Interferometry of Entangled Quantum States of Light in Curved Space-Time

Thomas B. Mieling

February 23, 2021

Work done in collaboration with Christopher Hilweg.

TM is a recipient of a DOC Fellowship of the Austrian Academy of Sciences.

Motivation

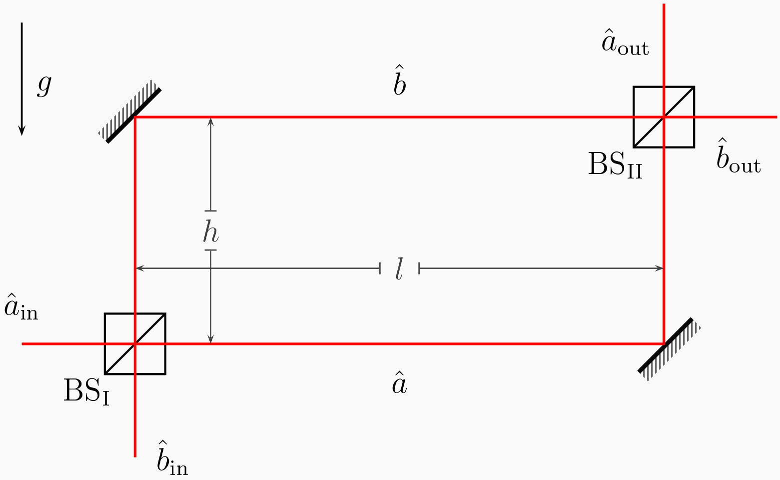
Context: quantum physics in the presence of external gravitational fields

Ongoing efforts to demonstrate gravitationally-induced quantum interference

- Matter-wave interferometry (Colella *et al.* 1975; Kasevich & Chu 1992; Sorrentino *et al.* 2012) – Newtonian gravity suffices
- Optical interferometry (Zych *et al.* 2012; Hilweg *et al.* 2017) – beyond Newtonian desciption, but insensitve to gravity gradients

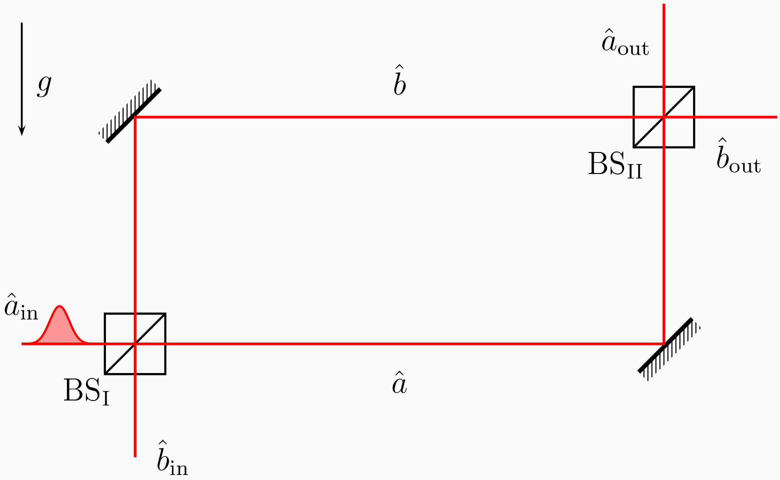
So far: no experimental realisation of quantum interference, where the phase shift is explicable only using general relativity

