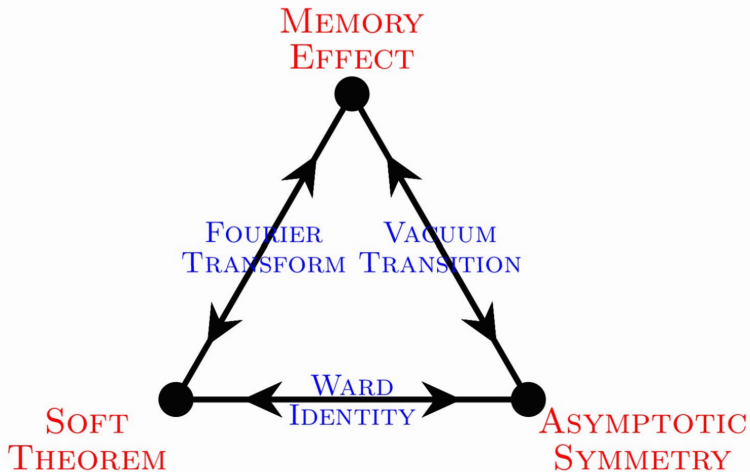


Asymptotic charges for spin-1 and spin-2 fields at the critical sets of null infinity (arxiv:2112.03890)

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February 18, 2022

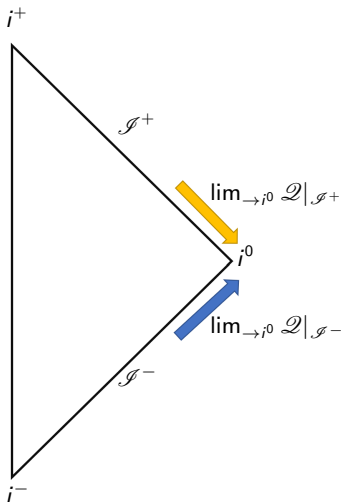


Asymptotic symmetries

Null Infinity

Spatial infinity

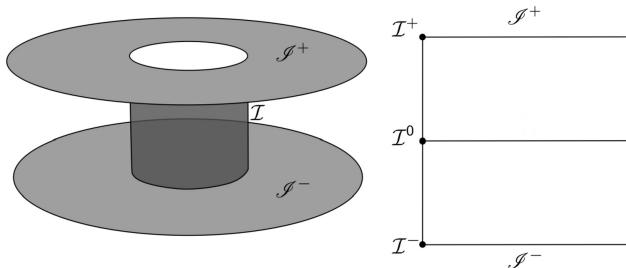
R. Geroch (1972), A.
Ashtekar, R.O. Hansen & J.D.
Romano (1978,1980), M.
Henneaux & C. Troessaert
(1992,2018)



Matching Problem

$$\mathcal{Q}|_{\mathcal{I}^+} = ? \mathcal{Q}|_{\mathcal{I}^-}$$

M. Campiglia & R. Eyheralde
(2017), M. Henneaux & C.
Troessaert (2018), Kartik
Prabhu (2018, 2019, 2021)



**Minkowski
physical metric**

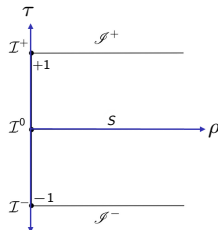
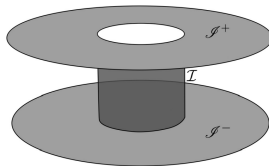
$$\begin{aligned} (t, r, x^i) &\rightarrow (\tau, \rho, x^i) \\ (\tilde{\mathcal{M}}, \tilde{\eta}) &\rightarrow (\bar{\mathcal{M}}, \bar{\eta}) \end{aligned}$$

**Blow up of
spatial infinity**

$$\mathcal{I} \equiv \{p \in \bar{\mathcal{M}} \mid |\tau(p)| < 1, \rho(p) = 0\}$$

$$\mathcal{I}^\pm \equiv \{p \in \bar{\mathcal{M}} \mid \tau(p) = \pm 1, \rho(p) = 0\}$$

$$\mathcal{I}^0 \equiv \{p \in \bar{\mathcal{M}} \mid \tau(p) = 0, \rho(p) = 0\}$$

