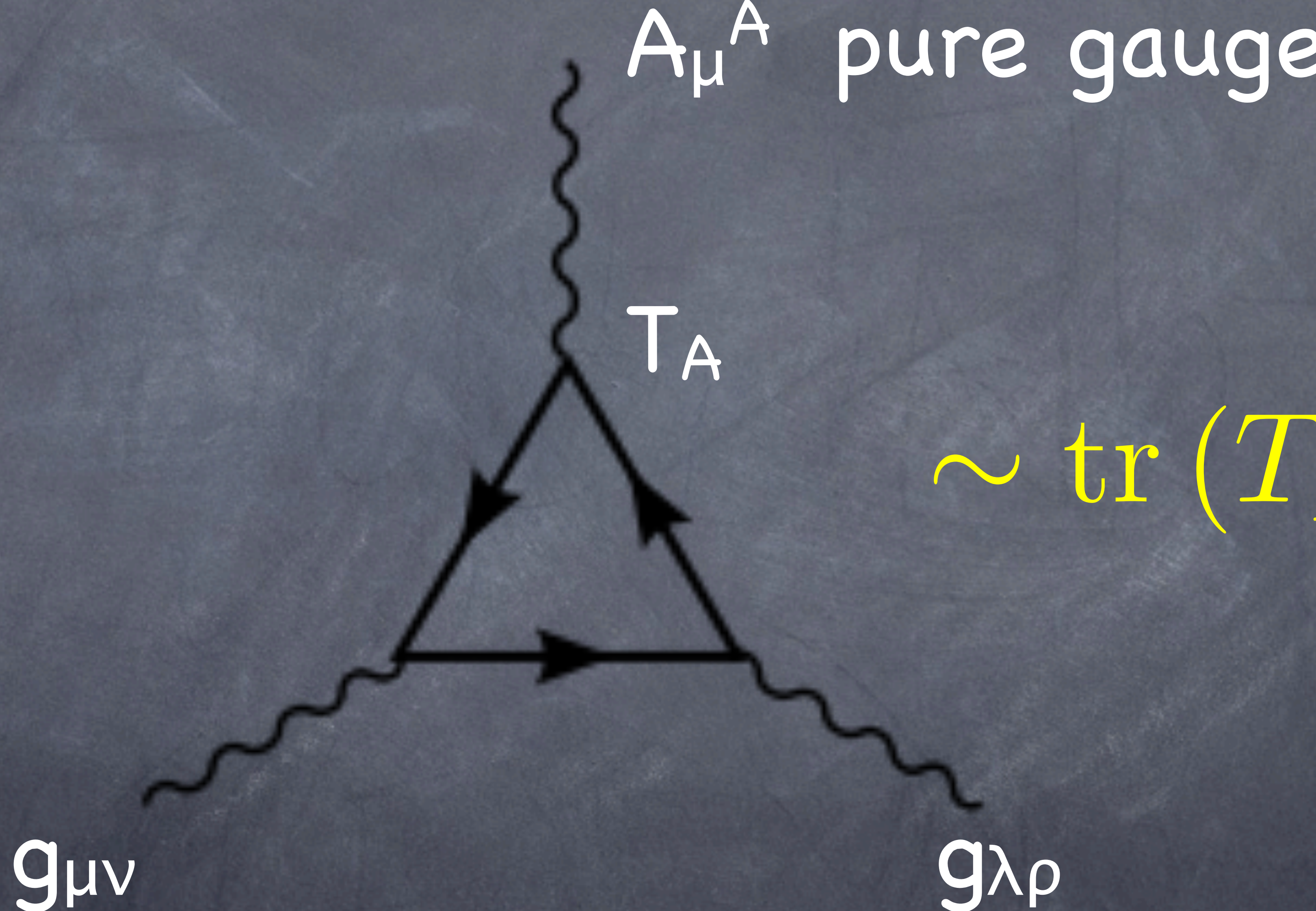
Integration constant

gravitational anomaly!