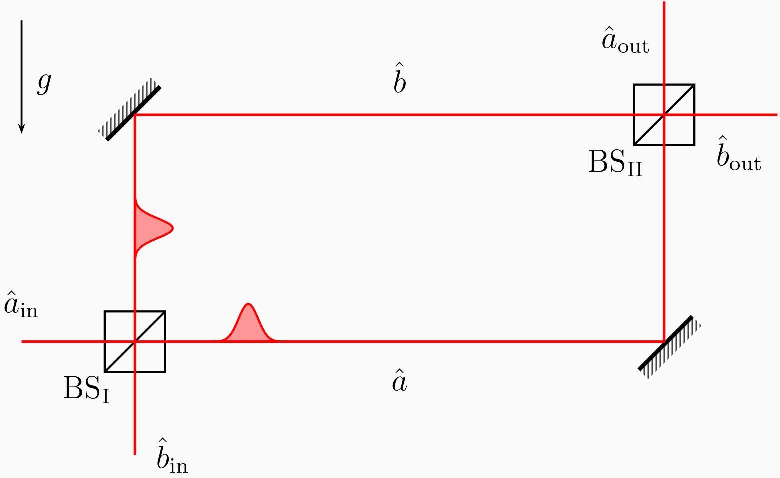
Schematics

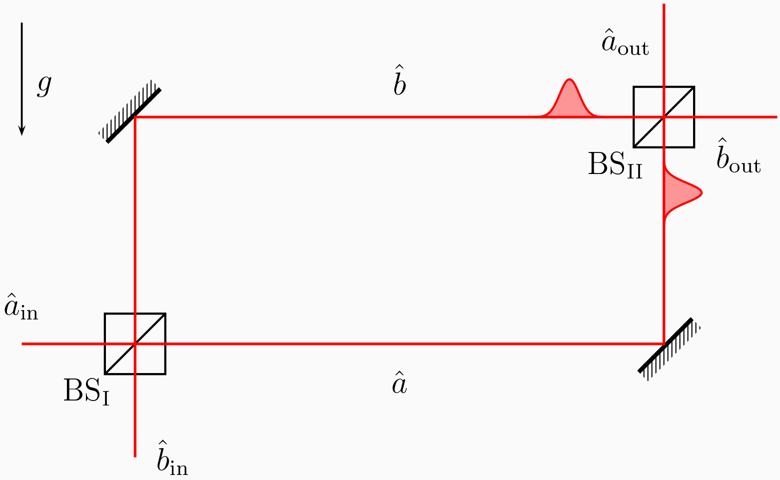


g
$\hat{a}_{\text{out}}$
$\hat{b}$
BS_{II}
$\hat{b}_{\text{out}}$
h
l
$\hat{a}_{\text{in}}$
BS_{I}
$\hat{a}$
$\hat{b}_{\text{in}}$

