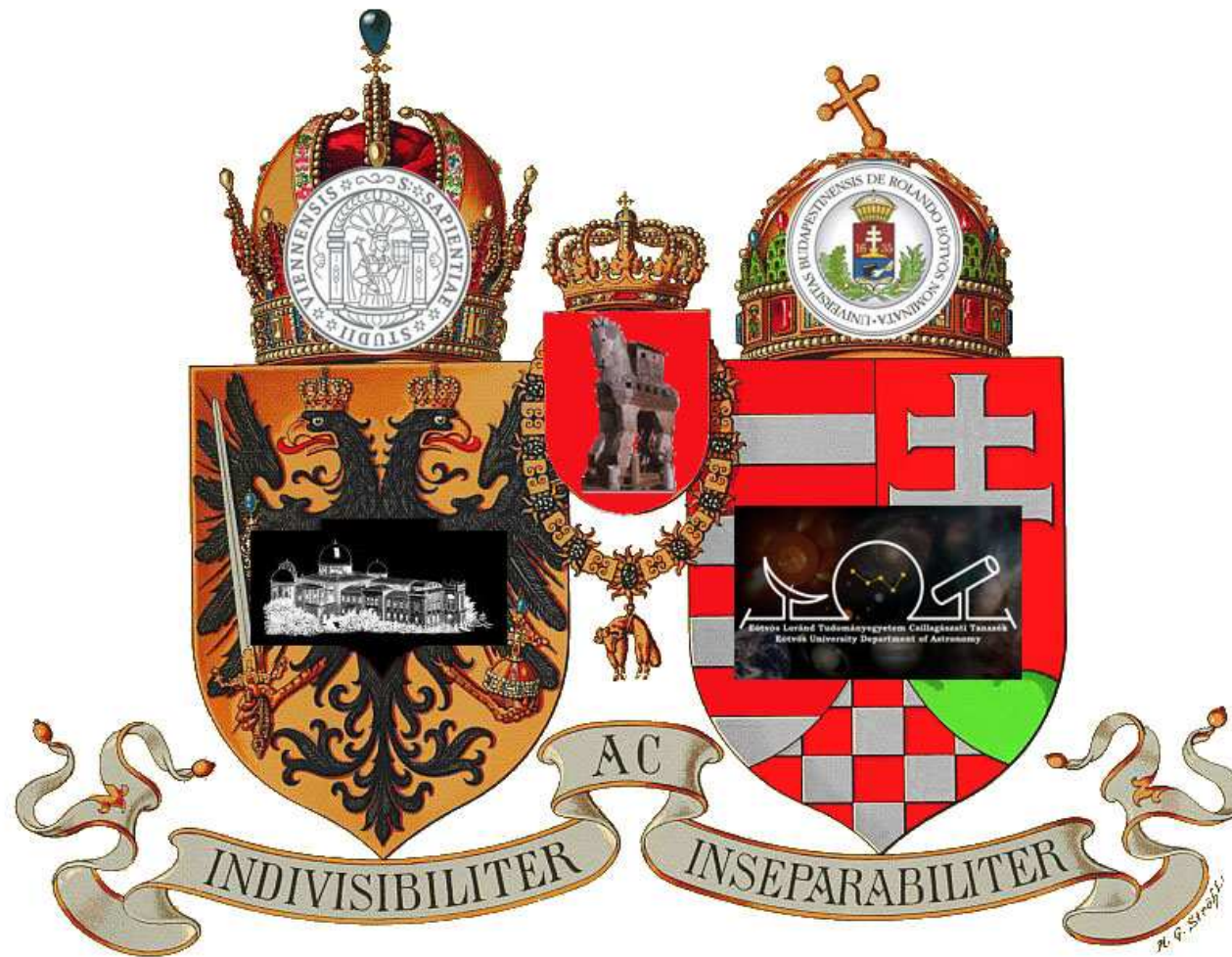


Dynamics of possible Trojan planets in double stars



Richard Schwarz

Former work:

Dynamics of possible Trojan planets in binary systems:
(MNRAS 2009)

We will investigate **two different types**:

Type1) binaries consist of two stellar objects and

Type2) binaries consist of a stellar and a substellar (brown dwarf) object

Former work:

Dynamics of possible Trojan planets in binary systems: (MNRAS 2009)

Table 1. Orbital elements of six binary candidates, with $\mu < 1/25$, m denotes the mass of the star expressed in solar mass units (M_{\odot}). In the upper part systems with known orbital parameters, whereas in the lower part systems with unknown orbital parameters are listed.

Name	$m (M_{\odot})$	μ	a (au)	e
51 θ Vir	2.98	0.026	–	–
51 θ Vir	0.08	0.026	9.91	–
53 ξ Uma	1.1	0.035	–	–
53 ξ Uma	0.04	0.035	15.99	0.39
HD 223099		0.03	–	–
SB 152		0.035	–	–
SB 667		0.03	–	–
SB 831		0.03	–	–

$$\mu = \frac{m_1}{m_1 + m_2}$$

Table 3. Orbital elements of all binaries with substellar candidates, sorted by increasing mass of the secondary.

Name	Spectral type	m_{star} (M_{\odot})	m_{planet} (M_{J})	$\sin(i)$	μ	a (au)	e	ω ($^{\circ}$)
18 Del	G6 III	2.3	–	–	–	–	–	–
18 Del b	–	0.0098	10.30	–	0.0043	2.60	0.08	166.1
NGC 24233	–	2.4	–	–	–	–	–	–
NGC 24233 b	–	0.0101	10.60	–	0.0042	2.10	0.21	18.0
XO–3	F5 V	1.213	–	–	–	–	–	–
XO–3 b	–	0.0113	11.79	–	0.0092	0.045	0.26	345.8
HD 162020	K2 V	0.8	–	–	–	–	–	–
HD 162020 b	–	0.0131	13.75	–	0.0162	0.072	0.28	–27.3
HD 13189	K2 II	4.5	–	–	–	–	–	–
HD 13189 b	–	0.0134	14.00	–	0.0030	1.85	0.28	160.7
HD 168443	G5	1.06	–	–	–	–	–	–
HD 168443 b	–	0.0077	8.02	–	–	0.3	0.5286	172.9
HD 168443 c	–	0.0173	18.10	–	0.0160	2.91	0.21	65.1
NGC 4349 No127	–	3.9	–	–	–	–	–	–
NGC 4349 No127 b	–	0.0189	19.80	–	0.0048	2.38	0.19	61.0
CoRoT-Exo3	G0 V	1.37	–	–	–	–	–	–
CoRoT-Exo3 b	–	0.0207	21.66	–	0.0149	0.057	0.00	0.0

Results

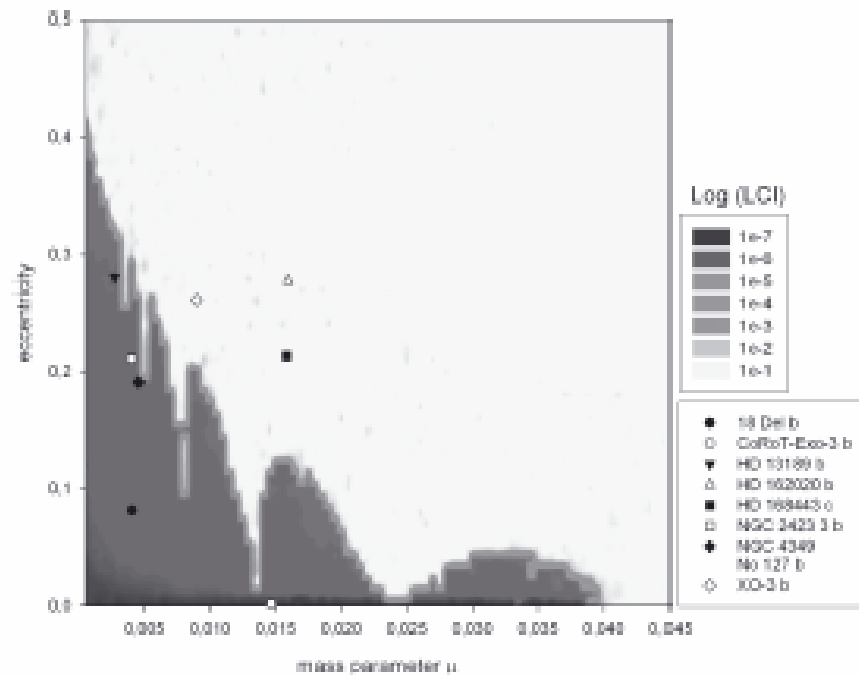


Figure 3. Stability of the Lagrangian point L_4 depending on the eccentricity and the mass parameter.

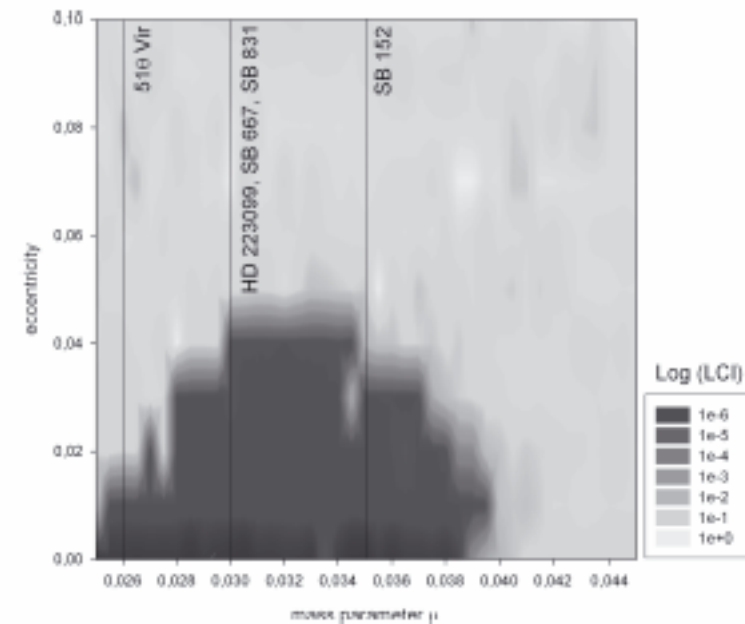
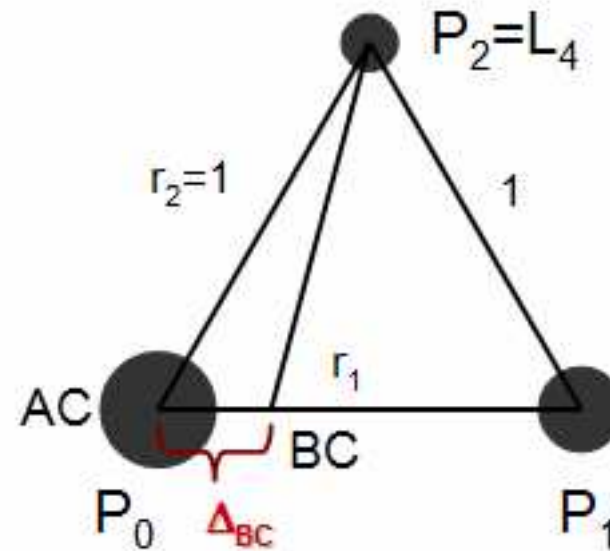


Figure 2. Stability in the Lagrangian point L_4 depending on the eccentricity and the mass parameter. 53ξ Uma is not presented, because the initial eccentricity is too high.

Expand investigations to the 3 dimensional case

Initial conditions

- .) R3BP and 3D-R3BP
- .) Starting at L_4
- .) AC, BC
- .) integration time 1000years



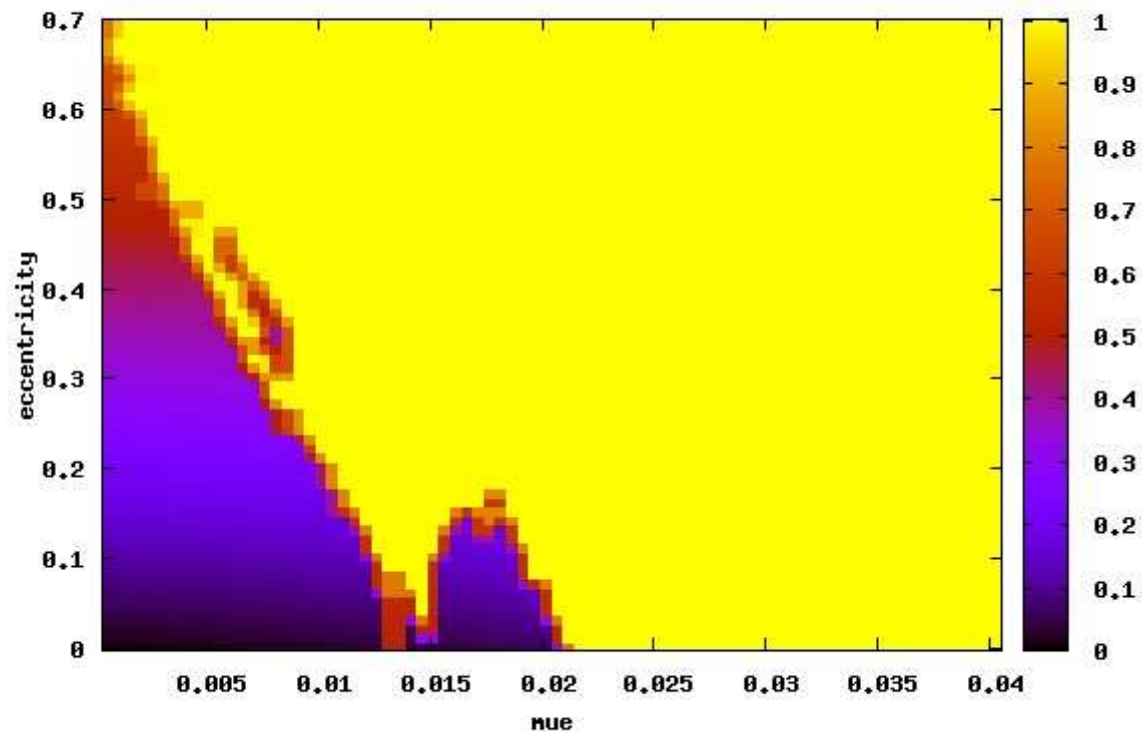
Stability

- .) maximum eccentricity
- .) Lyapunov indicator

Integrators:

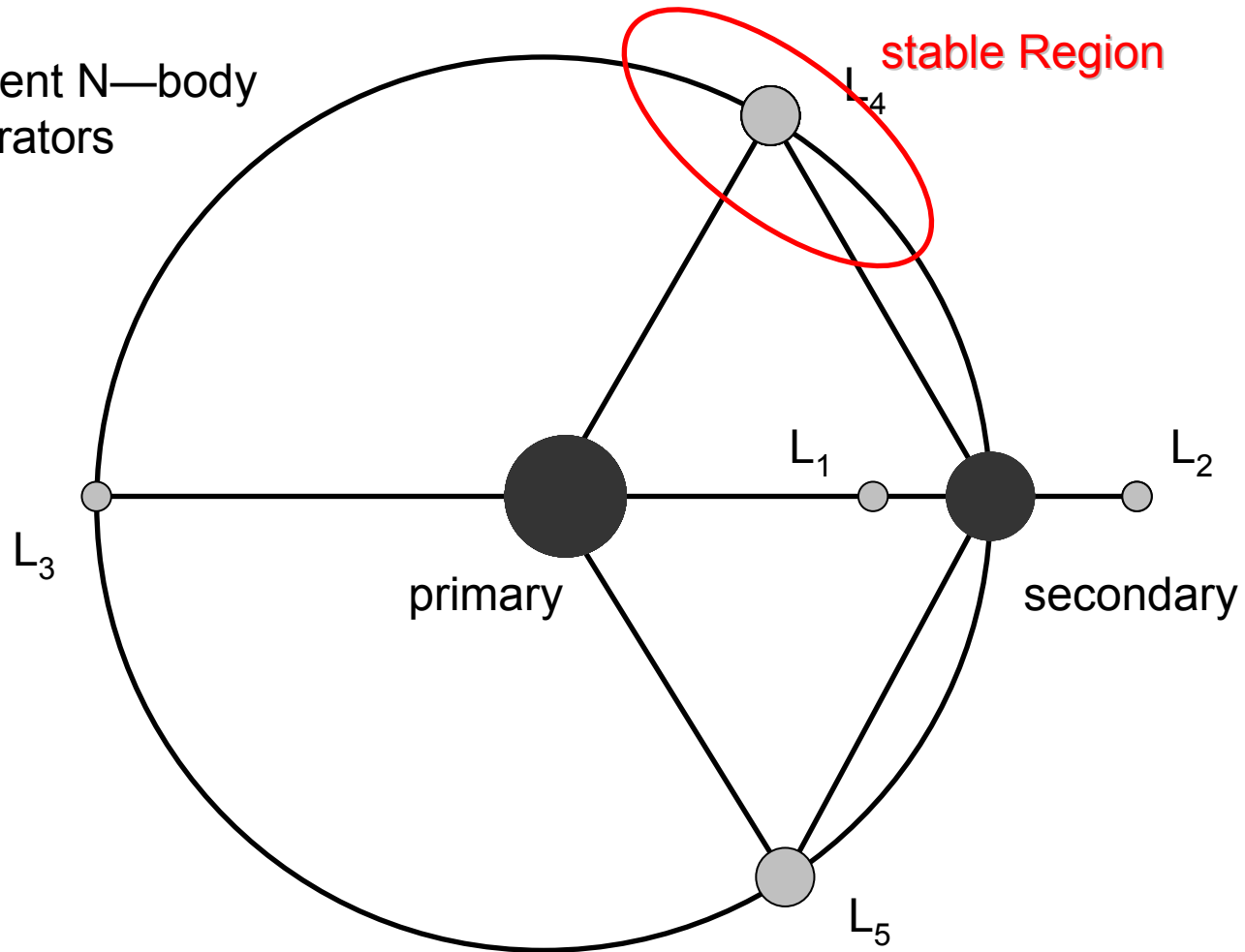
- .) Burlirsch-Stoer
- .) Lie
- .) Nordsieck

