By means of Morse theory we prove the existence of a nontrivial solution to a superlinear $p$-harmonic elliptic problem with Navier boundary conditions having a linking structure around the origin. Moreover, in case of both resonance near zero and nonresonance at $+\infty$ the existence of two nontrivial solutions is shown.

1. Introduction and main results

Let $p > 1$ and $\Omega \subset \mathbb{R}^n$ be a smooth bounded domain with $n \geq 2p + 1$. We are concerned with the existence of nontrivial solutions to the $p$-harmonic equation

$$\Delta(|\Delta u|^{p-2}\Delta u) = g(x,u) \text{ in } \Omega$$

with Navier boundary conditions

$$u = \Delta u = 0 \text{ on } \partial \Omega,$$

where $g : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function such that for some $C > 0$,

$$|g(x,s)| \leq C(1 + |s|^{q-1})$$

for a.e. $x \in \Omega$ and all $s \in \mathbb{R}$, being $1 \leq q < p_*$ and $p_* = np/(n-2p)$.

It is well known that the functional $\Phi : W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \to \mathbb{R}$

$$\Phi(u) = \frac{1}{p} \int_{\Omega} |\Delta u|^p \, dx - \int_{\Omega} G(x,u) \, dx,$$

with $G(x,s) = \int_0^s g(x,t) \, dt$, is of class $C^1$ and

$$\langle \Phi'(u), \varphi \rangle = \int_{\Omega} |\Delta u|^{p-2}\Delta u \Delta \varphi \, dx - \int_{\Omega} g(x,u) \varphi \, dx$$
for each \( \varphi \in W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) \). Moreover, the critical points of \( \Phi \) are weak solutions for (1.1). Notice that for the eigenvalue problem
\[
\Delta(|\Delta u|^{p-2}\Delta u) = \lambda |u|^{p-2}u \quad \text{in } \Omega
\]
(1.6)
with boundary data (1.2), as for the \( p \)-Laplacian eigenvalue problem with Dirichlet boundary data,
\[
\lambda_n = \inf_{A \in \Gamma_n} \sup_{u \in A} \int_\Omega |\Delta u|^p \, dx, \quad n = 1, 2, \ldots
\]
(1.7)
is the sequence of eigenvalues, where
\[
\Gamma_n = \{ A \subseteq W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) \setminus \{0\} : A = -A, \, \gamma(A) \geq n \},
\]
(1.8)
being \( \gamma(A) \) the Krasnoselski’s genus of the set \( A \). This follows by the Ljusternik-Schnirelman theory for \( C^1 \)-manifolds proved in [13] applied to the functional
\[
J(u) = \int_\Omega |\Delta u|^p \, dx,
\]
(1.9)
\[\mathcal{M} = \left\{ u \in W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) : \int_\Omega |u|^p \, dx = 1 \right\},\]
since \( \mathcal{M} \) is a \( C^1 \)-manifold with tangent space
\[
T_u \mathcal{M} = \left\{ w \in W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) : \int_\Omega |u|^{p-2}uw \, dx = 0 \right\}.
\]
(1.10)
The next remark is the starting point of our paper.

**Remark 1.1.** It has been recently proved by Drábek and Otani [4] that (1.6) with boundary data (1.2) has the least eigenvalue
\[
\lambda_1(p) = \inf \left\{ \int_\Omega |\Delta u|^p \, dx : u \in W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega), \, \|u\|^p_p = 1 \right\}
\]
(1.11)
which is simple, positive, and isolated in the sense that the solutions of (1.6) with \( \lambda = \lambda_1(p) \) form a one-dimensional linear space spanned by a positive eigenfunction \( \varphi_1(p) \) associated with \( \lambda_1(p) \) and there exists \( \delta > 0 \) so that \( (\lambda_1(p), \lambda_1(p) + \delta) \) does not contain other eigenvalues. The situation is actually more involved with Dirichlet boundary conditions
\[
u = \nabla u = 0 \quad \text{on } \partial \Omega
\]
(1.12)
and, to our knowledge, it is not clear whether the first eigenspace has the previous good properties; the fact is that while Navier boundary conditions allow to reduce the fourth-order problem into a system of two second-order problems, Dirichlet boundary conditions do not. Some pathologies are indeed known, for instance, the first eigenfunction of \( \Delta^2 u = \lambda u \) with boundary data (1.12) may change sign [12].
Remark 1.2. Let $V = \text{span}\{\phi_1\}$ be the eigenspace associated with $\lambda_1$, where $\|\phi_1\|_{2,p} = 1$. Taking a subspace $W \subset W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega)$ complementing $V$, that is, $W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) = V \oplus W$, there exists $\hat{\lambda} > \lambda_1$ with

$$
\int_{\Omega} |\Delta u|^p \, dx \geq \hat{\lambda} \int_{\Omega} |u|^p \, dx
$$

(1.13)

for each $u \in W$ (in case $p = 2$, one may take $\hat{\lambda} = \lambda_2$).