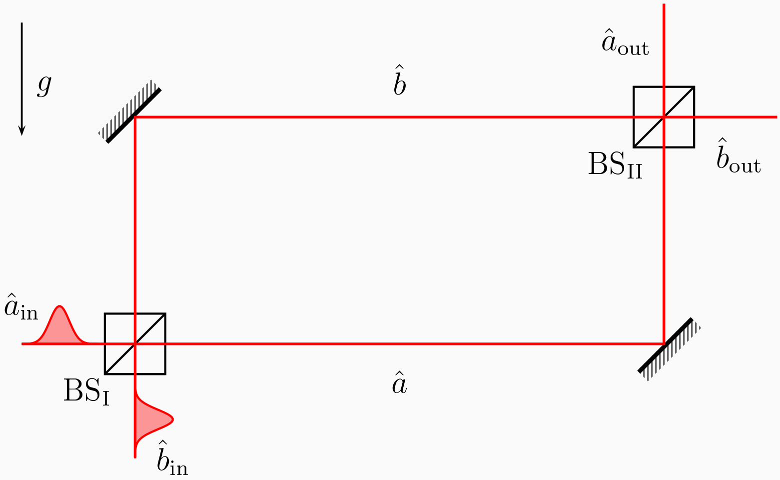
Single Photons

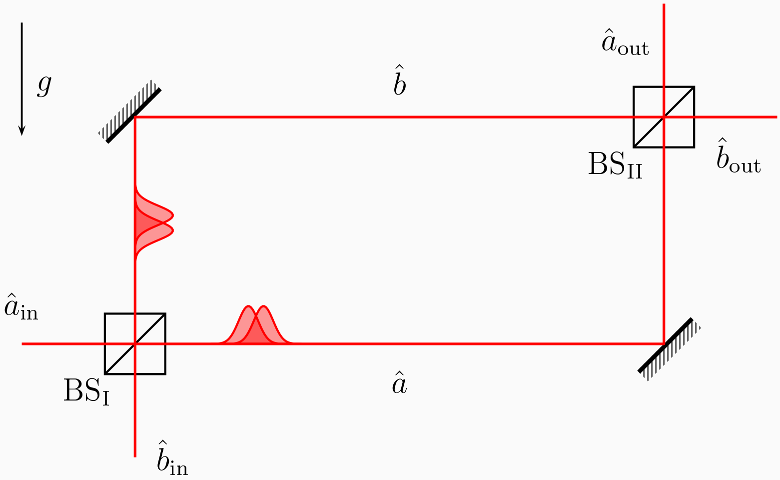


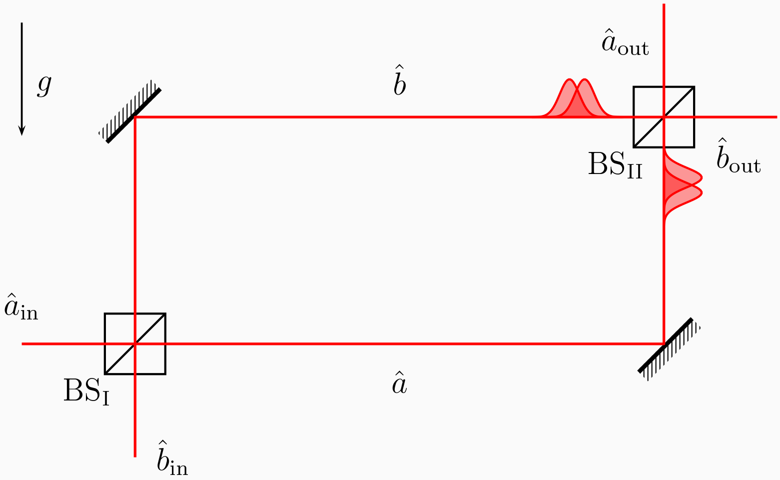

g
$\hat{a}_{\text{out}}$
$\hat{b}$
$\hat{b}_{\text{out}}$
BS_{II}
$\hat{a}_{\text{in}}$
BS_{I}
$\hat{a}$
$\hat{b}_{\text{in}}$



g
$\hat{b}$
$\hat{a}_{\mathrm{out}}$
$\hat{b}_{\mathrm{out}}$
$\mathrm{BS_{II}}$
$\hat{a}_{\mathrm{in}}$
$\mathrm{BS_{I}}$
$\hat{a}$
$\hat{b}_{\mathrm{in}}$


g
$\hat{b}$
$\hat{a}_{\text{out}}$
$\hat{b}_{\text{out}}$
BS_{II}
$\hat{a}_{\text{in}}$
BS_{I}
$\hat{a}$
$\hat{b}_{\text{in}}$


g
$\hat{b}$
$\hat{a}_{\mathrm{out}}$
$\mathrm{BS}_{\mathrm{II}}$
$\hat{b}_{\mathrm{out}}$
$\hat{a}_{\mathrm{in}}$
BS_{I}
$\hat{a}$
$\hat{b}_{\mathrm{in}}$


g
$\hat{b}$
$\hat{a}_{\mathrm{out}}$
$\mathrm{BS}_{\mathrm{II}}$
$\hat{b}_{\mathrm{out}}$
$\hat{a}_{\mathrm{in}}$
BS_{I}
$\hat{a}$
$\hat{b}_{\mathrm{in}}$

Setup

Static space-time metric

$$\mathbf{g} = -c^2 N(\boldsymbol{x})^2 \mathrm{d}t^2 + h_{ij}(\boldsymbol{x}) \mathrm{d}x^i \mathrm{d}x^j$$

Riemann curvature

$$\mathbf{R}_{0i0j} = c^2 N \nabla_i \nabla_j N \qquad \mathbf{R}_{ijkl} = R_{ijkl}$$

Gordon's optical metric

$$\tilde{\mathbf{g}} = -c^2 (N(\boldsymbol{x})/n(\boldsymbol{x}))^2 \mathrm{d}t^2 + h_{ij}(\boldsymbol{x}) \mathrm{d}x^i \mathrm{d}x^j$$

Lagrangian

$$L[F] = -\tfrac{1}{4} \tilde{\mathbf{g}}^{ab} \tilde{\mathbf{g}}^{cd} F_{ac} F_{bd} \qquad L[\varphi] = -\tfrac{1}{2} \tilde{\mathbf{g}}^{ab} (\nabla_a \varphi)(\nabla_b \varphi)$$

Setup

Neglecting polarisation degrees of freedom, consider scalar quantum field operator

$$\hat{\varphi}(t, \boldsymbol{x}) = \int \frac{\mathrm{d}\omega}{\sqrt{2\omega}} [\hat{a}(\omega) e^{-i\omega t} u_\omega(\boldsymbol{x}) + \hat{a}^\dagger(\omega) e^{+i\omega t} \bar{u}_\omega(\boldsymbol{x})]$$