Kubo formulas II



$$\sim \mathrm{tr}\,(T_A)$$

Anomalies

$$\nabla_\mu J^\mu_A = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{ABC}}{32\pi^2}F^B_{\mu\nu}F^C_{\rho\lambda} + \frac{b_A}{768\pi^2}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right)$$

$$d_{ABC} = \frac{1}{2}\mathrm{tr}\left(\{T_A, T_B\}T_C\right)_R - \frac{1}{2}\mathrm{tr}\left(\{T_A, T_B\}T_C\right)_L$$

$$b_A = \mathrm{tr}\left(T_A\right)_R - \mathrm{tr}\left(T_A\right)_L$$

Hydrodynamics

- also energy flux $\lim_{p_j \to 0} \frac{i}{2p_j} \sum_{i,k} \epsilon_{ijk} \left. \langle T_{0i} T_{0k} \rangle \right|_{\omega=0} \neq 0$

- hydrodynamics:

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \eta \sigma^{\mu\nu} + \zeta P^{\mu\nu} \partial_\lambda u^\lambda + Q^\mu u^\nu + Q^\nu u^\mu$$

$$J^\mu = n u^\mu + \sigma P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) u^\lambda + \nu^\mu$$

[Son,Surowka], [Eling, Neiman, Oz], [Erdmenger, Haack, Kaminski, Yarom], [Banerjee, Bhattacharya, Bahattacharya, Dutta Loganayagam, Surowka], [Loganayagam] [Kharzeev, Yee] [Sadovyev, Isachenkov, Zakharov]

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$$T^{\mu\nu}=(\epsilon+P)u^\mu u^\nu+Pg^{\mu\nu}+\eta\sigma^{\mu\nu}+\zeta P^{\mu\nu}\partial_\lambda u^\lambda+Q^\mu u^\nu+Q^\nu u^\mu$$

$$J^\mu=nu^\mu+\sigma P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)u^\lambda+\nu^\mu$$

anomalous contributions

[Son,Surowka], [Eling, Neiman, Oz], [Erdmenger, Haack, Kaminski, Yarom], [Banerjee, Bhattacharya, Bahattacharya, Dutta Loganayagam, Surowka], [Loganayagam] [Kharzeev, Yee] [Sadovyev, Isachenkov, Zakharov]

Hydrodynamics

$$\nu^\mu_A = \sigma^{\mathcal{B}}_{AB}\mathcal{B}^{B,\mu} + \sigma^{\mathcal{V}}_A\omega^\mu$$

$$Q^\mu = \rho^{\mathcal{B}}_A\mathcal{B}^{A,\mu} + \rho^{\mathcal{V}}_A\omega^\mu$$

$$\mathcal{B}^{A,\mu} = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}u_\nu\mathcal{F}^A_{\rho\lambda}$$

$$\omega^\mu = \epsilon^{\mu\nu\rho\lambda}u_\nu\partial_\rho u_\lambda$$

$$\sigma^{\mathcal{B}}_{AB} = \frac{1}{4\pi^2}d_{ABC}\mu^C$$

$$\sigma^{\mathcal{V}}_A = \rho^{\mathcal{B}}_A = \frac{1}{8\pi^2}d_{ABC}\mu^B\mu^C + \frac{T^2}{24}b_A$$

$$\rho^{\mathcal{V}} = \frac{1}{12\pi^2}d_{ABC}\mu^A\mu^B\mu^C + \frac{T^2}{12}b_A\mu^A$$

Holography

[H.U. Yee], [Gynther, KL,Pena-Benitez, Rebhan]
[Kalayhdzian, Kirsch], [Gorski, Zayakin]

- mixed gauge gravitational Chern Simons term

$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN} + \epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)\right]$$

- current

$$16\pi G J^\mu = \frac{\sqrt{-g}}{\sqrt{-g_0}}F^{r\mu}$$

- on-shell we recover the anomaly

$$D_\mu J^\mu = -\frac{1}{16\pi G}\epsilon^{\mu\nu\rho\lambda}\left(\kappa F_{\mu\nu}F_{\rho\lambda} + \lambda R^\alpha_{(4)\ \beta\mu\nu}R^\beta_{(4)\ \alpha\rho\lambda}\right)$$

Holography

- Kubo formulas: fluctuations

$$a_x(z), a_y(z), h^t_x(z), h^t_y(z), h^z_x(z), h^z_y(z)$$

- background: charged AdS black hole

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f(r)}dr^2 \qquad A_{(0)} = (\beta - \frac{\mu\, r_{\mathrm{H}}^2}{r^2})$$

- correlators are

$$\langle JJ\rangle = -ip_z\left(\frac{\kappa}{2\pi G}\mu - \frac{\kappa}{6\pi G}\beta\right)$$