We may now assume the following conditions:

(\mathcal{H}_1) there exist $R > 0$ and $\bar{\lambda} \in ]\lambda_1, \hat{\lambda}[\,$ such that

$$
|s| \leq R \implies \lambda_1 |s|^p \leq pG(x,s) \leq \bar{\lambda} |s|^p,
$$

(1.14)

for a.e. $x \in \Omega$ and each $s \in \mathbb{R}$;

(\mathcal{H}_2) there exist $\vartheta > p$ and $M > 0$ such that

$$
|s| \geq M \implies 0 < \vartheta G(x,s) \leq sg(x,s),
$$

(1.15)

for a.e. $x \in \Omega$ and each $s \in \mathbb{R}$.

Assumption (\mathcal{H}_1) corresponds to a resonance condition around the origin while (\mathcal{H}_2) is the standard condition of Ambrosetti-Rabinowitz type.

Theorem 1.3. Assume that conditions (\mathcal{H}_1) and (\mathcal{H}_2) hold. Then problem (1.1) with boundary conditions (1.2) admits a nontrivial solution in $W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega)$.

Now replace (\mathcal{H}_2) with a nonresonance condition at $+\infty$.

Theorem 1.4. Assume that condition (\mathcal{H}_1) holds and that for a.e. $x \in \Omega$

$$
\lim_{|s| \to +\infty} \frac{pG(x,s)}{|s|^p} < \lambda_1.
$$

(1.16)

Then problem (1.1) with boundary conditions (1.2) admits two nontrivial solutions in $W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega)$.

We use variational methods to prove Theorems 1.3 and 1.4. Usually, one uses a minimax type argument of mountain pass type to prove the existence of solutions of equations with a variational structure. However, it seems difficult to use minimax theorems in our situation. Thus we will adopt an approach based on Morse theory. Notice that there were a few works using Morse theory to treat $p$-Laplacian problems with Dirichlet boundary conditions (see [9] and the references therein). Moreover, to the authors' knowledge, (1.1) has a very poor literature; the only papers in which a $p$-harmonic equation is mentioned are [1, Section 8] and [4].

The existence of multiple solutions depends mainly on the behaviour of $G(x,s)$ near 0 and at $+\infty$. Without the above resonant or nonresonant conditions to obtain multiple solutions seems hard even in the semilinear case $p = 2$. 

Remark 1.5. Arguing as in [9], it is possible to prove Theorem 1.4 by replacing assumption (1.16) with the following conditions:
\[
\lim_{|s| \to +\infty} \frac{pG(x,s)}{|s|^p} = \lambda_1, \quad \lim_{|s| \to +\infty} \{g(x,s)s - pG(x,s)\} = +\infty
\] (1.17)
for a.e. \(x \in \Omega\) (resonance condition at \(+\infty\)).

Remark 1.6. The existence of solutions \(u \in W^{2,p}_0(\Omega)\) of the quasilinear problem
\[
\Delta(|\Delta u|^{p-2} \Delta u) = g(x,u) \quad \text{in } \Omega,
\]
\[
u = \nabla u = 0 \quad \text{on } \partial \Omega
\] (1.18)
under the previous assumptions (\(\mathcal{H}_j\)) is, to our knowledge, an open problem.

2. Proofs of Theorems 1.3 and 1.4

In this section, we give the proof of our main results. It is readily seen that
\[
\|u\|_{2,p} = \left( \int_{\Omega} |\Delta u|^p \, dx \right)^{1/p}
\] (2.1)
is an equivalent norm of the standard space norm of \(W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega)\). For \(\Phi\) a continuously Fréchet differentiable map, let \(\Phi'\) denote its Fréchet derivative.

