Staticity and regularity for spin-2 fields near spatial infinity on flat spacetime

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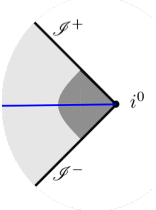
Motivation: the problem at spatial infinity



Peeling and the problem at spatial infinity i^0

- Penrose's conformal approach. $(\tilde{\mathcal{M}}, \tilde{\boldsymbol{g}})$ = physical spacetime and $(\mathcal{M}, \boldsymbol{g})$ = "unphysical" spacetime
$$\boldsymbol{g} = \Theta^2 \tilde{\boldsymbol{g}}$$
- Conformal structure degenerates at spatial infinity i^0 (Penrose, Friedrich).
- Classical Penrose's Peeling theorem ($n = 0, 1, 2, 3, 4$, "NP-gauge")
$$\psi_n = \mathcal{O}(\tilde{r}^{-5+n})$$
- Time-Sym ID $\Longrightarrow$ formal expansions close to i^0 [Friedrich 98, G. & Valiente Kroon 17]

$$\psi_{0,1,2} = \mathcal{O}(\tilde{r}^{-3}\ln(\tilde{r})),\ \psi_3 = \mathcal{O}(\tilde{r}^{-2}),\ \psi_4 = \mathcal{O}(\tilde{r}^{-1})$$

Connection to i^0

- CEFE-logs origin $\rightsquigarrow i^0$.
- Regularity condition at the level of initial data!

Other polyhm. exps

- Winicour, Chrusciel, Duarte et al ...

Spin-2 fields as toy models for the gravitational field

Linearised gravity as a spin-2 field (Penrose)

$(\mathbb{R}^4, \tilde{\eta}_{ab}, \tilde{\nabla})$ physical Minkowski spt

$\tilde{g}_{ab} = \tilde{\eta}_{ab} + \varkappa\, \tilde{\gamma}_{ab} + \mathcal{O}(\varkappa^2)$ γ_{ab}-pert

$\tilde{R}_{abcd} = 2\tilde{\nabla}_{[a}\tilde{\nabla}_{|[c}\tilde{\gamma}_{d]|b]}$ Weyl symm

Metric approach: $\tilde{R}_{ab} = 0 \implies \tilde{\partial}^2 \tilde{g}_{\mu\nu} = \ldots$

Curvature approach: $\tilde{R}_{ab} = 0 \implies \tilde{\nabla}^a \tilde{\phi}_{abcd} = 0.$ $+$ (str eqs)

Physical spin-2 field equation

$$\boxed{\tilde{\nabla}^a \tilde{\phi}_{abcd} = 0}$$ Bianchi

Spin-2 conformally invariance $\boldsymbol{g} = \Theta^2 \tilde{\boldsymbol{g}}$

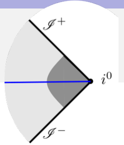
$$\phi_{abcd} = \Theta^{-1}\tilde{\phi}_{abcd} \implies \nabla^a \phi_{abcd} = 0.$$

The Conformal Einstein Field Equations (Friedrich)

Rescaled Weyl tensor

$\phi_{abcd} = \Theta^{-1} C_{abcd}$

F-gauge in Minkowski and the cylinder at i^0



Standard representations

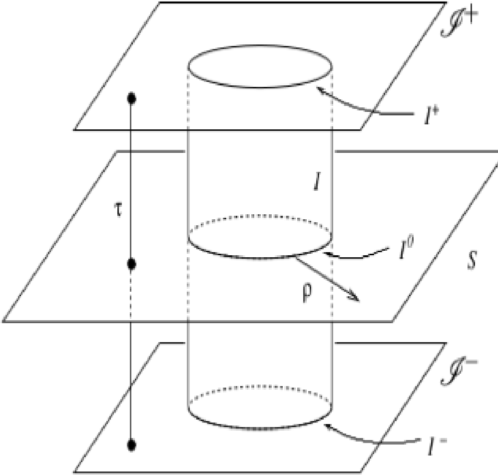
- $i^0 \rightarrow$ point

Cylinder at spatial infinity

- $\boldsymbol{\eta} = \Theta^2 \tilde{\boldsymbol{\eta}}, \quad \Theta = \rho(1-\tau^2)$
- $i^0 \rightarrow$ blown up to $I \approx \mathbb{R} \times \mathbb{S}^2$
- $\mathscr{I}^{\pm} = \{\tau = \pm 1\}$,
  $\mathcal{S} = \{\tau = 0\}$
- $I^{\pm}$: Critical sets, where i^0 "touches" $\mathscr{I}^{\pm}$!

