The Zeeman-Effect
Some Basics

- **Quantum numbers ($qn$)**

  - $n = 1, 2, 3, \ldots$ main qn (energy states -shell)
  - $l = 0, 1, \ldots (n-1)$ orbital angular momentum (s, p, d, \ldots-subshell)
  - $m_l = -l, \ldots, +l$ magnetic qn, projection of $l$ (energy shift)
  - $m_s = \pm \frac{1}{2}$ spin angular momentum
  - $j = \sqrt{l^2 - s}$ total angular momentum (if coupled, interactions of spin-orbit dominate over $B$)
Some Basics

- \( l = 0, m_l = 0 \)
- \( l = 1, m_l = 0, 1 \)
- \( l = 2, m_l = 0, 1, 2 \)
Some Basics

- **more than 1 e⁻**

\[
L = \sum l_i \quad (S, P, D, \ldots)
\]
\[
S = \sum s_i
\]
\[
J = \sum j_i \quad (\text{with } m_j)
\]

Eigenvalues:
\[
L = \hbar [l (l + 1)]^{\frac{1}{2}}
\]
\[
S = \hbar [s (s + 1)]^{\frac{1}{2}}
\]
\[
J = \hbar [j (j + 1)]^{\frac{1}{2}}
\]

\[
n^{2S+1} L_J
\]
Zeeman Effect

• discovered 1896 different splitting patterns in different spectral lines

• splitted lines are polarized

• splitting proportional to magnetic field
  \[ \Delta \lambda \sim g_L \lambda^2 B \]

• Interaction of atoms with external magnetic field due to magnetic moment \( m_j \)
Zeeman Effect

• normal Zeeman effect:
  \[ S = 0, \quad L = J \neq 0 \]

• split in 3 components
  components with \( \Delta m_J = 0 \) (\( \pi \))
  components with \( \Delta m_J = -1 \) and +1 (\( \pm \sigma \))

• 2 possibilities:
  • \( B \) parallel to line of sight
    \( \Rightarrow \) two \( \sigma \) circular polarized
  • \( B \) perpendicular to line of sight
    \( \Rightarrow \) two \( \sigma \) polarised vertical to field
    one \( \pi \) polarised horizontal to field
Zeeman Effect

- interaction with external field

- magnetic momentum of Atom

\[ V = -\vec{\mu} \cdot \vec{B} \]

\[ \vec{\mu}_J = g_J \frac{e_0}{2m_e} \vec{j} \]

\[ g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \]

- \( J \) and \( \mu_J \) antiparallel
  since \( S=0 \) and \( L=J \) \( \Rightarrow \) \( g_J = 1 \)
  \( \Rightarrow \) only 3 values of \( \Delta m \) possible:
  0, ± 1 \( \Rightarrow \) \( \pi \), ± \( \sigma \)

\[ \mu_B = \frac{(e\hbar)}{(2m_e)} \quad \text{Bohr magneton} \]

\[ V_{m_J} = -\mu_{J,z}B_0 = +m_J g_J \mu_B B_0 \]

\( \Rightarrow \) \( V = m_J \mu_B B_0 \)
Zeeman Effect

Selection rules:

- \( \Delta L = \pm 1 \)
- \( \Delta M = 0, \pm 1 \)

fine-structure depending on \( n, j \)
Fully Split Titanium Lines at 2.2\(\mu\)m

Rüedi et al. 1998
Anormal Zeeman Effect

- Anormal Zeeman effect (LS – coupling):
  \( S \neq 0, L \neq 0 \) ➞ usual case

\[
\vec{j} = \vec{l} + \vec{s}
\]

\[
j = 1 + 1/2 = 3/2 \\
j = 1 - 1/2 = 1/2
\]

- \( J \) and \( \mu_J \) not antiparallel

\( g_J \neq 1 \)

\( \Delta E = g_J \Delta m_J \mu_J B \)

more different energy levels ➞ more components possible

\[
\vec{\mu}_L = -g_L \frac{\mu_B}{\hbar} \vec{L}
\]

\[
\vec{\mu}_S = -g_S \frac{\mu_B}{\hbar} \vec{S}
\]

\[
\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S
\]
Anormal Zeeman Effect

Zeeman components for sodium D lines
green: pi components, red & blue: sigma components

\[ n^{2S+1}L_J \]

 JDJL-2006
Paschen-Back Effect

- for most transitions and $B < 50000 \text{ G}$ ($5 \text{ T} = \text{ upper limit for main sequence stars}$) lines are in Zeeman regime

- if $|\vec{B}_0| > |\vec{B}_f|$ \quad $L$ and $S$ decouple $\Rightarrow$ Paschen-Back effect

\[
V_{m_S,m_L} = (m_L + 2m_S)\mu_B B_0
\]
\[
\Delta E = (\Delta m_L + 2\Delta m_S)\mu_B B_0
\]
Paschen-Back Effect

weak field

strong field

$\Delta m_L = 0, \pm 1$

$\Delta m_S = 0$
Paschen-Back Effect

- Fine structure splitting varies a lot in atoms, so a few lines may be in Paschen-Back regime at much smaller B value than others.

- Zeeman splitting for sodium $D_1$ and $D_2$ in a field of 3 T

- Splitting for Li: 0.3 cm$^{-1}$

- 3 T is a strong field for Li but a weak field for Na.
  Na shows Zeeman splitting
  Li would show Paschen-Back splitting
Hyperfine structure

- fields of $B_0 \geq 0.1$ T often cause splitting
- magnetic momentum of a proton due to its spin:

$$\mu_I = \frac{e_0}{2m_p} I$$

$I$ ... spin of nucleus

$F = J + I$  spin coupling

$m_F = -F, F-1, ..., +F$
Hyperfine structure

- for stronger fields \(\rightarrow\) lines in the Paschen-Back regime
Hanle effect

- classical explanation:
  mercury atoms in absorption-cell – polarized light excites $e^-$
damped oscillation until $e^-$ is ground state (lifetime of upper state) –
emits polarised light (fluorescence resonance) – depending on
observing angle light is not always visible – if external $B_0$ present

$\Rightarrow$ intensity of fluorescence resonance changes (= Hanle effect)
Hanle effect

- quantum mechanical explanation: level-crossing spectroscopy
- a Zeeman sublevel (energy decreases with increasing $B_0$) may cross with another magnetic sublevel from a lower state (energy increases with increasing magnetic field)
  - if atoms excited coherently at fields corresponding to crossing points, changes of fluorescence intensity
  - “non-zero field level crossing” occurs in atoms that already show hyperfine structure without an external $B_0$. 

![Diagram of energy levels and magnetic field effects](image)
Hanle effect

• unpolarised light is scattered through ~90° scattered beam is linearly polarised perpendicular to scattering plane

• if scattering atom is in magnetic field $J$ vector precesses with period of $\frac{4\pi mc}{eB}$ (Lamor frequency)

• if this frequency is comparable to the decay lifetime of upper state, the polarisation plane of re-emitted photon will be rotated from non-magnetic case

Intensity of the fluorescent light proportional to $B \implies$ allows detection of weak fields (<10G) in situations with large-angle scattering
Applications