Modelling stellar atmospheres with full Zeeman treatment

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Outline of the problem

Properties of magnetic CP stars:
- upper main sequence stars
- \( T_{\text{eff}} \) ranging from 8,000 to 15,000 K
- spectrum and photometric variability
- peculiar and stratified abundances
- magnetic fields, \( |B| \) ranging from \( \sim 100 \text{ G} \) to 35 kG, up to 100 kG at magnetic poles

Bagnulo et al., 2001, A&A 369, 889
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requirements specification for the atmospheric model code

- temperature range (of interest): 8.000 to 15.000 K
- peculiar and stratified abundances
- arbitrary inclination of the field in plane parallel models
- full Zeeman treatment, polarised Feautrier solver
- hydrostatic equilibrium with magnetic pressure
- VALD line data, CoCoS
- simplifications: plane parallel, Kurucz continuum routines, no dynamic phenomena considered, no microturbulence.
The CAMAS Code

Program features:

- **ATLAS12** continua for comparability with standard models
- Consistent with spectral synthesis (COSSAM) and radiative diffusion (CARAT) code
- Written in Ada95
- Thread parallel
- Modularised, object oriented
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Polarised radiation transfer equation

\[ \frac{d}{dz} I = -K I + K (S, 0, 0, 0) \dagger \]

Stokes vector

\[ I = (I, Q, U, V) \dagger \]

absorption matrix

\[ K = \kappa_c I + \kappa_o \Phi \]

line absorption matrix

\[ \Phi = \begin{pmatrix}
\phi_I & \phi_Q & \phi_U & \phi_V \\
\phi_Q & \phi_I & \phi_V & -\phi_U \\
\phi_U & -\phi_V & \phi_I & \phi_Q \\
\phi_V & \phi_U & -\phi_Q & \phi_I
\end{pmatrix} \]
Zeeman Feautrier solver

- Feautrier equation can be generalised to the magnetic case in the presence of blends (see Alecian and Stift, 2004, A&A 416, 703)

\[
\frac{d\vec{J}}{d\tau_{5000}} = \mathbf{X}\vec{H} \quad \text{and} \quad \frac{d\vec{H}}{d\tau_{5000}} = \mathbf{X}(\vec{J} - \vec{S}) \quad \text{with} \quad \mathbf{X} := \frac{\mathbf{K}}{\kappa 5000 \mu}
\]

\[
\frac{d}{d\tau_{5000}}(\mathbf{X}^{-1}\frac{d\vec{J}}{d\tau_{5000}}) = \mathbf{X}(\vec{J} - \vec{S}) \quad \text{(inner points)}
\]

\[
\frac{d\mathbf{X}^{-1}}{d\tau_{5000}}\frac{d\vec{J}}{d\tau_{5000}} + \mathbf{X}^{-1}\frac{d^2\vec{J}}{d\tau_{5000}^2} = \mathbf{X}(\vec{J} - \vec{S}) \quad \text{(boundary condition)}
\]

- \(N\) equations, \(N\) unknowns

\[
\begin{align*}
B_1 \vec{J}_1 - C_1 \vec{J}_2 &= \vec{L}_1 \\
- A_n \vec{J}_{n-1} + B_n \vec{J}_n - C_n \vec{J}_{n+1} &= \vec{L}_n \\
- A_N \vec{J}_{N-1} + B_N \vec{J}_N &= \vec{L}_N
\end{align*}
\]
Temperature correction

Dreizler’s Lucy Unsöld scheme adapted for polarised radiation transport equations:

(see Dreizler, 2003, ASPC 288, 69)

- momenta of the polarised radiation transport equation
- differences \( \Delta X = X_{\text{Equilibrium}} - X_{\text{Model}} \) of flux, intensity and radiation pressure
- two flux criteria
  - local balance of emitted versus absorbed energy
    \[
    \frac{d(H_I)}{d\tau_{5000}} = 0 \quad \text{(constant flux)}
    \]
    cannot be used in deep layers where the atmosphere becomes diffusive and \( S \sim J \sim \) local Planck function.
  - nonlocal condition of constant flux:
    \[
    \int_{0}^{\infty} \int H_I d\Omega d\nu - \frac{\sigma}{4\pi} T_{\text{eff}}^4 = 0 \quad \text{(desired value of the flux)}
    \]
    inefficient in regions with small opacities
Combining the flux criteria

\[ \Delta T = \frac{\pi}{4\sigma T^3} \left( d_1 \left( \frac{S \int_0^\infty [K_\nu \bar{J}_\nu] I d\nu}{\int_0^\infty [K_\nu] I I I S_\nu d\nu} - S \right) \right) \]

- \( d_1 \) \( \left( \frac{S \int_0^\infty [K_\nu \bar{J}_\nu] I d\nu}{\int_0^\infty [K_\nu] I I I S_\nu d\nu} - S \) \) local energy conservation

+ \( d_2 \frac{S \int_0^\infty [K_\nu \bar{J}_\nu] I d\nu}{J_l \int_0^\infty [K_\nu] I I I S_\nu d\nu} f_0 \Delta H_l(0) \) surface flux

+ \( d_3 \frac{S \int_0^\infty [K_\nu \bar{J}_\nu] I d\nu}{J_l \int_0^\infty [K_\nu] I I I S_\nu d\nu} \frac{1}{f} \)

\[ \int_0^\tau \int_0^\infty [K_\nu \bar{H}_\nu] I d\nu \frac{\Delta H_l d\tau}{H_l \kappa_{5000}} \]

- global energy conservation
Results and comparisons

Two effects
- enhanced line blanketing
- magnetic pressure

Comparison with the results of
Comparison with Carpenter’s results

CAMAS results

Comparisons with LLMODELS

Differences between the fieldless and the “isotropic” model

\[ T_{\text{eff}} = 15000 \text{K}, [\text{M/H}] = 0.0 \]
Comparisons with LLMODELS

Differences between the field less and the “isotropic” model

CAMAS results

Kochukhov et al., 2005, A&A 433, 671
\( \tau_{\text{ross}} \) is affected by (magnetic) line blanketing, \( \tau_{5000} \) is not directly affected.