**Lemma 2.1.** The functional \(\Phi\) satisfies the Palais-Smale condition.

**Proof.** Let \((u_h) \subset W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega)\) be such that \(|\Phi(u_h)| \leq B\), for some \(B > 0\) and \(\Phi'(u_h) \to 0\) as \(h \to +\infty\). Let \(d = \sup_{h \geq 0} \Phi(u_h)\). Then we have
\[
\vartheta d + \|u_h\|_{2,p} \geq \vartheta \Phi(u_h) - \langle \Phi'(u_h), u_h \rangle
\]
\[
= \left( \frac{\vartheta}{p} - 1 \right) \|u_h\|_{2,p}^p - \int_{|u_h| \geq M} \left[ \vartheta G(x, u_h) - g(x, u_h) u_h \right] \, dx
\]
\[
- \int_{|u_h| \leq M} \left[ \vartheta G(x, u_h) - g(x, u_h) u_h \right] \, dx
\]
\[
\geq \left( \frac{\vartheta}{p} - 1 \right) \|u_h\|_{2,p}^p - \int_{|u_h| \leq M} \left[ \vartheta G(x, u_h) - g(x, u_h) u_h \right] \, dx
\]
\[
\geq \left( \frac{\vartheta}{p} - 1 \right) \|u_h\|_{2,p}^p - D,
\]
for some \(D \in \mathbb{R}\). Thus \((u_h)\) is bounded and, up to a subsequence, we may assume that \(u_h \rightharpoonup u\) is, for some \(u\), in \(W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega)\). Since the embedding \(W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) \hookrightarrow L^q(\Omega)\) is compact, then a standard argument shows that \(u_h \to u\) strongly and the proof is complete. \(\square\)

Now recall the notion of “Local Linking,” which was initially introduced by Liu and Li [8] and has been used in a vast amount of literature (cf. [2, 5, 6, 11]).
Definition 2.2. Let $E$ be a real Banach space such that $E = V \oplus W$, where $V$ and $W$ are closed subspaces of $E$. Let $\Phi : E \to \mathbb{R}$ be a $C^1$-functional. We say that $\Phi$ has a local linking near the origin $0$ (with respect to the decomposition $E = V \oplus W$), if there exists $\varrho > 0$ such that

\begin{align}
  u \in V : \|u\| \leq \varrho & \implies \Phi(u) \leq 0, \\
  u \in W : 0 < \|u\| \leq \varrho & \implies \Phi(u) > 0.
\end{align}

(2.3)

We now show that our functional $\Phi$ has a local linking near the origin with respect to the space decomposition $W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) = V \oplus W$, according to Remark 1.2.

Lemma 2.3. There exists $\varrho > 0$ such that conditions (2.3) hold with respect to the decomposition $W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) = V \oplus W$.

Proof. For $u \in V$, the condition $\|u\|_{2,p} \leq \varrho$ implies $u(x) \leq R$ for a.e. $x \in \Omega$ if $\varrho > 0$ is small enough, being $R > 0$ as in assumption ($H_{1}$). Thus for $u \in V$,

\begin{align}
  \Phi(u) &= \frac{1}{P} \int_{\Omega} |\Delta u|^p \, dx - \int_{\Omega} G(x, u) \, dx \\
  &= \frac{1}{P} \int_{\Omega} |u|^p \, dx - \int_{\Omega} G(x, u) \, dx = \int_{\{\|u\| \leq R\}} \left[ \frac{\lambda_1}{P} |u|^p - G(x, u) \right] \, dx \leq 0
\end{align}

(2.4)

provided that $\|u\|_{2,p} \leq \varrho$ and $\varrho$ is small.

To prove the second assertion, take $u \in W$. In view of (1.3) and (1.13) we have

\begin{align}
  \Phi(u) &= \frac{1}{P} \int_{\Omega} |\Delta u|^p \, dx - \int_{\Omega} G(x, u) \, dx \\
  &= \frac{1}{P} \int_{\Omega} (|\Delta u|^p - \hat{\lambda}|u|^p) \, dx \\
  &\quad - \left( \int_{\{\|u\| \leq R\}} + \int_{\{\|u\| \geq R\}} \right) (G(x, u) - \frac{\hat{\lambda}}{P} |u|^p) \, dx \\
  &\geq \frac{1}{P} \left( 1 - \frac{\hat{\lambda}}{\lambda} \right) \|u\|_{2,p}^p - c \int_{\Omega} |u|^s \, dx \geq \frac{1}{P} \left( 1 - \frac{\hat{\lambda}}{\lambda} \right) \|u\|_{2,p}^p - C \|u\|_{2,p}^s,
\end{align}