Starting in Lagrangian point L_4

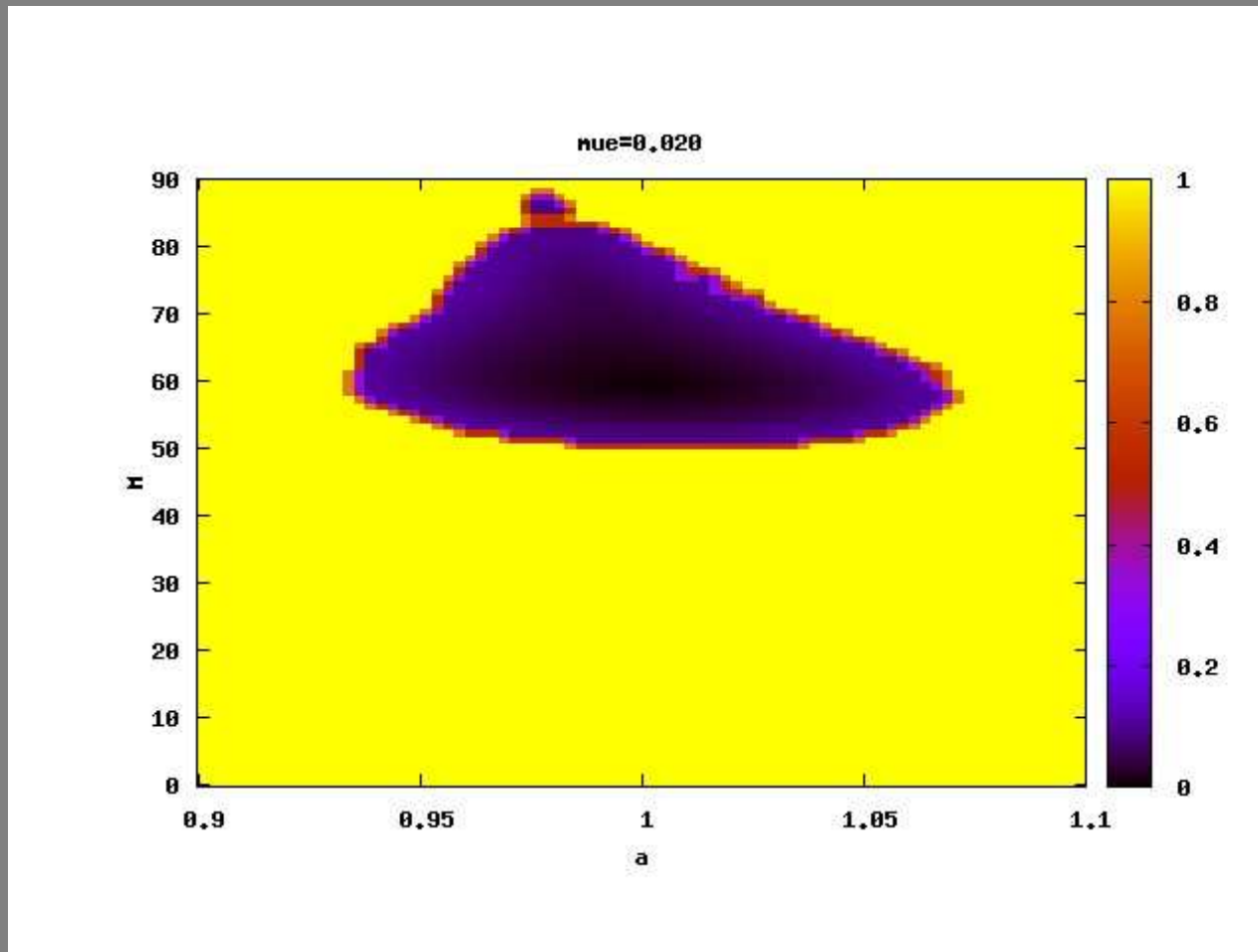


Trojan configuration of the Planar circular case

Used: Different N—body
integrators



Stability map



Initial conditions:

$M = 0-90^\circ \Delta M = 1^\circ$

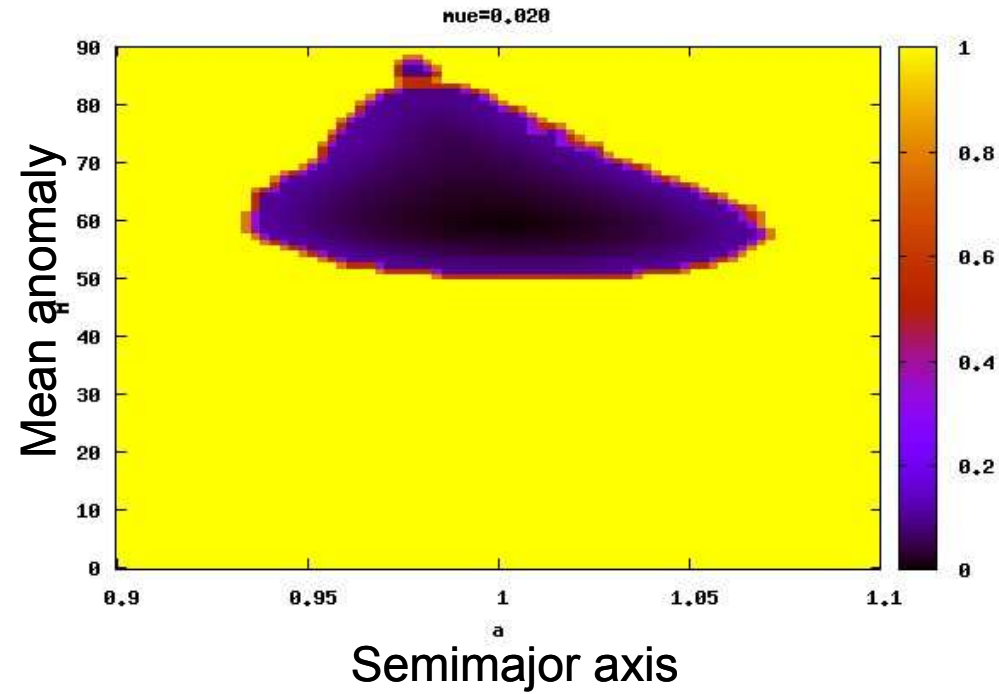
$a = 0.85-1.1 \quad a_1 = 0.85-1.15 \quad \Delta a = 0.0025$