Mode functions satisfy

$$(n/N)\Delta(u_\omega N/n) + (n\omega/cN)^2 u_\omega = 0$$

Normalisation gives standard commutation relation

$$[a(\omega), a^\dagger(\omega')] = \delta(\omega - \omega')$$

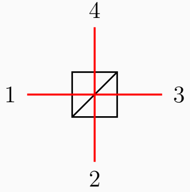
Vacuum state $|0\rangle$

$$\hat{a}(\omega)\,|0\rangle = 0$$

First approximation: eikonal equation

$$\phi(\boldsymbol{x}_B) = \phi(\boldsymbol{x}_A) + \int_\gamma \frac{n\omega}{cN}\hat{k}_i \mathrm{d}x^i$$

Beam splitter coupling at $\boldsymbol{x}_*$:



$$\hat{\varphi}_3(t,\boldsymbol{x}_*) = \mathcal{T}\hat{\varphi}_1(t,\boldsymbol{x}_*) + \mathcal{R}\hat{\varphi}_2(t,\boldsymbol{x}_*)$$
$$\hat{\varphi}_4(t,\boldsymbol{x}_*) = \mathcal{R}\hat{\varphi}_1(t,\boldsymbol{x}_*) + \mathcal{T}\hat{\varphi}_2(t,\boldsymbol{x}_*)$$
$$|\mathcal{T}|^2 + |\mathcal{R}|^2 = 1 \qquad \bar{\mathcal{T}}\mathcal{R} + \bar{\mathcal{R}}\mathcal{T} = 0$$

$$\begin{pmatrix} \hat{a}_1^\dagger(\omega)\bar{u}_{1,\omega}(\boldsymbol{x}_*) \\ \hat{a}_2^\dagger(\omega)\bar{u}_{2,\omega}(\boldsymbol{x}_*) \end{pmatrix} = \begin{pmatrix} \mathcal{T} & \mathcal{R} \\ \mathcal{R} & \mathcal{T} \end{pmatrix} \begin{pmatrix} \hat{a}_3^\dagger(\omega)\bar{u}_{3,\omega}(\boldsymbol{x}_*) \\ \hat{a}_4^\dagger(\omega)\bar{u}_{4,\omega}(\boldsymbol{x}_*) \end{pmatrix}$$

Quasi-Schrödinger Picture

The mode-couplings can be realised by unitary transformations $\hat{U}_{\text{BS}}$ and $\hat{U}_{\phi}$:

$$\hat{U}_{\text{BS}} |0\rangle = |0\rangle \qquad \hat{U}_{\phi} |0\rangle = |0\rangle$$

$$\hat{U}_{\text{BS}} \hat{a}^{\dagger} \hat{U}_{\text{BS}}^{\dagger} = \frac{1}{\sqrt{2}}(a^{\dagger} + ib^{\dagger}) \qquad \boxed{\begin{aligned}\mathcal{T} &= 1/\sqrt{2}\\ \mathcal{R} &= i/\sqrt{2}\end{aligned}} \qquad \hat{U}_{\text{BS}} \hat{b}^{\dagger} \hat{U}_{\text{BS}}^{\dagger} = \frac{1}{\sqrt{2}}(\hat{b}^{\dagger} + i\hat{a}^{\dagger})$$

$$\hat{U}_{\phi} \hat{a}^{\dagger} \hat{U}_{\phi}^{\dagger} = a^{\dagger} e^{-i\phi'} \qquad \hat{U}_{\phi} \hat{b}^{\dagger} \hat{U}_{\phi}^{\dagger} = \hat{b}^{\dagger} e^{-i\phi''}$$

Mach-Zehnder Interferometer

Consider single modes and symmetric beam splitters

$$
\begin{aligned}
|\psi\rangle &= |1,1\rangle = a^\dagger b^\dagger |0\rangle \\
\hat{U}_{\text{BS}} |\psi\rangle &= \tfrac{i}{2}(a^\dagger a^\dagger + b^\dagger b^\dagger)|0\rangle = \tfrac{i}{\sqrt{2}}(|2,0\rangle + |0,2\rangle) \\
\hat{U}_\phi \hat{U}_{\text{BS}} |\psi\rangle &= \tfrac{i}{\sqrt{2}}(e^{-2i\phi'}|2,0\rangle + e^{-2i\phi''}|0,2\rangle) \\
\hat{U}_{\text{BS}} \hat{U}_\phi \hat{U}_{\text{BS}} |\psi\rangle &= \tfrac{i}{2\sqrt{2}}(e^{-2i\phi'} - e^{-2i\phi''})(|2,0\rangle - |0,2\rangle) - \tfrac{1}{2}(e^{-2i\phi'} + e^{-2i\phi''})|1,1\rangle
\end{aligned}
$$

Probability of finding both photons in the same mode

$$
p = 2\left|\tfrac{i}{2\sqrt{2}}(e^{2i\phi'} - e^{2i\phi''})\right|^2 = \tfrac{1}{2}(1 - \cos(2\Delta\phi))
$$

Compared to coherent states or single photons: doubled fringe frequency

The intermediate state $\frac{1}{\sqrt{2}}(|2,0\rangle + |0,2\rangle)$ is a special case of a maximally path-entangled “NOON” state $\frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle)$.