(2.5)

where $p < s \leq p_*$ and $c, C$ are positive constants. Since $s > p$, it follows that $\Phi(u) > 0$ for $\varrho > 0$ sufficiently small. \qed

Assume that $u$ is an isolated critical point of $\Phi$ such that $\Phi(u) = c$. We define the critical group of $\Phi$ at $u$ by setting for each $q \in \mathbb{Z}$

\begin{align}
  C_q(\Phi, u) = H_q(\Phi_c, \Phi_c \backslash \{u\}),
\end{align}

(2.6)

being $H_q(X, Y)$ the $q$th homology group of the topological pair $(X, Y)$ over the ring $\mathbb{Z}$ and $\Phi_c$ the $c$-sublevel of $\Phi$. For the detail of Morse theory and critical groups, we refer the reader to [3].
Since \( \dim V = 1 < +\infty \), by combining Lemma 2.3 and [7, Theorem 2.1], we obtain the following result.

**Lemma 2.4.** The point 0 is a critical point of \( \Phi \) and \( C_1(\Phi, 0) \neq \{0\} \).

We now investigate the behavior of \( \Phi \) near infinity.

**Lemma 2.5.** There exists a constant \( A > 0 \) such that

\[
a < -A \implies \Phi_a \simeq S^\infty,
\]

where \( S^\infty = \{ u \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) : \|u\|_{2,p} = 1 \} \).

*Proof.* By integrating inequality (1.15), we obtain a constant \( C_1 > 0 \) with

\[
|s| \geq M \implies G(x, s) \geq C_1 |s|^\theta
\]
a.e. in \( \Omega \) and for each \( s \in \mathbb{R} \). Thus, for \( u \in S^\infty \), we have \( \Phi(tu) \to -\infty \), as \( t \) goes to +\( \infty \). Set

\[
A = \left( 1 + \frac{1}{p} \right) M \mathcal{L}^n(\Omega) \max_{\Omega \times [-M, M]} |g(x, s)| + 1,
\]

being \( \mathcal{L}^n \) the Lebesgue measure. As in the proof of [10, Lemma 2.4] we obtain

\[
\int_{\Omega} G(x, u) \, dx - \frac{1}{p} \int_{\Omega} g(x, u) \, u \, dx \\
\leq \left( \frac{1}{\theta} - \frac{1}{p} \right) \int_{\|u\| \geq M} g(x, u) \, u \, dx + A - 1.
\]

For \( a < -A \) and

\[
\Phi(tu) = \frac{|t|^p}{p} - \int_{\Omega} G(x, tu) \, dx \leq a \quad (u \in S^\infty),
\]

in view of (2.8) and (2.10), arguing as in the proof of [10, Lemma 2.4],

\[
\frac{d}{dt} \Phi(tu) < 0.
\]

By the implicit function theorem, there is a unique \( T \in C(S^\infty, \mathbb{R}) \) such that

\[
\forall u \in S^\infty, \quad \Phi(T(u)u) = a.
\]

For \( u \neq 0 \), set \( \tilde{T}(u) = (1/\|u\|_{2,p}) T(u/\|u\|_{2,p}) \). Then \( \tilde{T} \in C(W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\}, \mathbb{R}) \) and

\[
\forall u \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\}, \quad \Phi(\tilde{T}(u)u) = a.
\]
We define now a functional $\hat{T} : W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) \setminus \{0\} \to \mathbb{R}$ by setting
\[
\hat{T}(u) = \begin{cases} 
\hat{T}(u) & \text{if } \Phi(u) \geq a, \\
1 & \text{if } \Phi(u) \leq a.
\end{cases} 
\tag{2.15}
\]
Since $\Phi(u) = a$ implies $\hat{T}(u) = 1$, we conclude that
\[
\hat{T} \in C(W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) \setminus \{0\}, \mathbb{R}). 
\tag{2.16}
\]
Finally, let $\eta : [0, 1] \times W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) \setminus \{0\} \to W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) \setminus \{0\}$,
\[
\eta(s, u) = (1-s)u + s\hat{T}(u)u. 
\tag{2.17}
\]
It results that $\eta$ is a strong deformation retract from $W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) \setminus \{0\}$ to $\Phi_a$. Thus $\Phi_a \simeq W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega) \setminus \{0\} \simeq S^\infty$. \hfill \Box