$e, i, \omega, \Omega = 0$

~ 11000 orbits

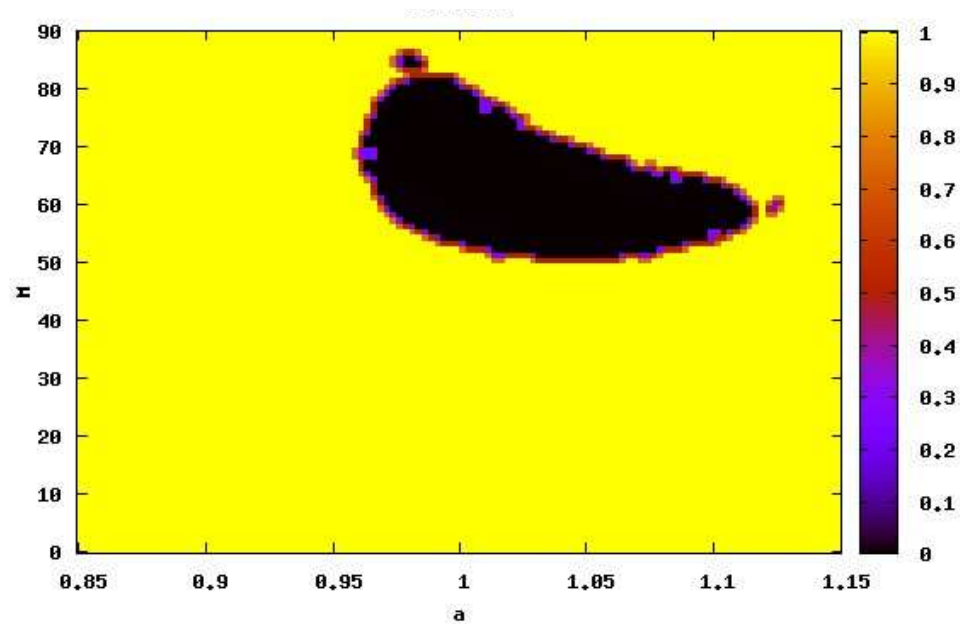
Integration-time 1000 years

2D-starting
in the AC



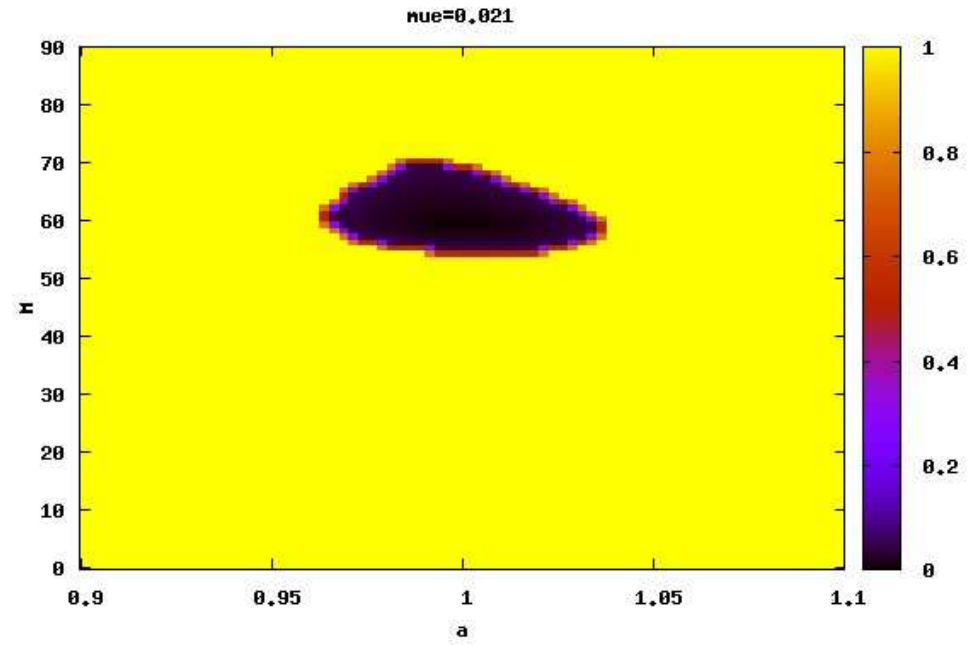
$\mu=0.020$

3D-starting
BC



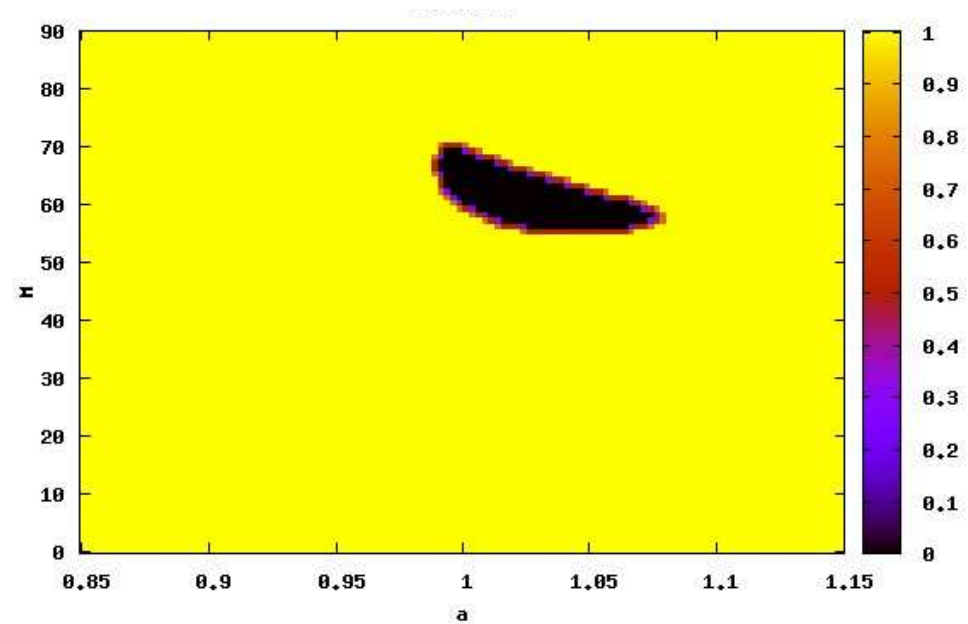
Integration time
= 1000 periods

2D-starting
in the AC

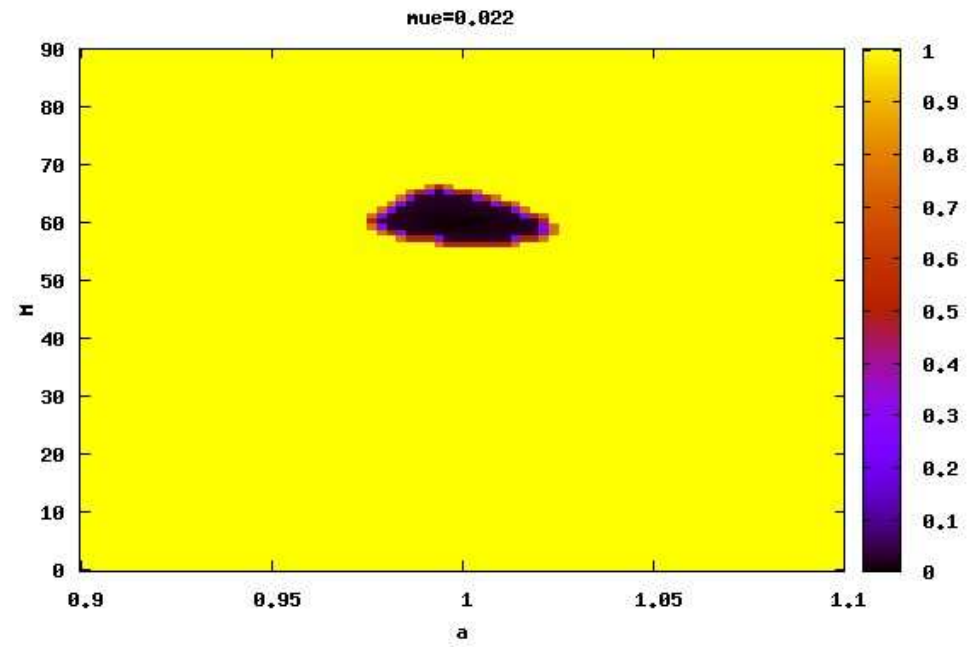


$\mu=0.021$

3D-starting
BC

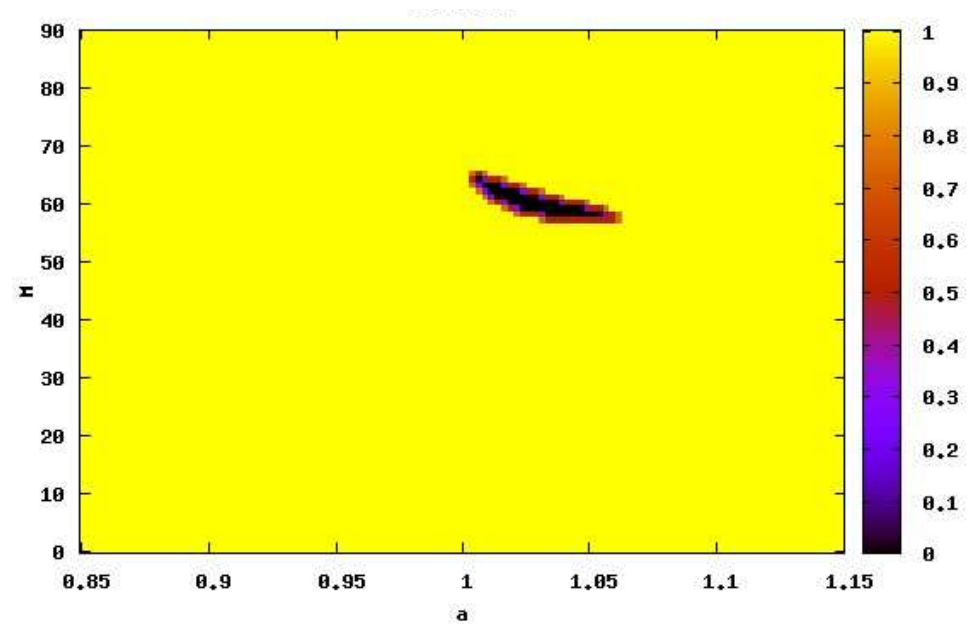


2D-starting
in the AC

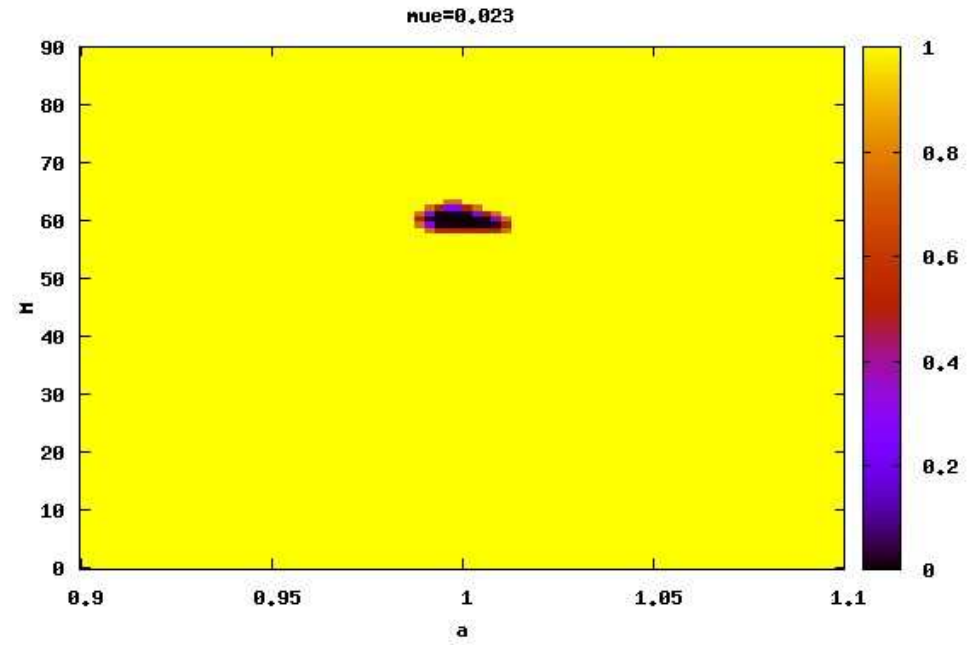


$\mu=0.022$

3D-starting
BC

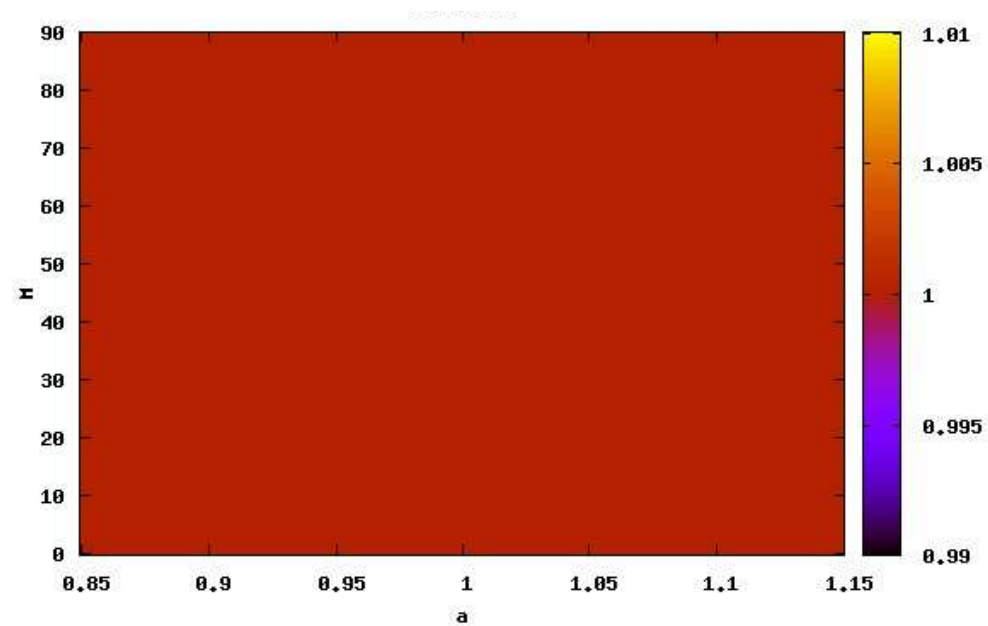


2D-starting
in the AC

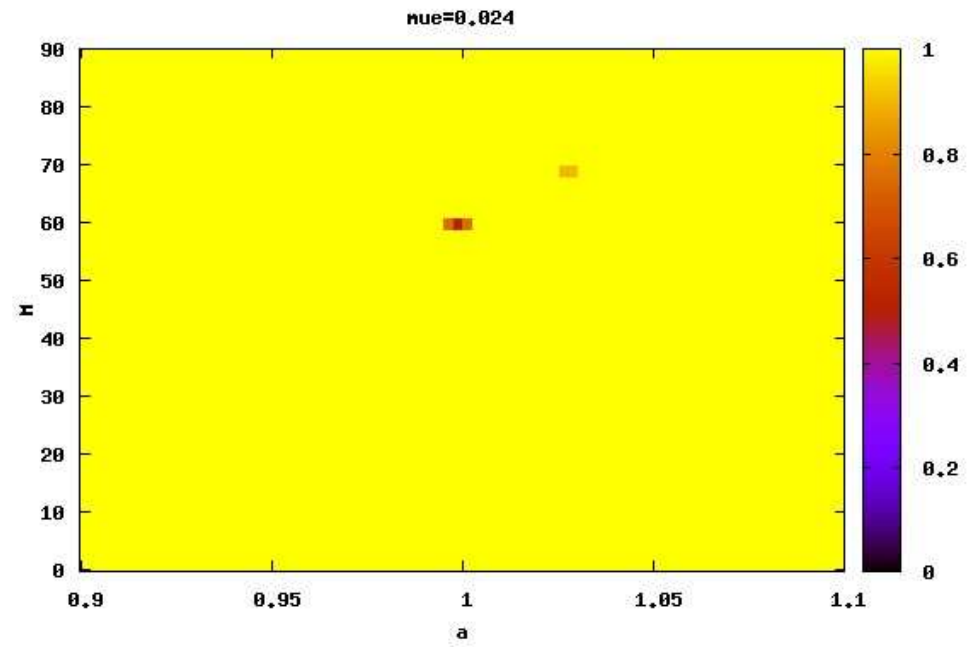


$$\mu=0.023$$

3D-starting
BC

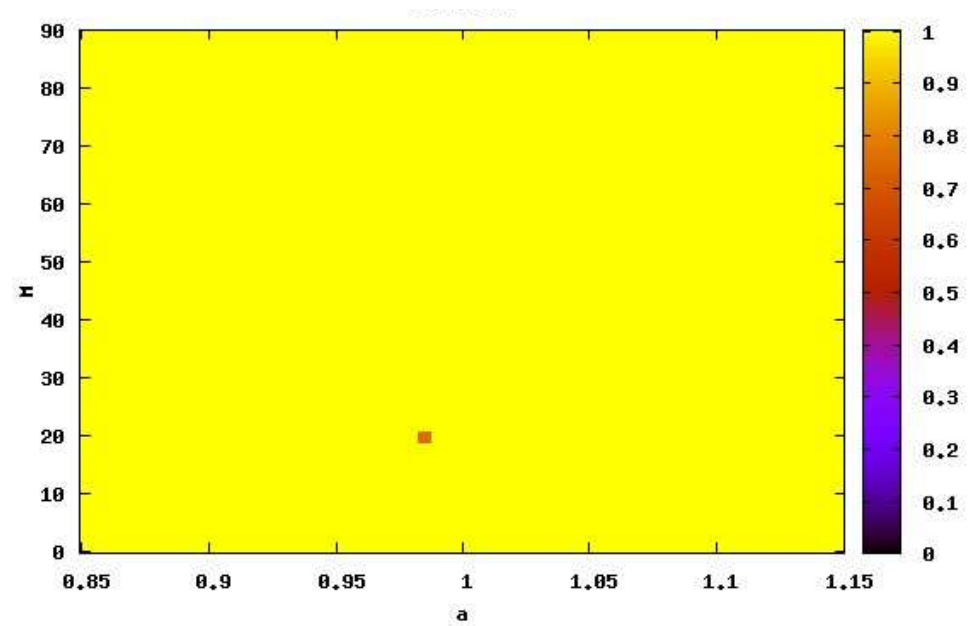