Prop (Valiente Kroon 02)

- Spin-2
  $\boldsymbol{\phi} \sim (\tau \pm 1)^{p \pm 2} \log(\tau \pm 1)$
- Regularity condition (no-log) at ID level:
- $\boldsymbol{D} \cdots \boldsymbol{D} \boldsymbol{B}(i^0) = 0$ on $\mathcal{S}$



A staticity condition

Physical staticity field $\tilde{Z}$

- $(\mathbb{R}^4, \tilde{\boldsymbol{\eta}}, \tilde{\nabla})$ physical Minkowski st.
- $\tilde{\nabla}^a \tilde{\phi}_{abcd} = 0$ physical spin-2 eq.
- $\tilde{\boldsymbol{\xi}} = \boldsymbol{\partial}_{\tilde{t}}$ timelike KV.
- "staticity zero-quantity"

$$\tilde{Z}_{abcd} \equiv \mathcal{L}_{\tilde{\xi}} \tilde{\phi}_{abcd}$$

- Staticity notion:

$$\tilde{\phi} \text{ is static } \iff \tilde{\boldsymbol{Z}} = 0$$

$\tilde{Z}$ propagates

- $\tilde{\nabla}^a \tilde{Z}_{abcd} = 0$
- Symmetric Hyperbolic. Uniqueness. Zero-quantity propagates:
- $\tilde{\boldsymbol{Z}}|_{\mathcal{S}} = 0 \implies \tilde{\boldsymbol{Z}} = 0$

ID-Staticity condition

- ID-physical-staticity cond

$$\boxed{\tilde{Z}_{abcd}|_{\mathcal{S}} = 0}$$

- Translate to the unphysical $(\mathcal{M}, \boldsymbol{g}) \rightsquigarrow i^0$-cylinder.

A staticity condition

Unphysical picture $(\mathcal{M}, \boldsymbol{g} = \Theta^2\tilde{\boldsymbol{g}})$

- $\tilde{\phi}_{abcd} = \Theta^{-1}\phi_{abcd}$

$$\boxed{\nabla^a\phi_{abcd} = 0}$$

- $\tilde{\nabla}_{(a}\tilde{\xi}_{b)} = 0$ KV $\Longrightarrow$ CKV:
- $\xi^a = \tilde{\xi}^a$, $\xi_a = \Theta^{-2}\tilde{\xi}_a$

$$\boxed{\nabla_{(a}\xi_{b)} - \frac{1}{4}g_{ab}\nabla_c\xi^c = 0}$$

- $\boldsymbol{\xi} = \Theta\boldsymbol{\partial}_\tau + 2\rho^2\tau\boldsymbol{\partial}_\rho$
- $Z_{abcd} \equiv \Theta^{-2}\tilde{Z}_{abcd}$

Spinorialisation

$$Z_{ABCD} = \Theta\nabla_{\boldsymbol{\xi}}\phi_{ABCD} + \phi_{ABCD}\nabla_{\boldsymbol{\xi}}\Theta + 2\phi_{Q(ABC}\nabla_{D)Q'}\xi^{QQ'}.$$

- $1+3$ spinor split adapted to $\boldsymbol{\partial}_\tau$
- Use Spin-2 and CKV eqs to remove ders in the $\boldsymbol{\tau}(=\boldsymbol{\ell}+\boldsymbol{n})$ direction.
- $\boxed{Z_{ABCD}|_{\mathcal{S}} = \sqrt{2}\Omega B_{ABCD}}$

Main result

Theorem (G. & Valiente Kroon 21)

A necessary and sufficient condition for a spin-2 field ϕ_{ABCD} over $(\mathcal{M}, \boldsymbol{\eta})$ to be static is that it satisfies the conformally invariant condition

$$B_{ABCD} = 0 \quad \text{on} \quad \mathcal{S}.$$

Corollary (G. & Valiente Kroon 21)

Static initial data for the spin-2 field gives rise to a solution ϕ_{ABCD} that extends analytically to the the critical sets $\mathcal{I}^{\pm}$. In particular, the solution is smooth at $\mathscr{I}^{\pm}$.

Corollary (G. & Valiente Kroon 2021)

Initial data satisfying the regularity condition does not correspond, in general, to static initial data for the spin-2 field ϕ_{ABCD} in a neighbourhood of i^0.