Remark 2.6. A result similar to Lemma 2.5 has been proved for the Laplacian $-\Delta$ in [3, 14], under the additional conditions
\[
g \in C^1(\Omega \times \mathbb{R}, \mathbb{R}), \quad g_t(x, 0) = \frac{\partial g(x, t)}{\partial t} \bigg|_{t=0} = 0. 
\tag{2.18}
\]
We recall the following topological result due to Perera [11].

Lemma 2.7. Let $Y \subset B \subset A \subset X$ be topological spaces and $q \in \mathbb{Z}$. If
\[
H_q(A, B) \neq \{0\}, \quad H_q(X, Y) = \{0\}, 
\tag{2.19}
\]
then it results that
\[
H_{q+1}(X, A) \neq \{0\} \quad \text{or} \quad H_{q-1}(B, Y) \neq \{0\}. 
\tag{2.20}
\]

Proof of Theorem 1.3. By Lemma 2.1, $\Phi$ satisfies the Palais-Smale condition. Note that $\Phi(0) = 0$, by [3, Chapter I, Theorem 4.2], there exists $\varepsilon > 0$ with
\[
H_1(\Phi_\varepsilon, \Phi_{-\varepsilon}) = C_1(\Phi, 0) \neq \{0\}. 
\tag{2.21}
\]
If $A$ is as in Lemma 2.5, for $a < -A$ we have $\Phi_a \simeq S^\infty$, which yields
\[
H_1(W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega), \Phi_a) = H_1(W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega), S^\infty) \neq \{0\}. 
\tag{2.22}
\]
Therefore, being $\Phi_a \subset \Phi_{-\varepsilon} \subset \Phi_\varepsilon$, Lemma 2.7 yields
\[
H_2(W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega), \Phi_\varepsilon) \neq \{0\} \quad \text{or} \quad H_0(\Phi_{-\varepsilon}, \Phi_a) \neq \{0\}. 
\tag{2.23}
\]
It follows that $\Phi$ has a critical point $u$ for which
\[
\Phi(u) > \varepsilon \quad \text{or} \quad -\varepsilon > \Phi(u) > a. 
\tag{2.24}
\]
Therefore, $u \neq 0$ and (1.1), (1.2) possess a nontrivial solution. \hfill \Box
Recall from [9] the following three-critical point theorem.

**Lemma 2.8.** Let $X$ be a real Banach space and let $\Phi \in C^1(X, \mathbb{R})$ be bounded from below and satisfying the Palais-Smale condition. Assume that $\Phi$ has a critical point $u$ which is homologically nontrivial, that is, $C_j(\Phi, u) \neq \{0\}$ for some $j$, and it is not a minimizer for $\Phi$. Then $\Phi$ admits at least three critical points.

**Proof of Theorem 1.4.** By Lemma 2.8, taking into account Lemma 2.4, it suffices to show that $\Phi$ is bounded from below. Indeed, by (1.16) there exist $\epsilon > 0$ small and $C > 0$ such that

$$G(x, s) \leq \frac{\lambda_1 - \epsilon}{p} |s|^p + C$$

(2.25)

for a.e. $x \in \Omega$ and each $s \in \mathbb{R}$. This, by (1.11), immediately yields

$$\Phi(u) \geq \frac{1}{p} \|u\|_{L^p}^p - \frac{1}{p} (\lambda_1 - \epsilon) \|u\|_{L^p}^p - C\mathcal{L}^n(\Omega)$$

$$\geq \frac{1}{p} \left( 1 - \frac{\lambda_1 - \epsilon}{\lambda_1} \right) \|u\|_{L^p}^p - C\mathcal{L}^n(\Omega) \to +\infty$$

(2.26)

as $\|u\|_{L^p} \to +\infty$. Then $\Phi$ is coercive and satisfies the Palais-Smale condition. In particular Lemma 2.8 provides the existence of at least two nontrivial critical points of $\Phi$. □

**Acknowledgment**

The authors wish to thank Prof. Pavel Drábek for his useful comments about the spectrum of the $p$-harmonic eigenvalue problem.

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