2D-starting
in the AC

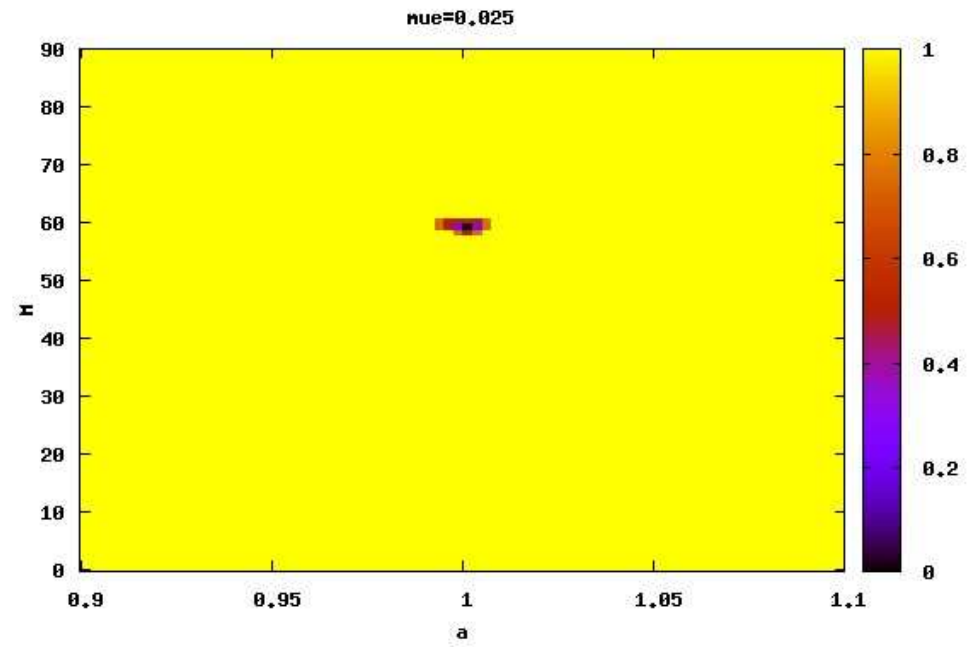


$\mu=0.024$

3D-starting
BC

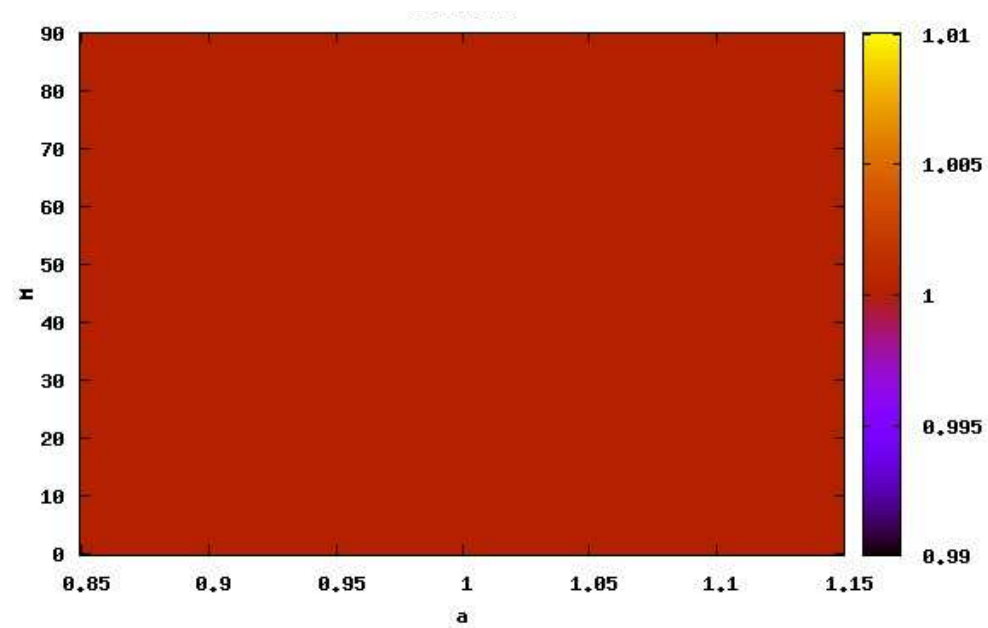


2D-starting
in the AC

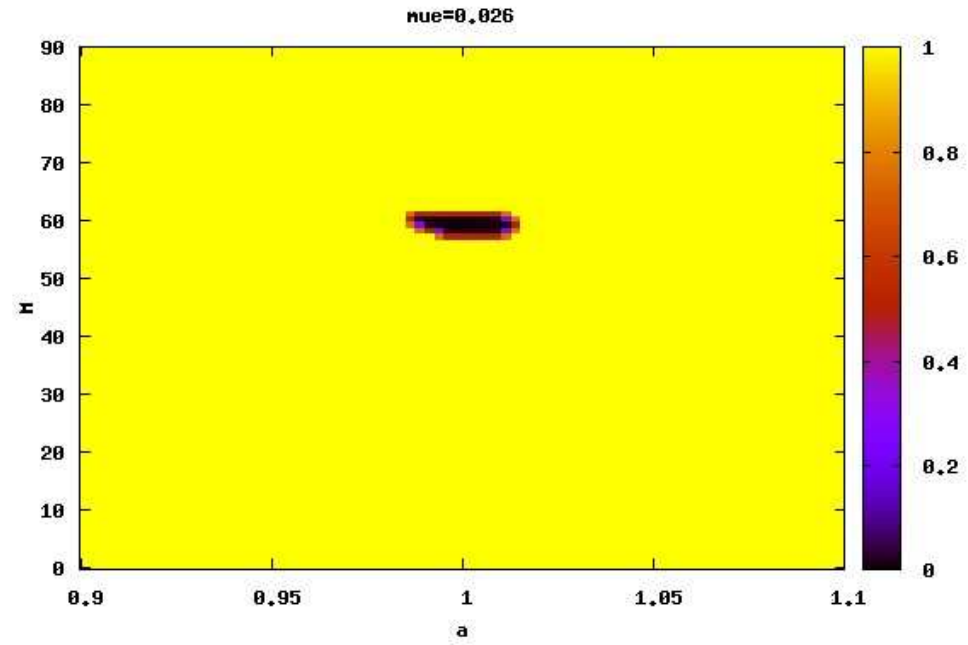


$\mu=0.025$

3D-starting
BC

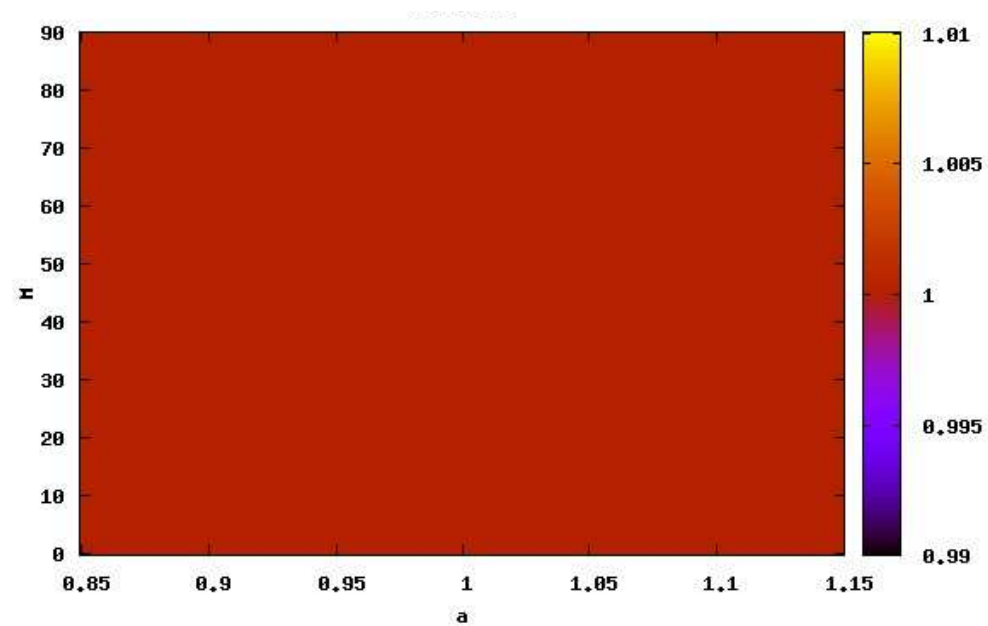


2D-starting
in the AC

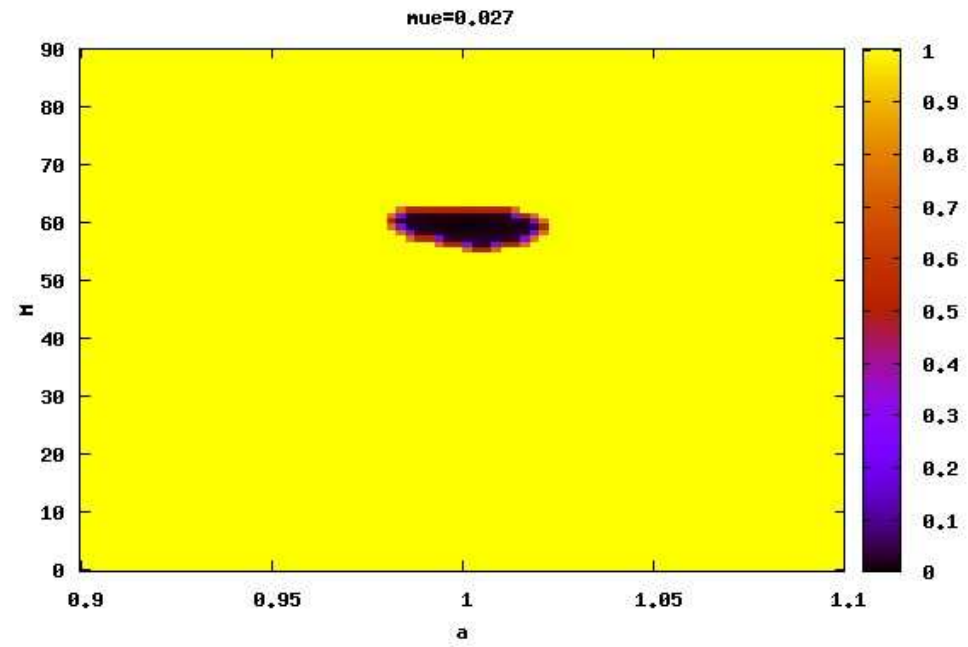


$$\mu=0.026$$

3D-starting
BC

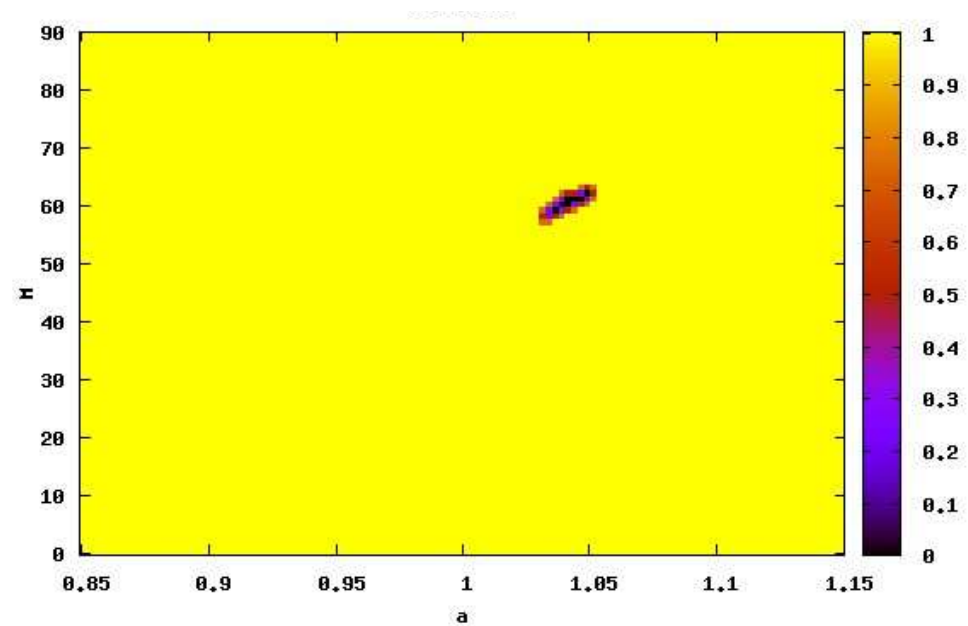


2D-starting
in the AC

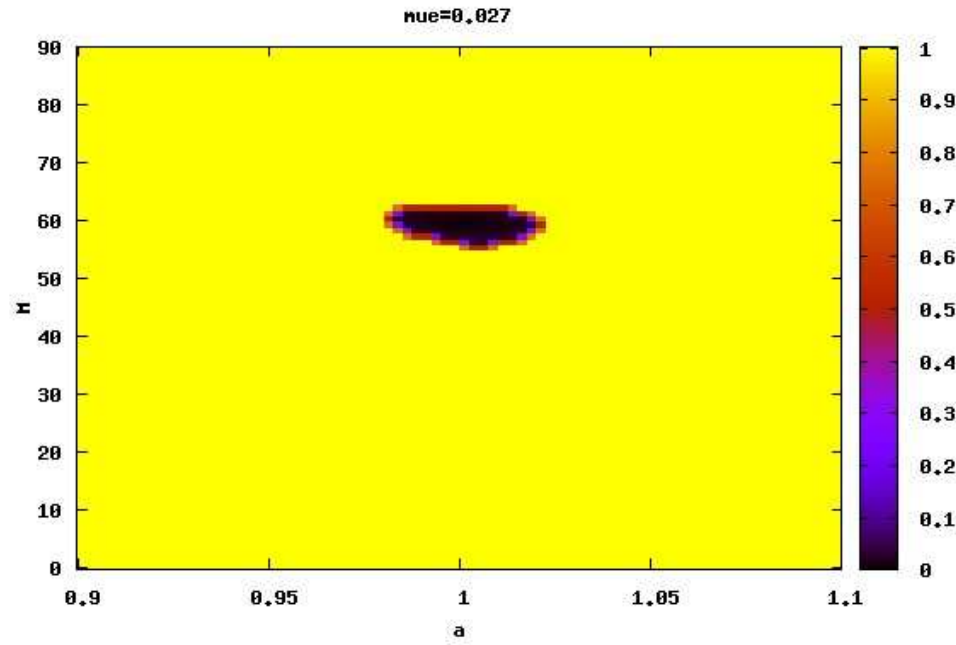


$\mu=0.027$

3D-starting
BC

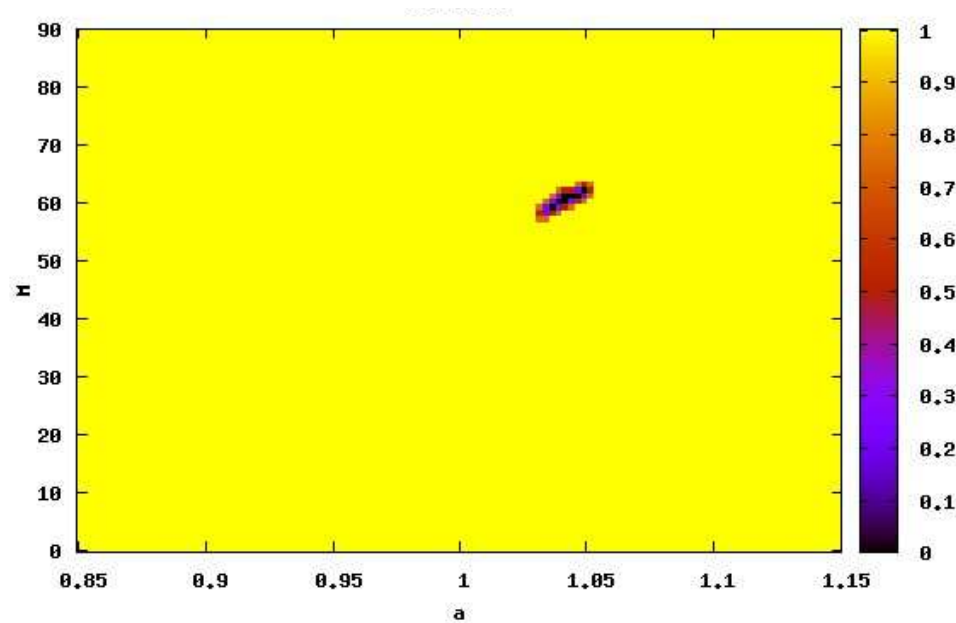


2D-starting
in the AC

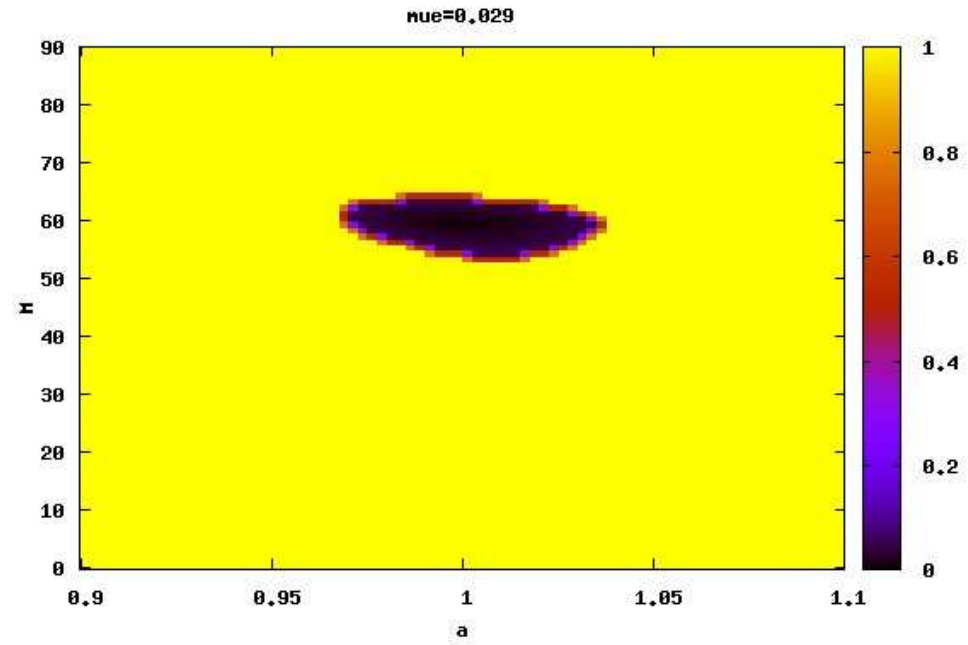


$\mu=0.028$

3D-starting
BC

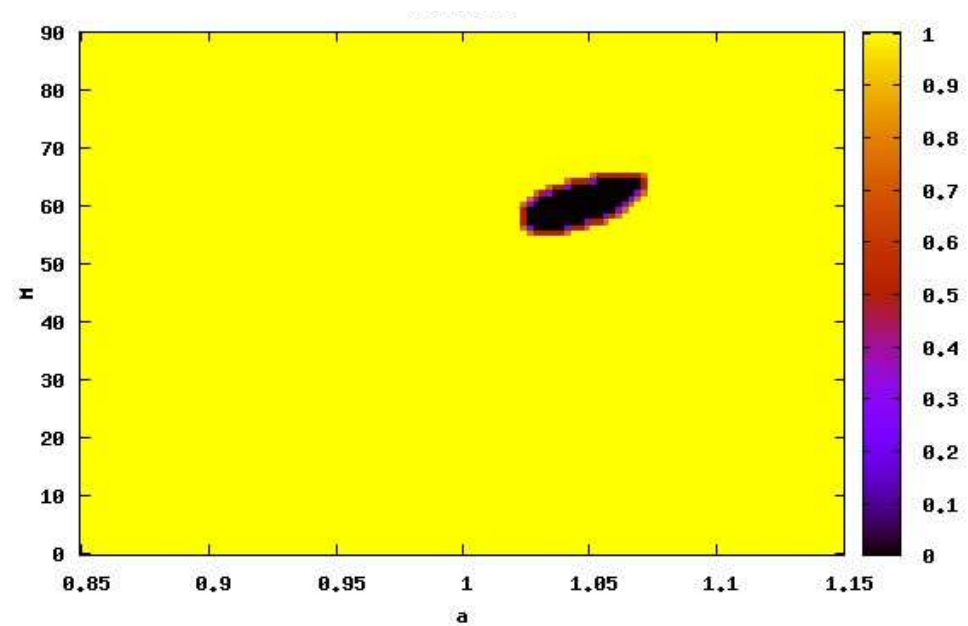