Sensitivity to Curvature

Previous proposals for NOON-state interferometry: redshift (Anastopoulos & Hu 2021), frame dragging (Brady & Haldar 2021), PPN parameters (Rivera-Tapia *et al.* 2021) assumed linear dependence of $\Delta\phi \propto \Delta h$.

Linear dependence $\Delta N \propto \Delta h$ is explicable using the weak equivalence principle: $N = 1 + gz/c^2$ and $h_{ij} = \delta_{ij}$ is flat space-time.

Excess probability

$$\delta p = p - \tilde{p}$$

$-GM_\oplus/(R_\oplus + h)$ potential

$\text{const.} + gh$ potential

Sensitivity to Curvature

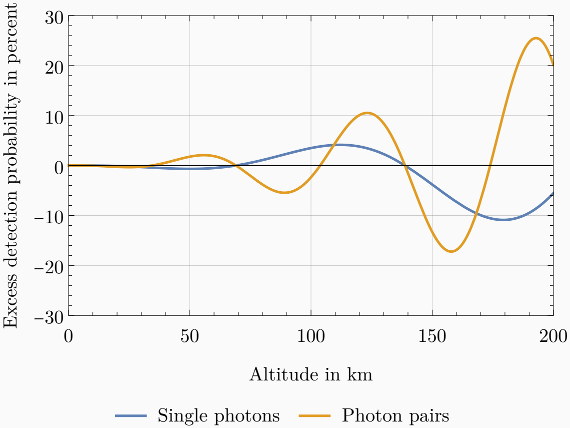


Figure 1: Excess probability, δp, for single photons and NOON states with wavelength $\lambda = 1500$ nm, spectral with $\delta\lambda = 1$ nm, and arm length $l = 100$ km.

Conclusion

Optical NOON-state interferometry experiments at sattelite altitudes …

- are beyond a Newtonian description of gravity
- are sensitive to gravity gradients
- simultaneously demonstrate signatures of quantum physics and general relativity

Anastopoulos, C. & Hu, B.-L. *Relativistic Particle Motion and Quantum Optics in a Weak Gravitational Field.* June 2021. arXiv: 2106.12514 [quant-ph].

Brady, A. J. & Haldar, S. Frame dragging and the Hong-Ou-Mandel dip: Gravitational effects in multiphoton interference. *Physical Review Research* **3,** 023024. doi:10.1103/PhysRevResearch.3.023024. arXiv: 2006.04221 [quant-ph] (Apr. 2021).

Colella, R., Overhauser, A. W. & Werner, S. A. Observation of Gravitationally Induced Quantum Interference. *Phys. Rev. Lett.* **34,** 1472–1474. doi:10.1103/PhysRevLett.34.1472 (June 1975).

Hilweg, C. *et al.* Gravitationally induced phase shift on a single photon. *New Journal of Physics* **19,** 033028. doi:10.1088/1367-2630/aa638f. arXiv: 1612.03612 [quant-ph] (Mar. 2017).

Kasevich, M. & Chu, S. Measurement of the gravitational acceleration of an atom with a light-pulse atom interferometer. *Applied Physics B: Lasers and Optics* **54,** 321–332. doi:10.1007/BF00325375 (May 1992).

Rivera-Tapia, M., Yáñez Reyes, M. I., Delgado, A. & Rubilar, G. *Outperforming classical estimation of Post-Newtonian parameters of Earth's gravitational field using quantum metrology.* Jan. 2021. arXiv: 2101.12126 [gr-qc].

Sorrentino, F. *et al.* Simultaneous measurement of gravity acceleration and gravity gradient with an atom interferometer. *Applied Physics Letters* **101,** 114106. doi:10.1063/1.4751112 (Sept. 2012).

References iii

Zych, M., Costa, F., Pikovski, I., Ralph, T. C. & Brukner, Č. General relativistic effects in quantum interference of photons. *Classical and Quantum Gravity* **29,** 224010. doi:10.1088/0264-9381/29/22/224010. arXiv: 1206.0965 [quant-ph] (Nov. 2012).