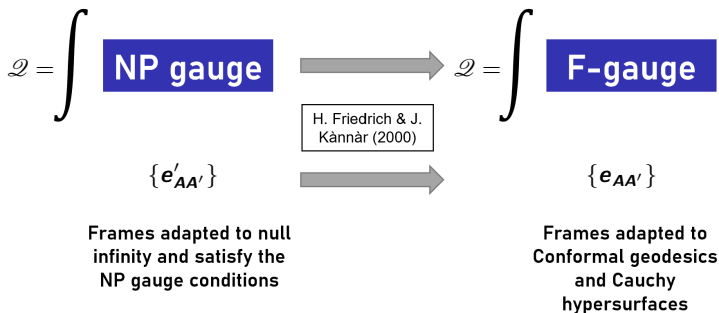


Initial data on the
Cauchy hypersurface

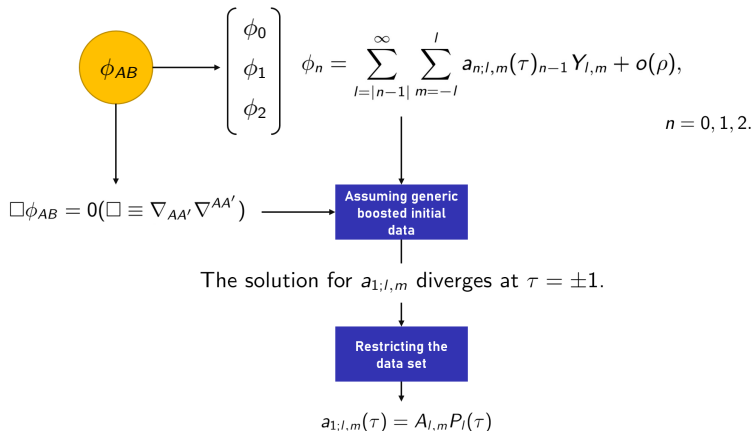
Juan A. Valiente-Kroon & Edgar Gasperin
(2020)

H. Friedrich & J. K annr (2000)

- 1 The F-gauge and the NP gauge
- 2 The asymptotic charges for spin-1 fields
- 3 The asymptotic charges for spin-2 fields
- 4 Main result



The asymptotic charges for spin-1 fields



The asymptotic charges for spin-1 fields

“Supertranslation” charges



Smooth functions on \mathbb{S}^2

If $\lambda = Y_{l,m}$



$$\mathcal{Q}|_{\mathcal{I}^\pm} \propto \bar{a}_{1;l,m}(\pm 1)$$

$$\mathcal{Q}|_{\mathcal{I}^+} = (-1)^l \mathcal{Q}|_{\mathcal{I}^-}$$

Given data that
satisfy the
regularity
conditions

The asymptotic charges for the spin-2 fields

A yellow circle containing the symbol ψ_{ABCD} is connected by a vertical line to a horizontal line. From this horizontal line, two arrows point downwards to the left. The first arrow points to the equation $\square\psi_{ABCD} = 0$. The second arrow points to the asymptotic expansion equation:
$$\psi_n = \sum_{l=|2-n|}^{\infty} \sum_{m=-l}^l a_{n;l,m}(\tau) 2^{-n} Y_{l,m} + o(\rho), \quad n = 0, \dots, 4.$$
 Below this equation, a vertical arrow points down to a blue rectangular box containing the text "Given data that satisfy the regularity conditions".

The charges are expressed in terms of the freely specifiable data and there is a natural correspondence between charges at \mathcal{I}^+ and \mathcal{I}^- .

- The main result of our work can be summarised in the following:

Theorem

For generic boosted initial data for the Maxwell and spin-2 fields, the asymptotic charges at \mathcal{I}^\pm are well defined if and only if the freely specifiable data on the initial hypersurface \mathcal{S}_* satisfy certain regularity conditions. Moreover, for a given harmonic $Y_{l,m}$ the charges at \mathcal{I}^+ and \mathcal{I}^- are expressible in terms of the same piece of freely specifiable data.

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- Next: The asymptotic charges at the critical sets in full gravity setting (with Kartik Prabhu).