2D-starting
in the AC

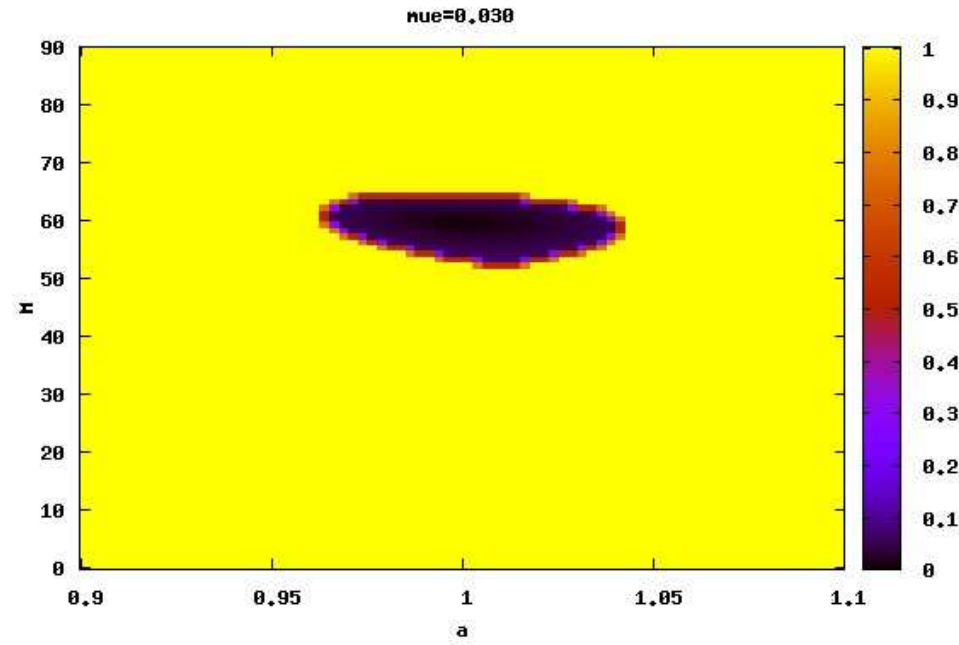


$$\mu=0.029$$

3D-starting
BC

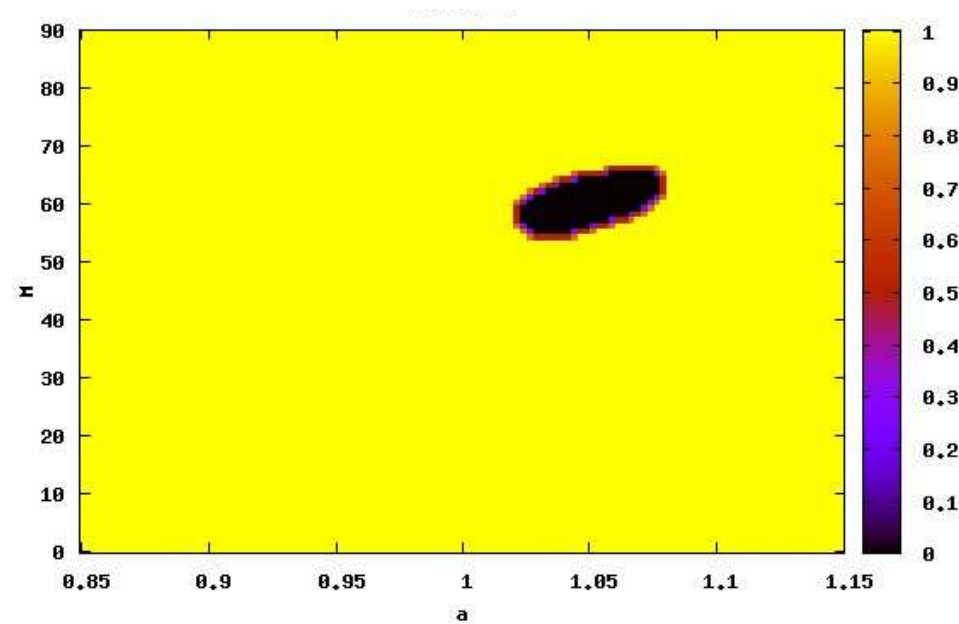


2D-starting
in the AC

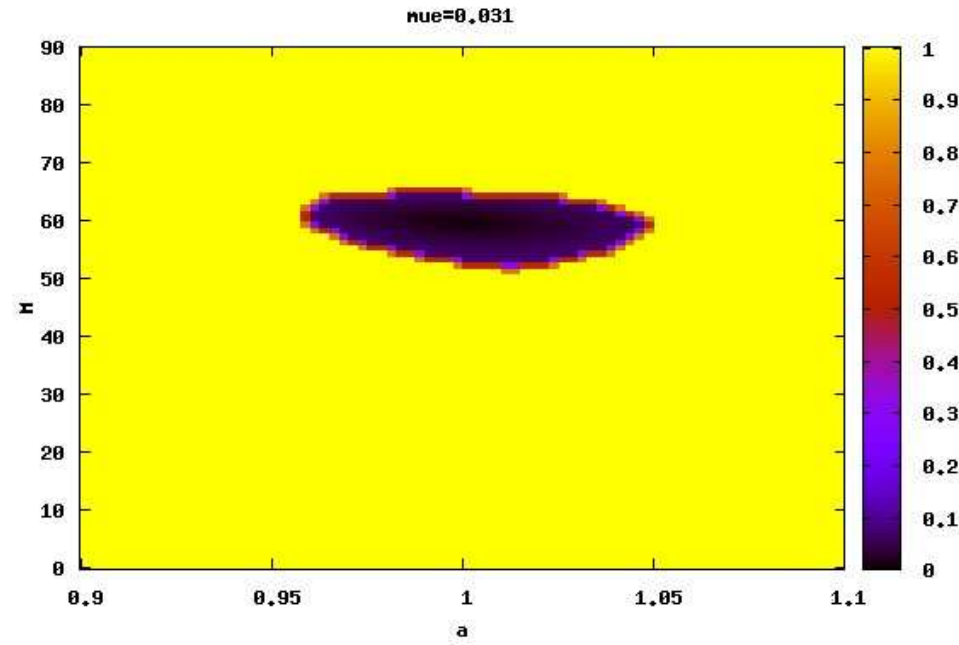


$$\mu=0.030$$

3D-starting
BC

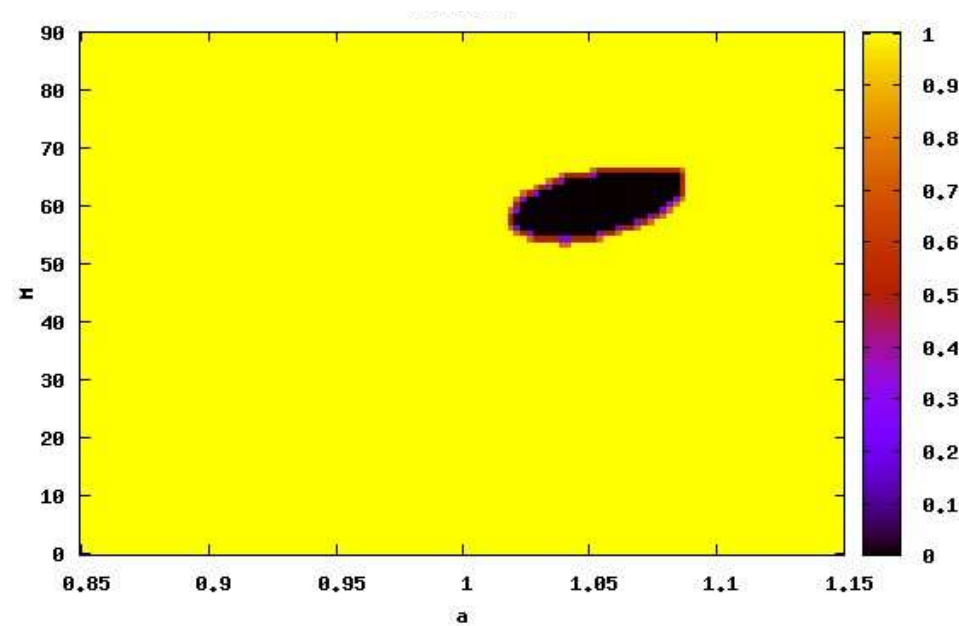


2D-starting
in the AC

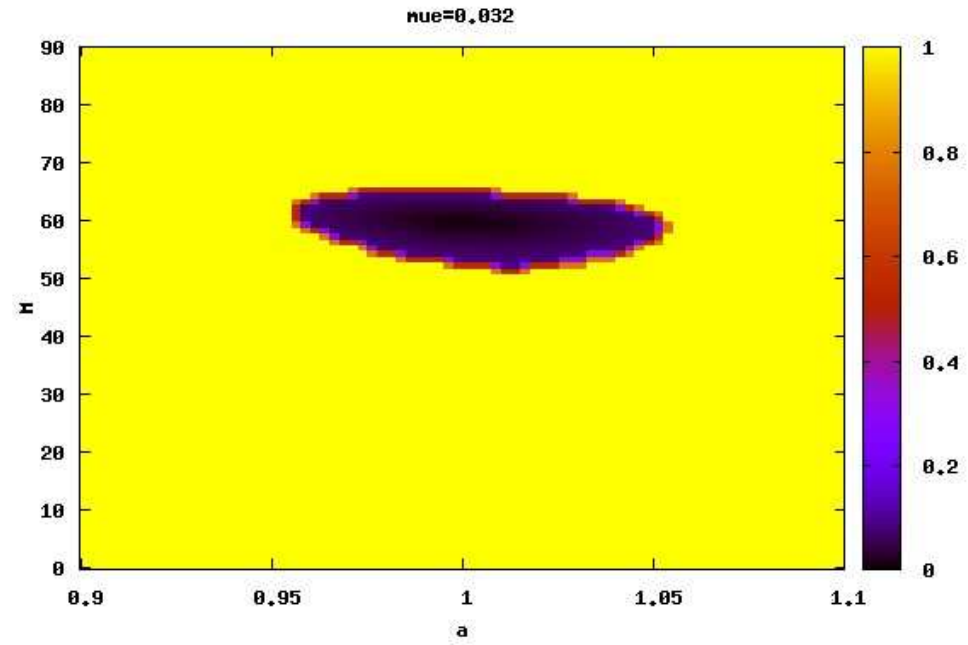


$$\mu=0.031$$

3D-starting
BC

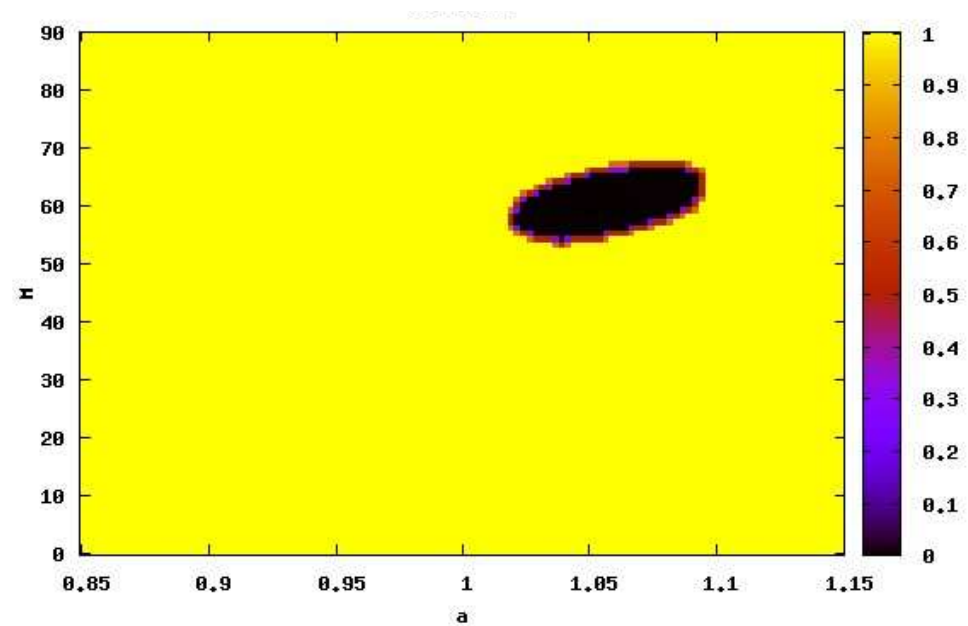


2D-starting
in the AC

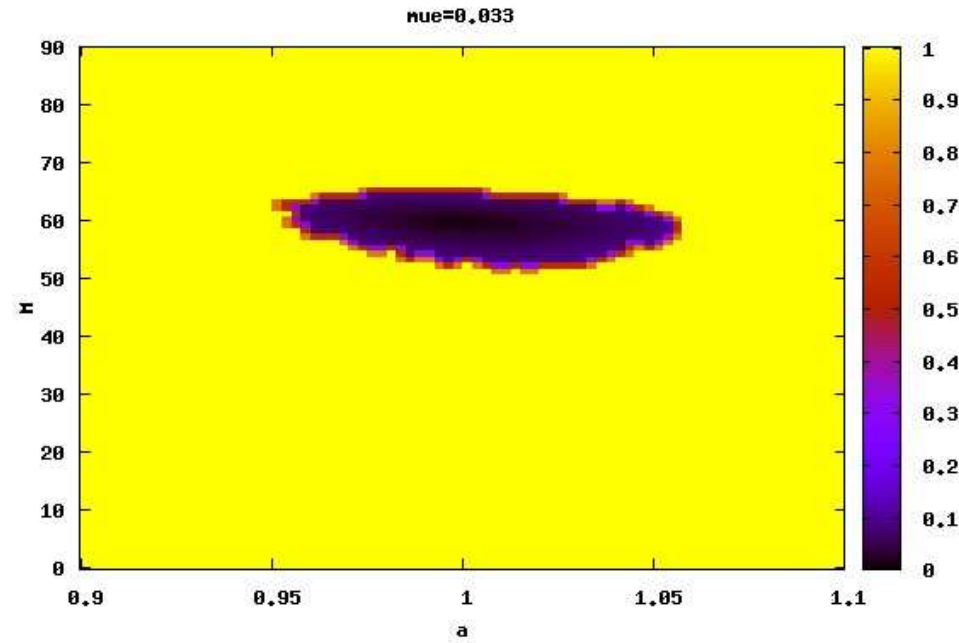


$$\mu=0.032$$

3D-starting
BC

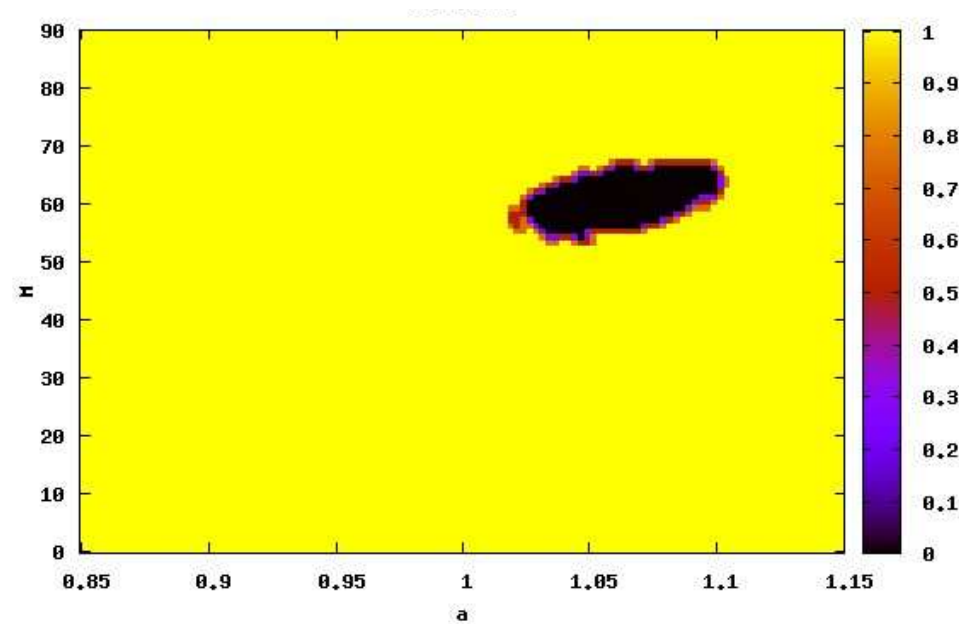


2D-starting
in the AC

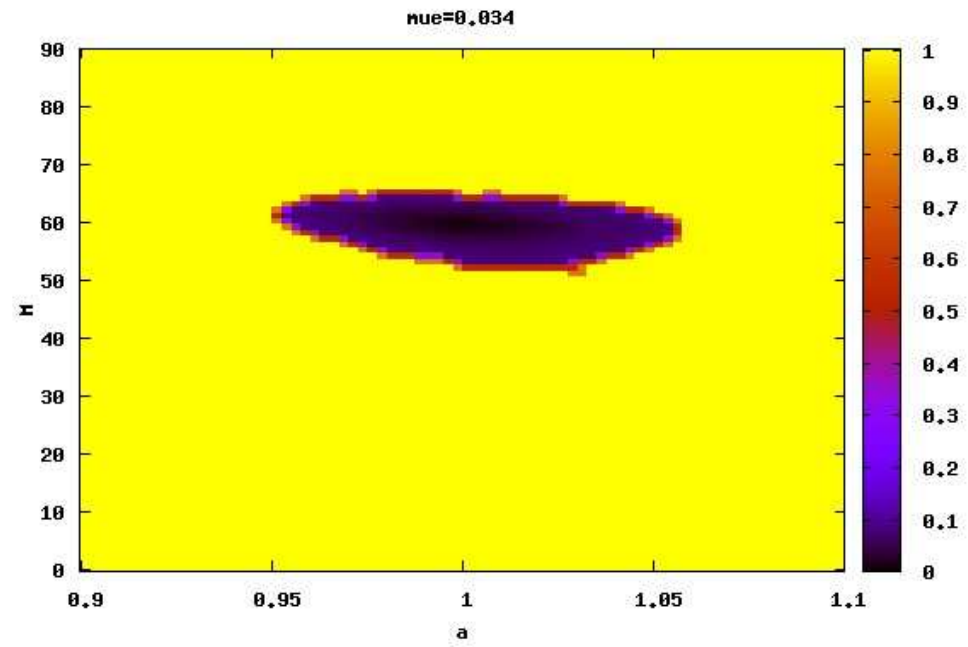


$$\mu=0.033$$

3D-starting
BC