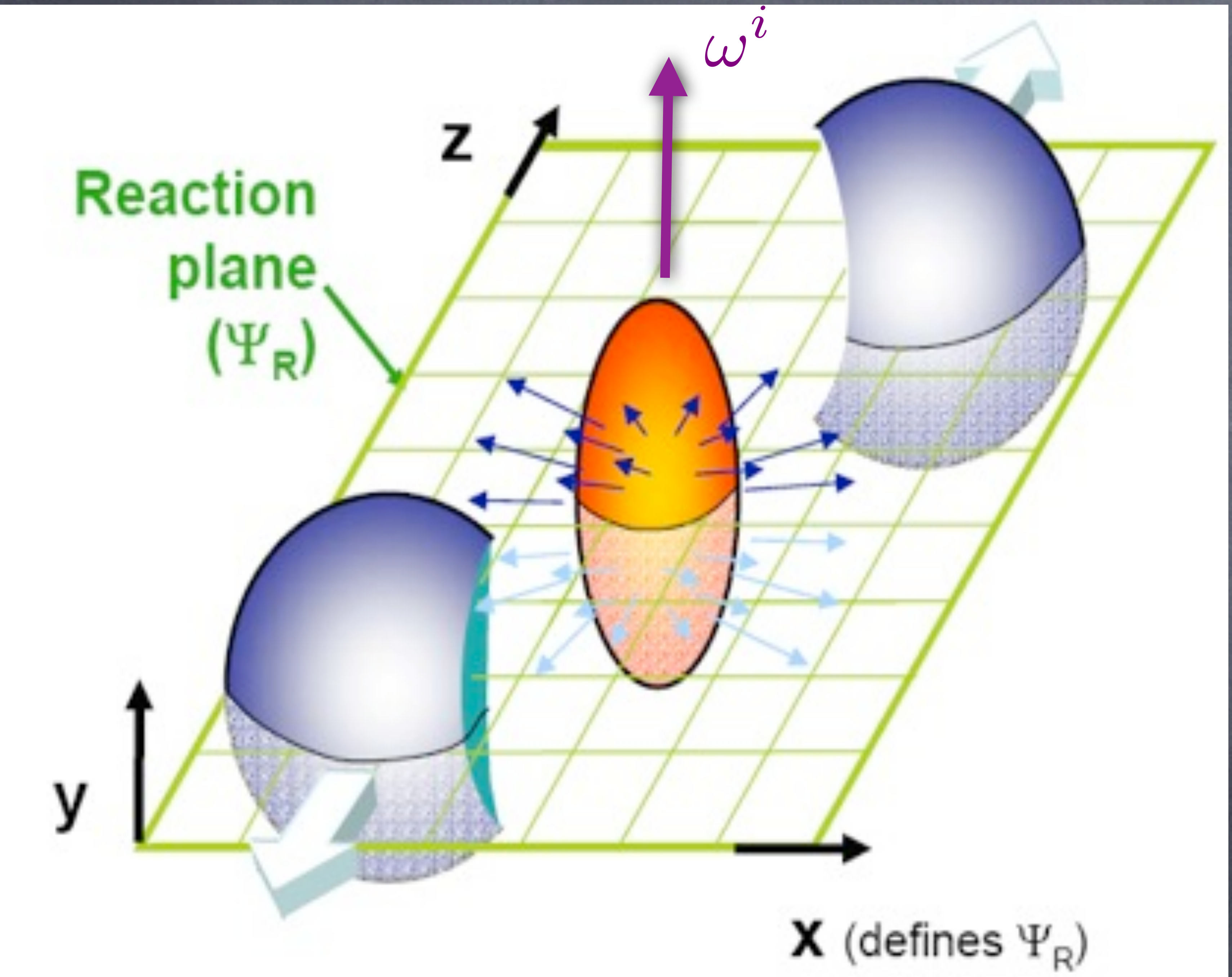
$$\langle JT\rangle = -ip_z\left(\frac{\kappa}{4\pi G}\mu^2 + \frac{2\lambda\pi}{G}T^2\right)$$

$$\langle TT\rangle = -ip_z\left(\frac{\kappa}{6\pi G}\mu^3 + \frac{4\lambda\pi}{G}\mu T^2\right)$$

coeffs consistent
with weak coupling

no T^3 terms !

Observe grav. Anomaly?



$$J_5^i = \left(\frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{6}\right)\omega^i$$

Enhanced Ω_ Production

[Karen-Zur, Oz]

Better Observable? Energy!

$$T^{0i} = \frac{\mu_5}{6} T^2 \omega^i$$

$E_{up} > E_{down}$

measurable effect?

Summary

- Anomalies -> parity violating transport
- Magnetic fields or vortices
- We have derived Kubo formulas
- (non)-renormalizaton
- Surprise: mixed gauge gravitational anomaly contributes
- Hydrodynamics and derivative expansion? (fluid/gravity)
  [Oz, Neiman], [Son], [Kharzeev, Yee], [Loganayagam]
- Holography with gravitational CS term
- Observable effects? [Karen-Zur, Oz], [Kharzeev, Son], [Kharzeev, Yee]