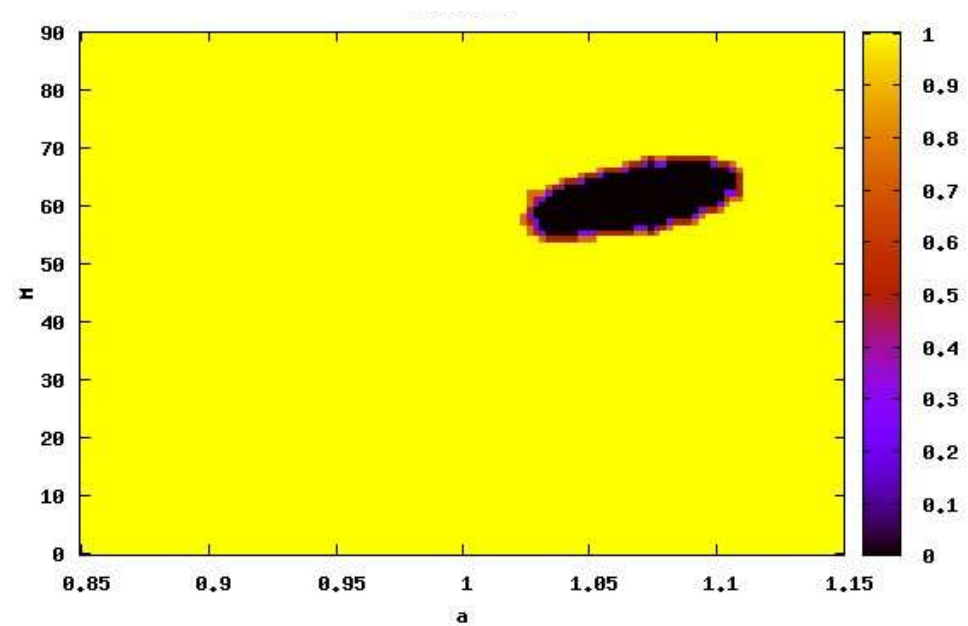


2D-starting
in the AC

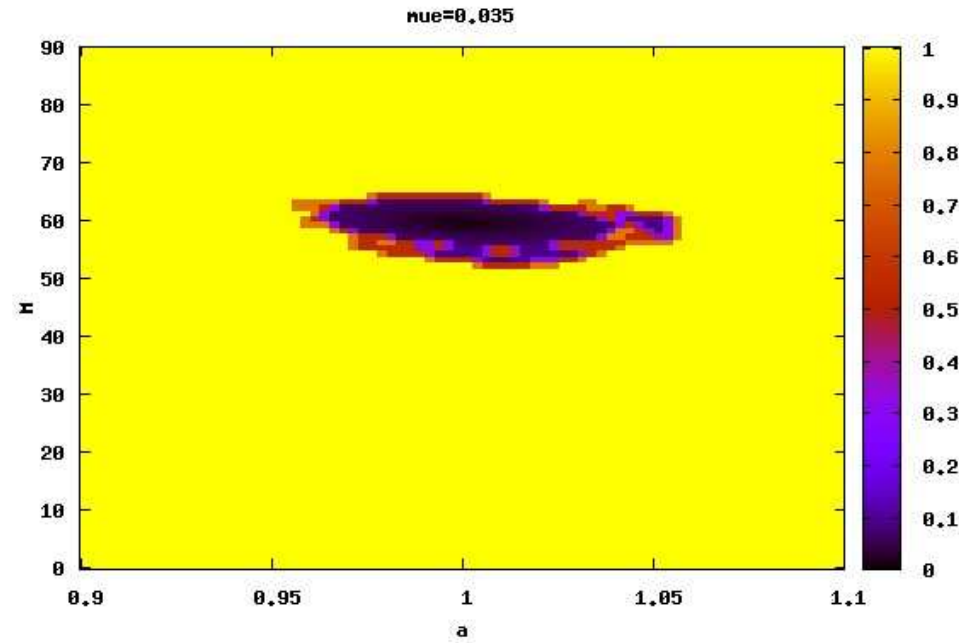


$$\mu=0.034$$

3D-starting
BC

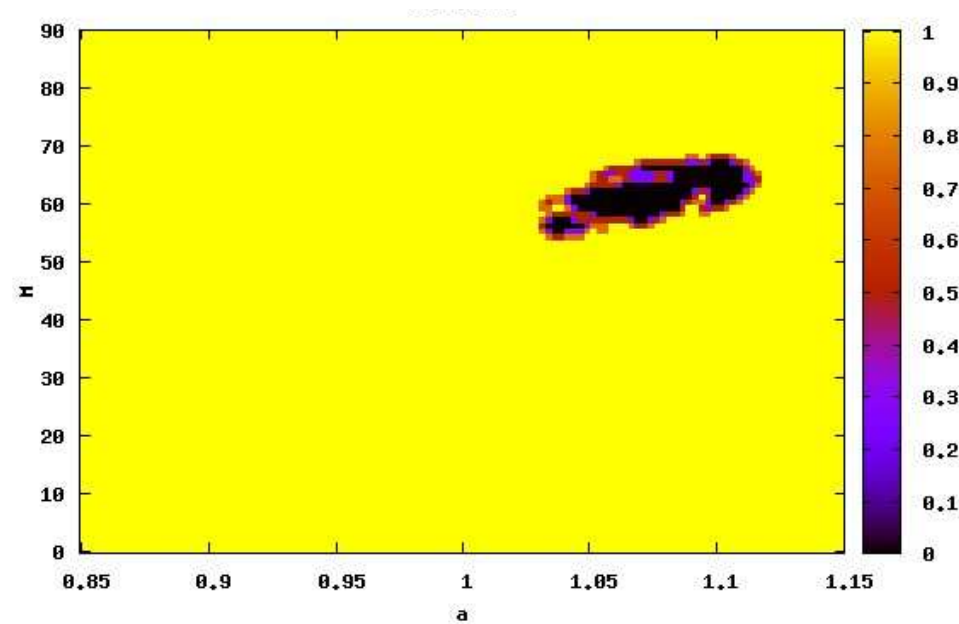


2D-starting
in the AC

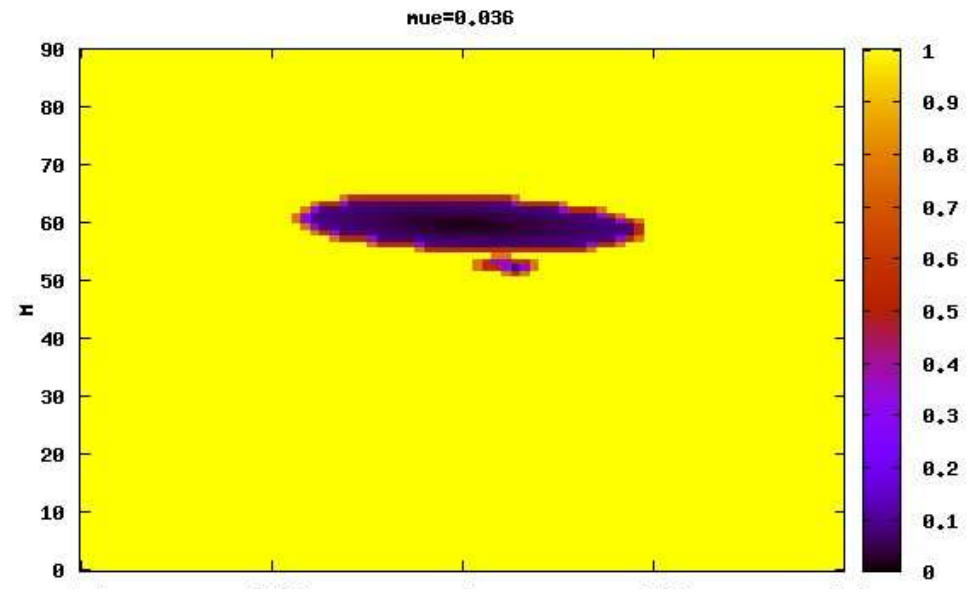


$\mu=0.035$

3D-starting
BC

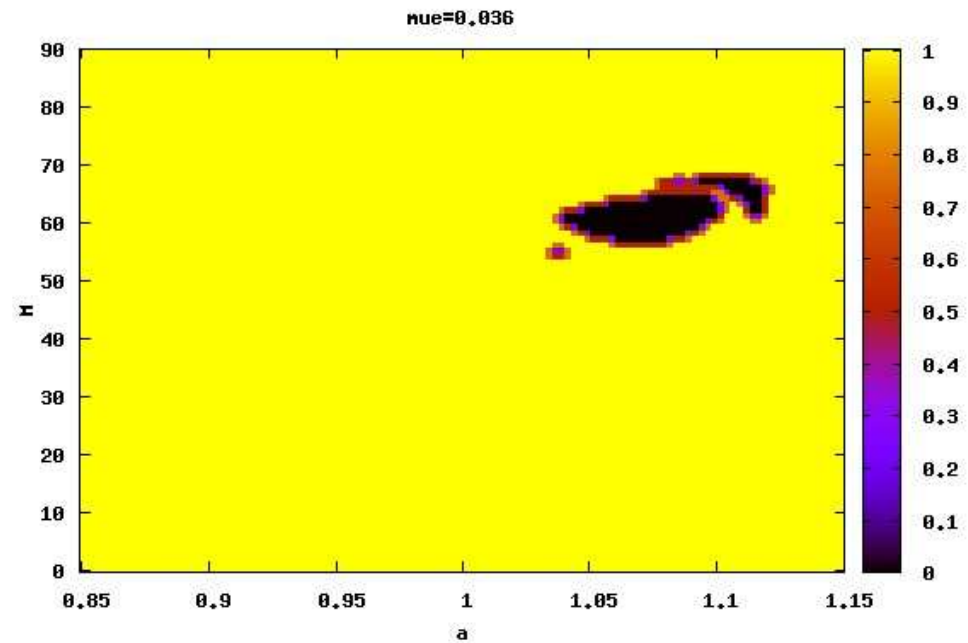


2D-starting
in the AC

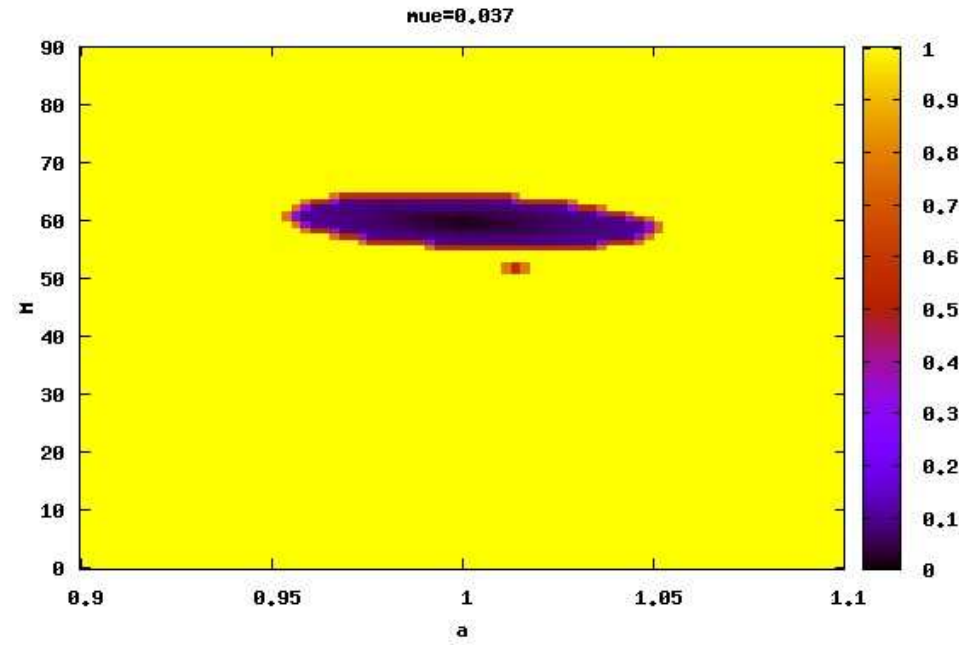


$$\mu=0.036$$

3D-starting
BC

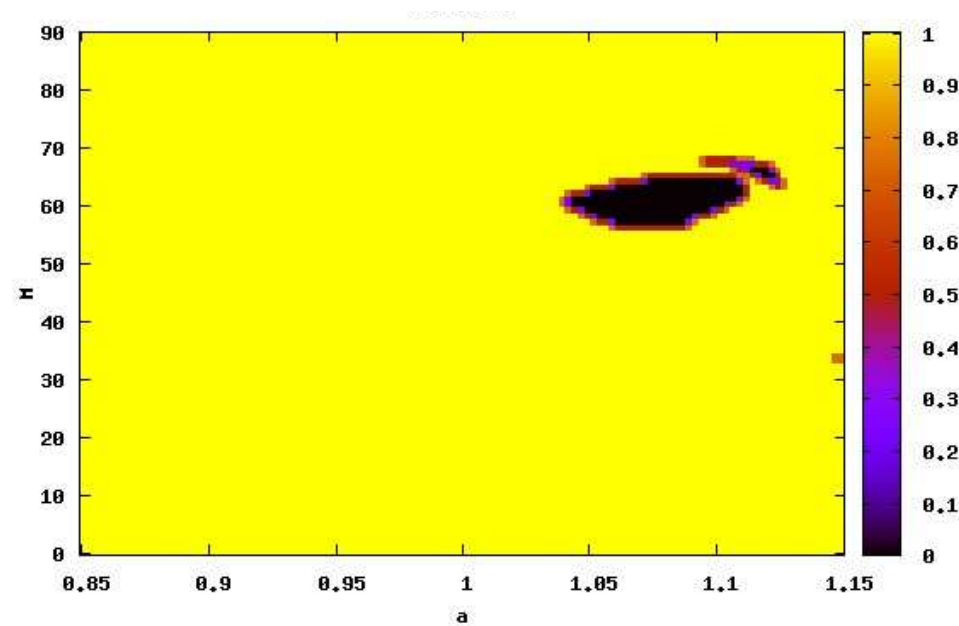


2D-starting
in the AC

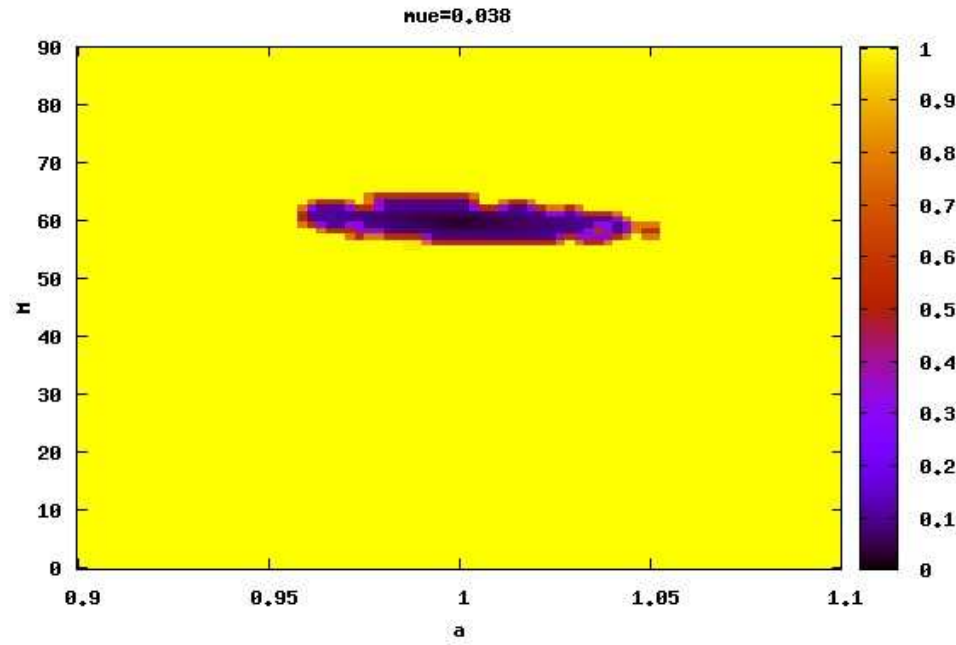


$$\mu=0.037$$

3D-starting
BC

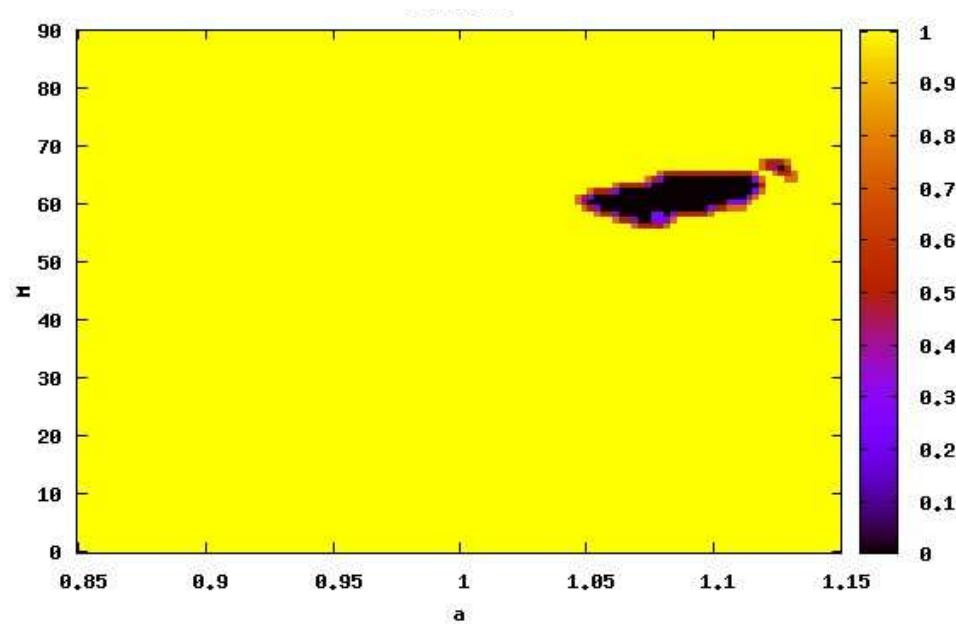


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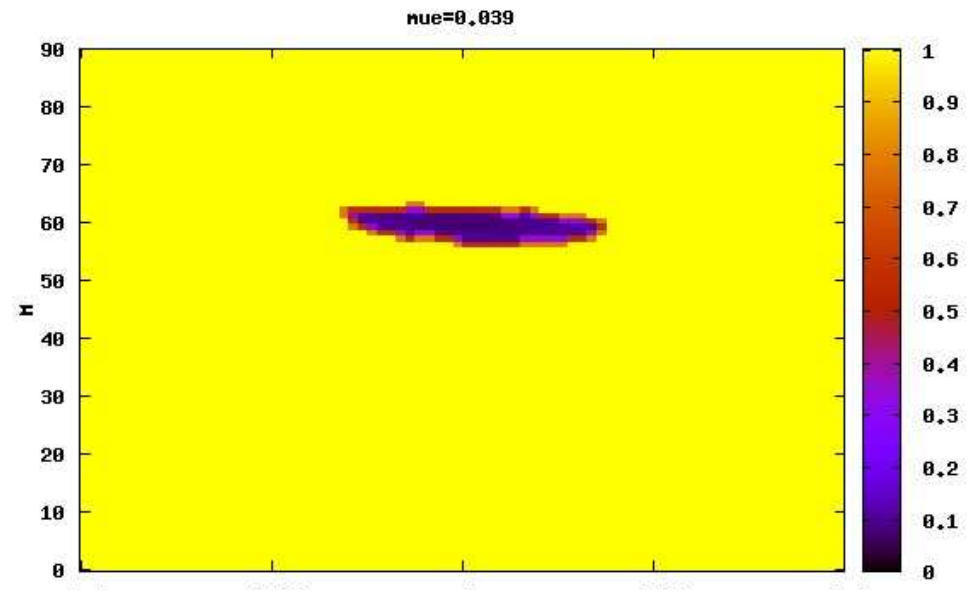


$$\mu=0.038$$

3D-starting
BC

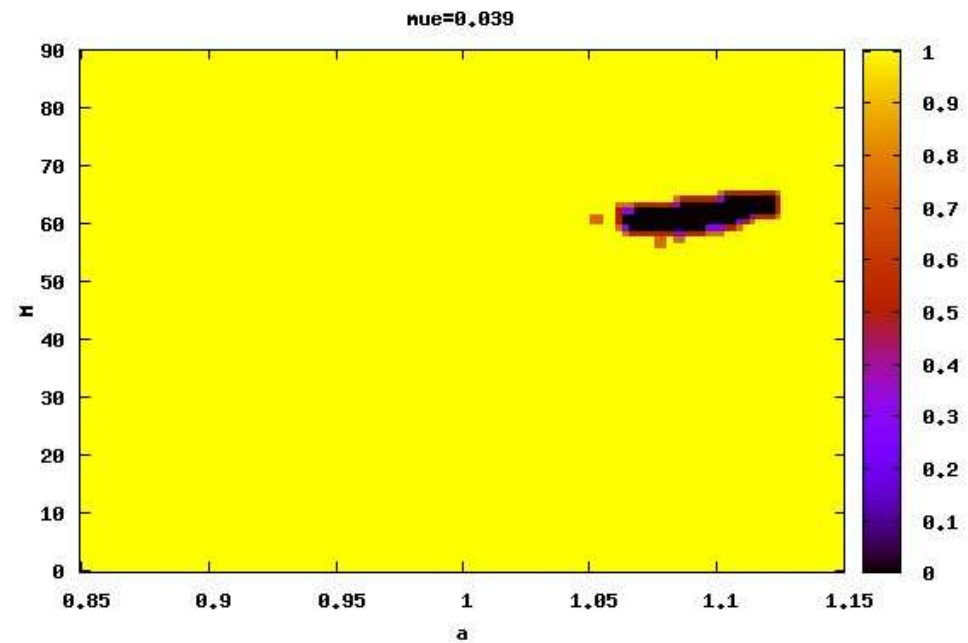


2D-starting
in the AC

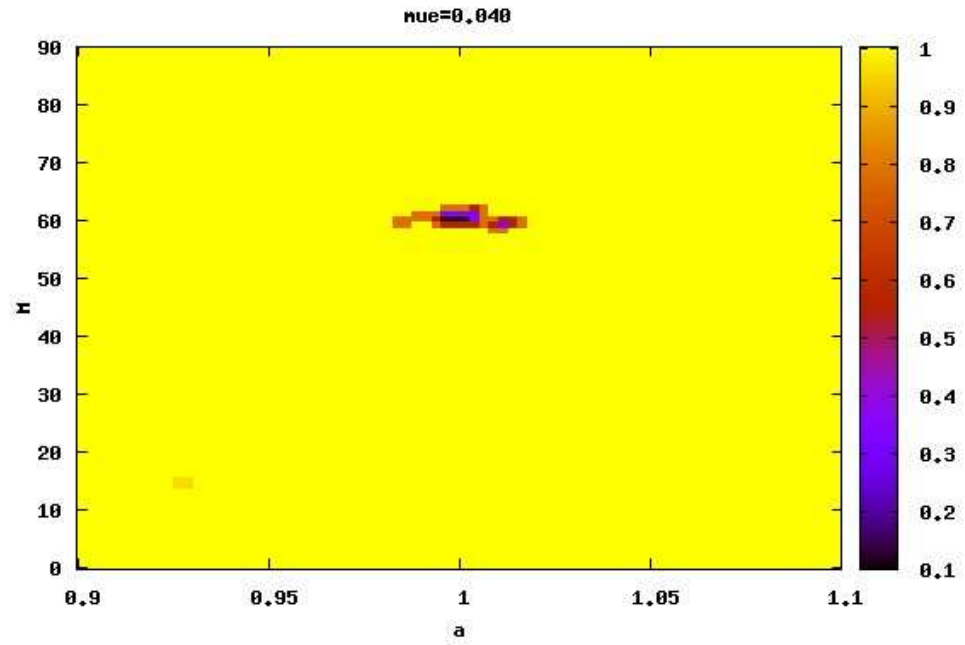


$$\mu=0.039$$

3D-starting
BC

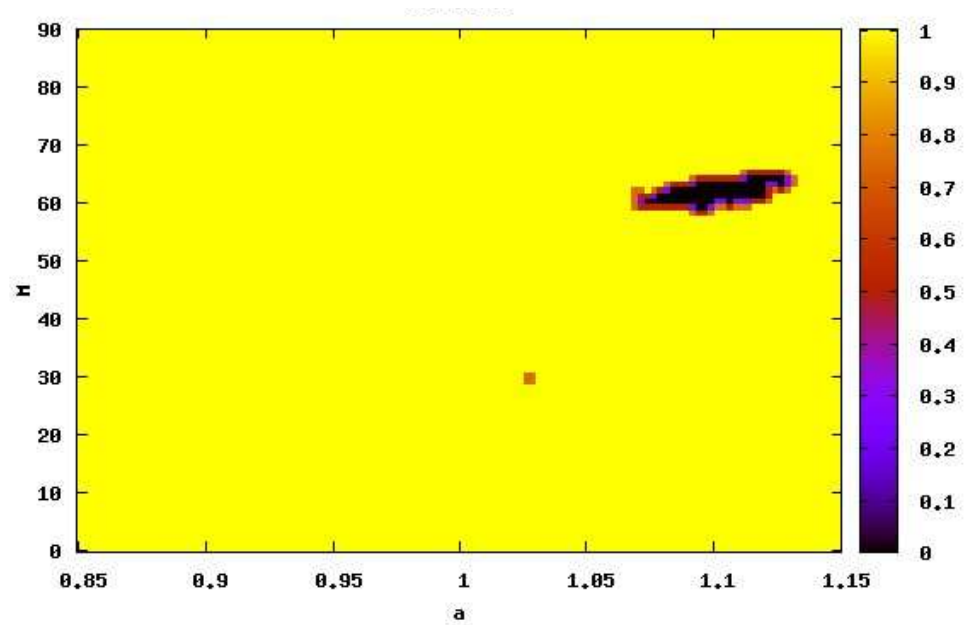


2D-starting
in the AC

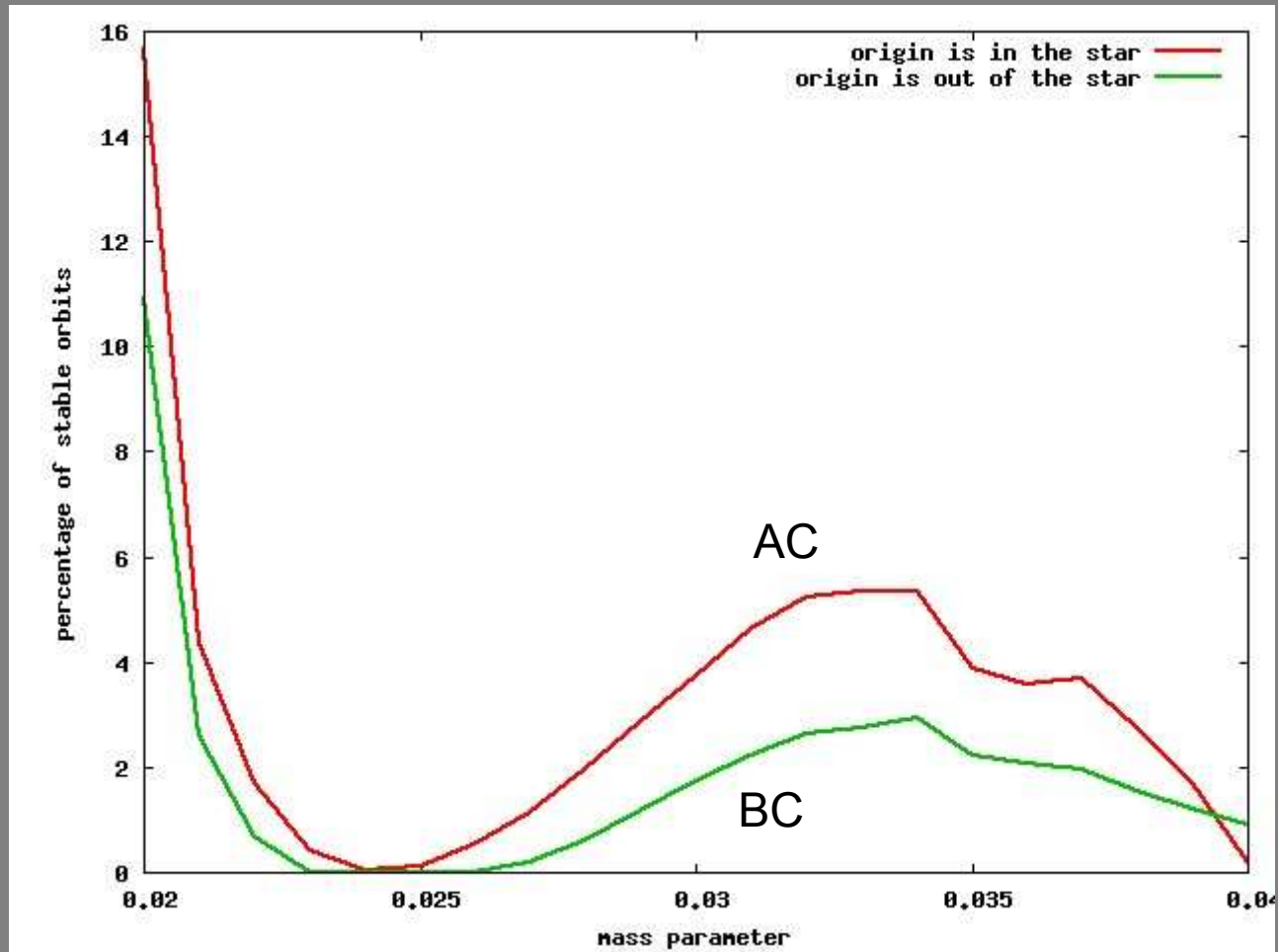


$\mu=0.040$

3D-starting
BC

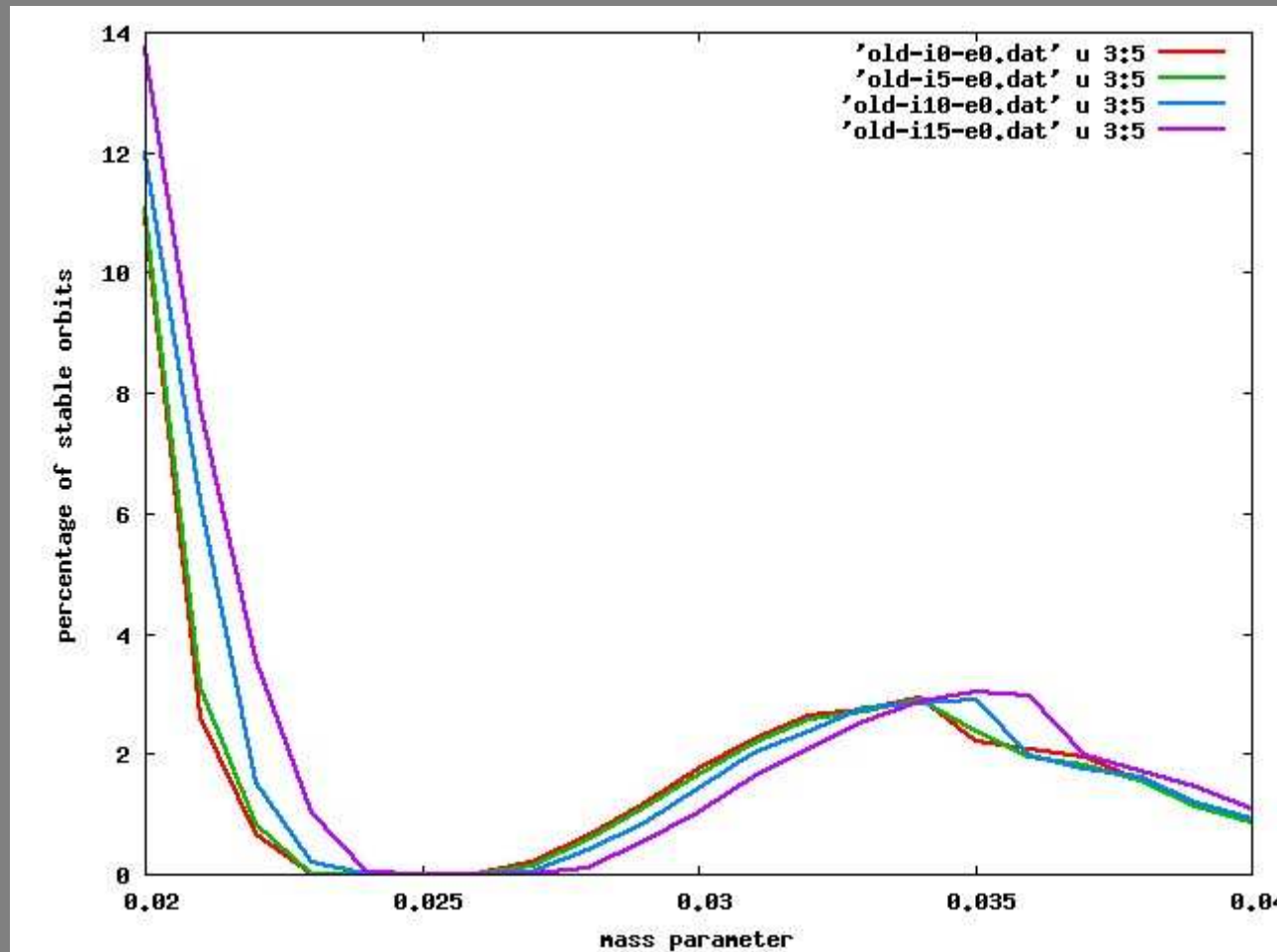


Stable region around L_4 for different mass ratios μ



Integration time = 1000 periods

Starting in the Barycenter



Some tests for 1Myrs

Barycentric shift = initial condition problem

1) AC $\dot{\vec{r}}_2, \dot{\vec{r}}_2'$

2) BC

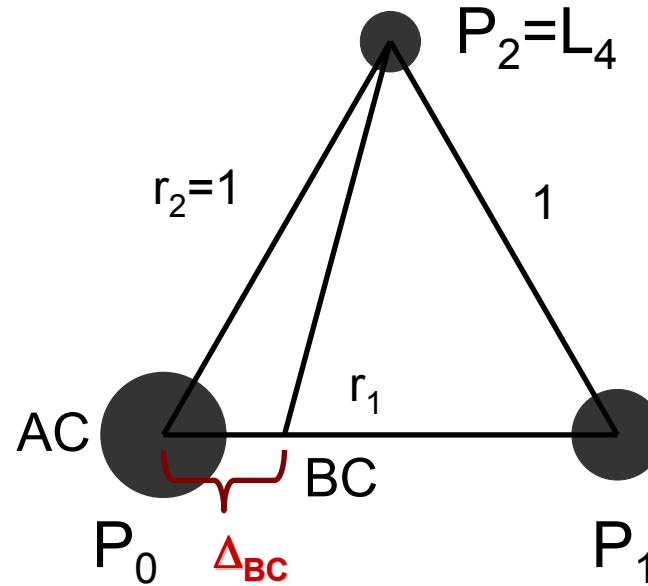
$$\vec{r}_2 = m_1 \cdot \vec{r}_1 + r_2'$$

$$\dot{\vec{r}}_2 = m_1 \cdot \dot{\vec{r}}_1 + \dot{r}_2'$$

$$\dot{x}_2 = m_1 \cdot \dot{x}_1 + \dot{x}'_2$$

$$\dot{y}_2 = m_1 \cdot \dot{y}_1 + \dot{y}'_2$$

$$\dot{z}_2 = 0$$



Barycentric shift

1) AC $\vec{r}'_2, \dot{\vec{r}}'_2$

2) BC

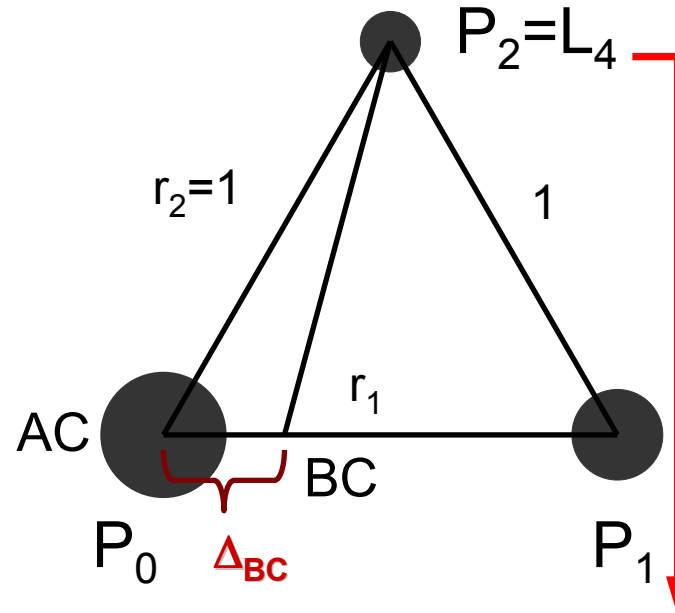
$$\vec{r}_2 = m_1 \cdot \vec{r}_1 + r'_2$$

$$\dot{\vec{r}}_2 = m_1 \cdot \dot{\vec{r}}_1 + \dot{r}'_2$$

$$\dot{x}_2 = m_1 \cdot \dot{x}_1 + \dot{x}'_2$$

$$\dot{y}_2 = m_1 \cdot \dot{y}_1 + \dot{y}'_2$$

$$\dot{z}_2 = 0$$



BC $x = \frac{1}{2} - r_1, y = \pm \frac{\sqrt{3}}{2}$

AC $r_1 = 1$

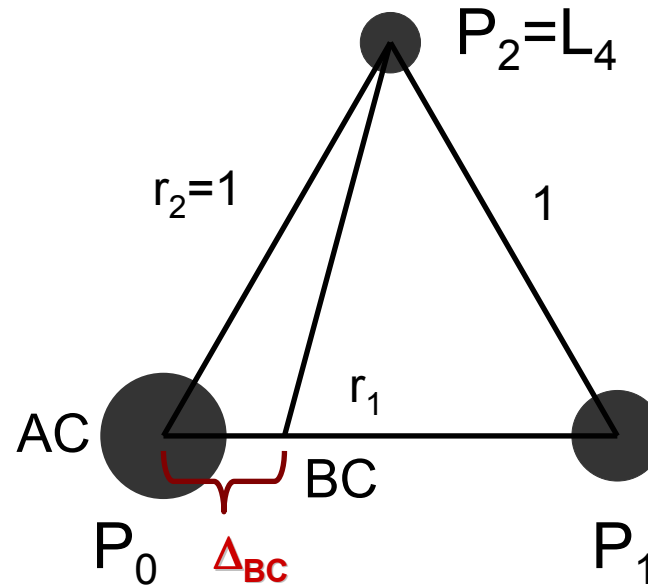
Barycentric shift

1) AC $\dot{\vec{r}}_2, \dot{\vec{r}}_2'$

2) BC

$$\vec{r}_2 = m_1 \cdot \vec{r}_1 + r_2'$$

$$\dot{\vec{r}}_2 = m_1 \cdot \dot{\vec{r}}_1 + \dot{r}_2'$$



Example: Jupitermass

$$\dot{x}_2 = m_1 \cdot \dot{x}_1 + \dot{x}'_2 = 0.001 \cdot 0 + (-0.866025)$$

$$\dot{y}_2 = m_1 \cdot \dot{y}_1 + \dot{y}'_2 = 0.001 \cdot 1 + 0.5 = 0.501$$

$$\dot{z}_2 = 0$$

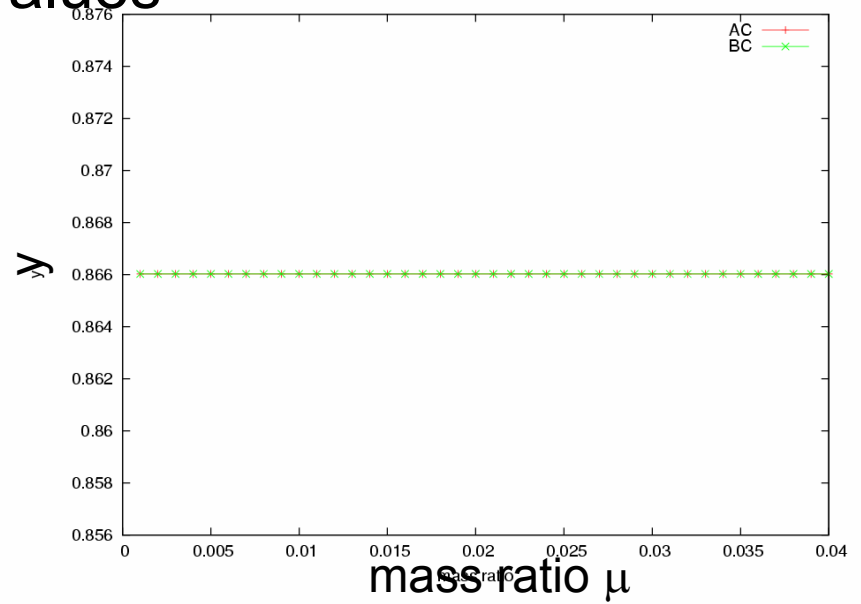
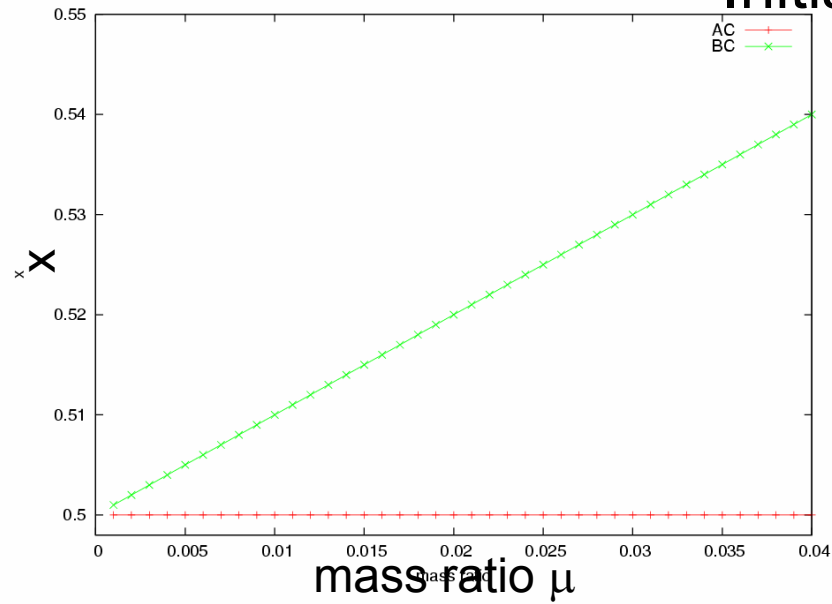
BC $x = \frac{1}{2} - r_1, y = \pm \frac{\sqrt{3}}{2}$

AC $r_1 = 1$

Initial values

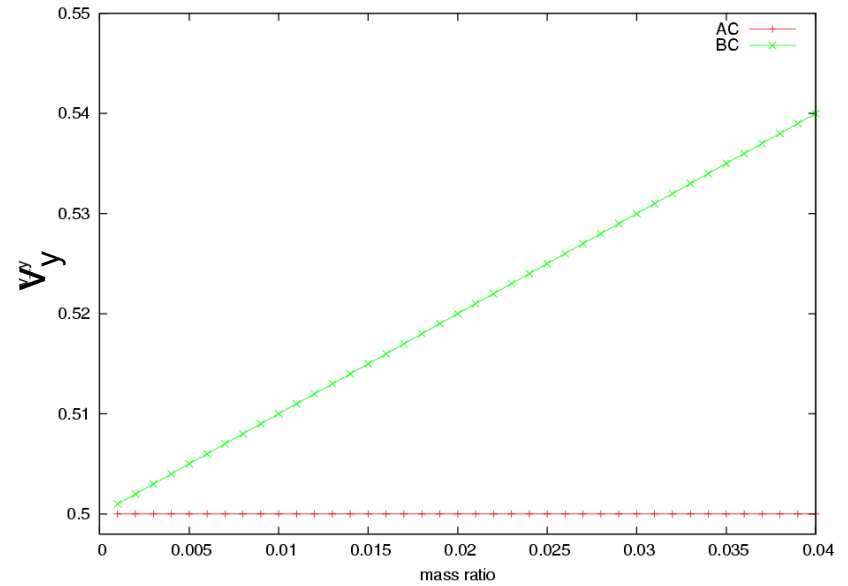
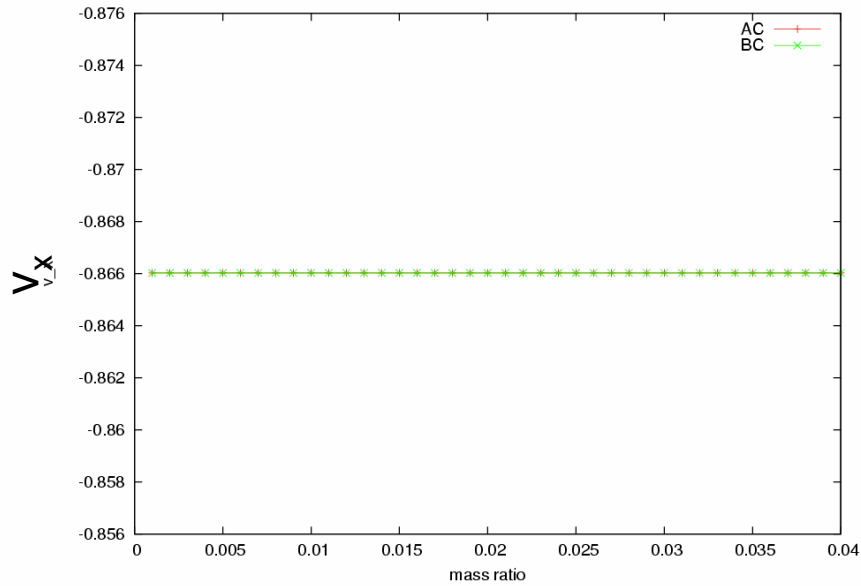
The values changes with the mass!

Initial values

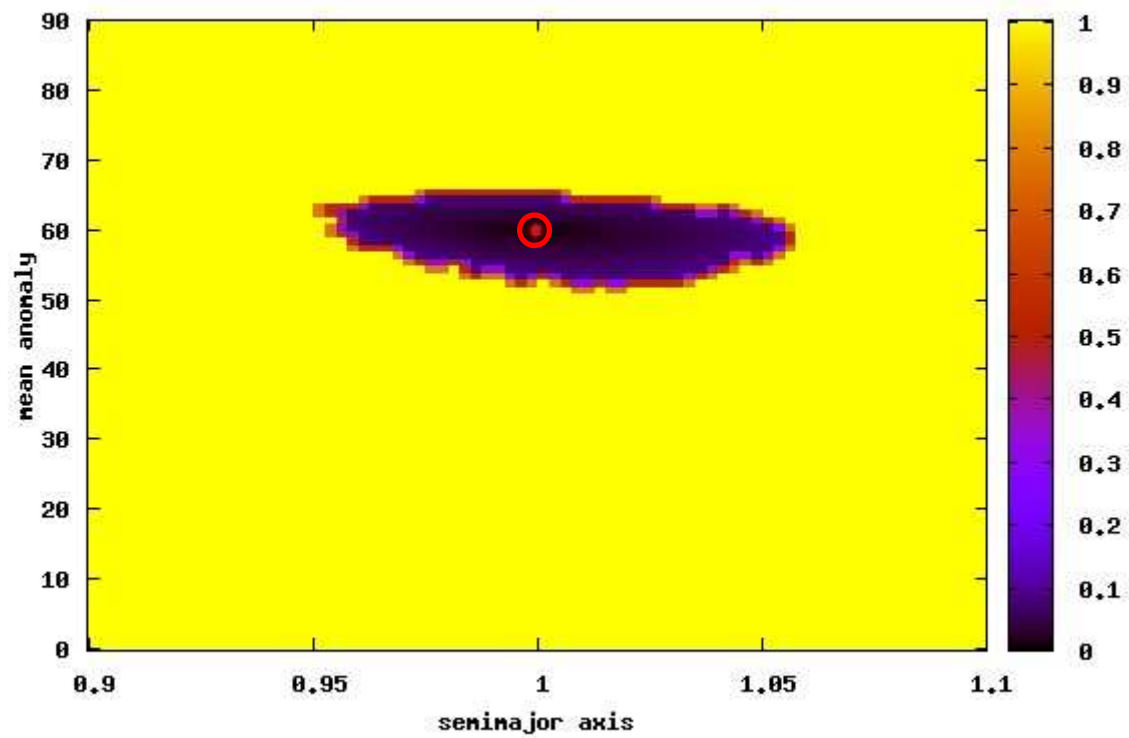


The values changes with the mass!

velocity

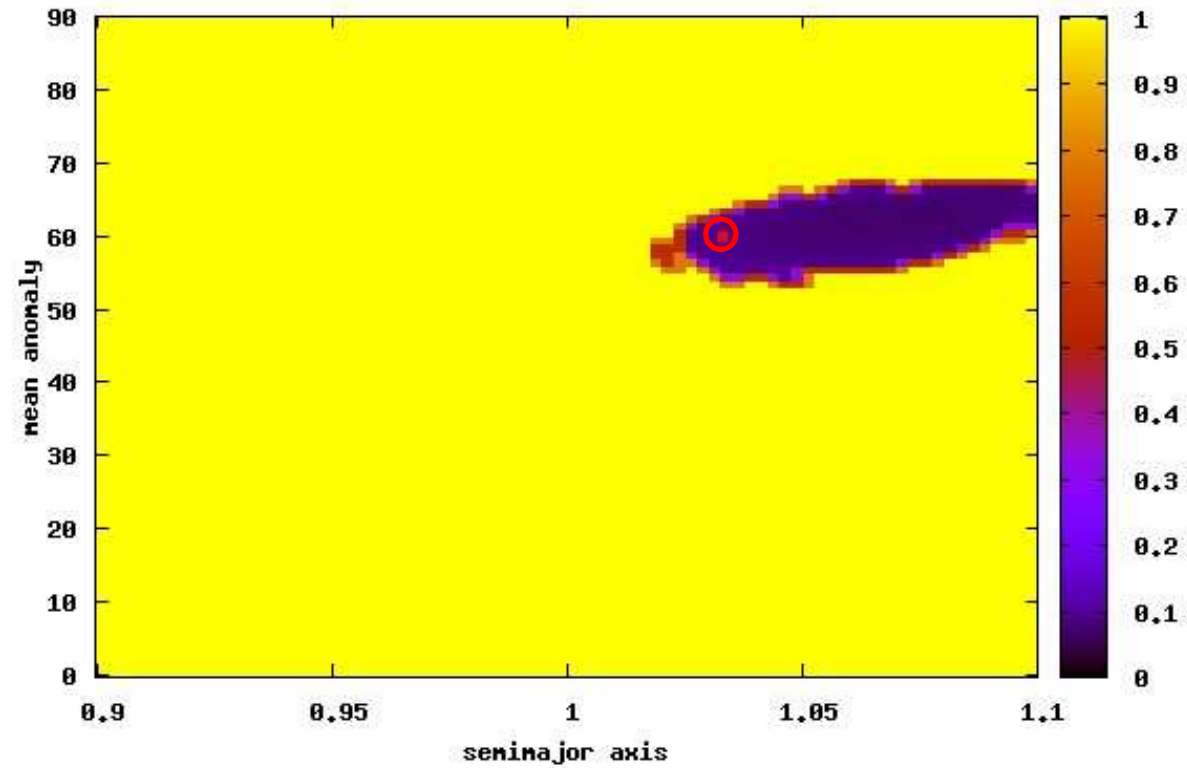


Lagrange point L_4 (AC)



Mass ratio $\mu=0.033$

Possible Lagrange point L_4 (BC)



Mass ratio $\mu=0.033$

Conclusions

- .) Barycentric shift is an initial condition problem
- .) The investigations showed that the initial conditions for AC and BC have large variations, for large mass ratios μ
- .) The differences are expressed at x and v_y
- .) The difference grow with the mass ratio.
- .) The stable regions shifted out (BC) and the stability limit at 0.04 also shifted out

Thank you for your attention!



Menelaus.

Paris.

Diomedes.

Odiseus.

Nestor.

Achilles.

Agamemnon.