**Graph 1:**
- \( T_{\text{eff}} = 15000 \text{K}, [\text{M/H}] = 0.0 \)
- \( \Delta \lg P \) vs. \( \lg \tau_{\text{ross}} \)

**Graph 2:**
- \( T_{\text{eff}} = 15000 \text{K}, [\text{M/H}] = 0.0 \)
- \( \Delta \lg P \) vs. \( \lg \tau_{5000} \) for different magnetic fields:
  - 1kG (red)
  - 5kG (green)
  - 10kG (blue)
  - 20kG (purple)
  - 40kG (cyan)
**Depth scales**

\[ T_{\text{eff}} = 15000K, [\text{M/H}] = 0.0 \]

\[ \tau_{\text{ross}} \text{ is affected by (magnetic) line blanketing, } \tau_{5000} \text{ is not directly affected} \]
Comparison with LLMODELS

differences between the anisotropic and the “isotropic” model

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Khan and Shulyak, 2006b, A&A 454, 933
Comparison with LLMODELS

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Summary

• The Zeeman effect enhances the opacity. This affects the optical depth and the temperature structure.

• CAMAS essentially confirms the results of LLMODELS. The isotropic models are largely identical, however the anisotropic models with strong magnetic fields exhibit notable differences that need to be clarified.

• The atmospheres computed with CAMAS will be used (among others) for the modelling of diffusion processes.

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Bagnulo, S., Wade, G. A., Donati, J.-F., Landstreet, J. D.,
Columbus.)
(eds.), ASP Conf. Ser. 288: Stellar Atmosphere Modeling,
Vol. 288 of ASP Conference Series, p. 69, Review of
temperature correction schemes, Damping
Comparison with LL MODELS

differences between the anisotropic and the “isotropic” model

CAMAS results

Khan and Shulyak, 2006b, A&A 454, 933
first depth point

\[
\frac{d\vec{J}}{d\tau_{5000}}|_1 = X_1(\vec{J}_1 - (1 - e^{-\tau_1}X_1)\vec{S}_1)
\]

\[
\vec{J}_1 = \vec{J}_2 - \Delta \tau_{(2,1)} \frac{d\vec{J}}{d\tau}|_1 - \frac{(\Delta \tau_{(2,1)})^2}{2} \frac{d^2\vec{J}}{d\tau^2}|_1 - \ldots \text{ (“forward” Taylor series)}
\]

\[
A_1 = 0
\]

\[
B_1 = 1 + \Delta \tau_{(2,1)}X_1 - \frac{(\Delta \tau_{(2,1)})^2}{2}X_1 \frac{dX^{-1}}{d\tau_{5000}}|_1 X_1 + \frac{(\Delta \tau_{(2,1)})^2}{2}X_1
\]

\[
C_1 = 1
\]

\[
\vec{L}_1 = (\Delta \tau_{(2,1)}) - \frac{(\Delta \tau_{(2,1)})^2}{2}X_1 \frac{dX^{-1}}{d\tau_{5000}}|_1 \frac{d\vec{X}^{-1}}{d\tau_{5000}}|_1 X_1 (1 - e^{-\tau_1}X_1)\vec{S}_1 + \frac{(\Delta \tau_{(2,1)})^2}{2}X_1^2\vec{S}_1
\]
inner depth points

\[
\frac{d}{d\tau} \mathbf{x}^{-1} \frac{d\mathbf{J}}{d\tau} \bigg|_n = \frac{\mathbf{x}^{-1} \frac{\mathbf{J}_{n+1} - \mathbf{J}_n}{\Delta \tau(n+1,n)} \mathbf{x}^{-1} \frac{\mathbf{J}_n - \mathbf{J}_{n-1}}{\Delta \tau(n,n-1)}}{\frac{\Delta \tau(n+1,n) + \Delta \tau(n,n-1)}{2}}
\]

\[
\mathbf{A}_n = \frac{(\mathbf{x}_{n-1} + \mathbf{x}_n)^{-1}}{\Delta \tau(n,n-1) \Delta \tau(n+1,n-1)}
\]

\[
\mathbf{B}_n = \frac{(\mathbf{x}_{n-1} - \mathbf{x}_n)^{-1}}{\Delta \tau(n,n-1) \Delta \tau(n+1,n-1)} + \frac{(\mathbf{x}_{n+1}^{-1} + \mathbf{x}_n)^{-1}}{\Delta \tau(n+1,n) \Delta \tau(n+1,n-1)} + \mathbf{x}_n
\]

\[
\mathbf{C}_n = \frac{(\mathbf{x}_{n+1} + \mathbf{x}_n)^{-1}}{\Delta \tau(n+1,n) \Delta \tau(n+1,n-1)}
\]

\[
\mathbf{L}_n = \mathbf{x}_n \tilde{\mathbf{S}}_n
\]
Flowchart of CAMAS

- read input, initialise
- enter radiative equilibrium loop

- check convection
- recompute equidistant $\tau_{5000}$ and interpolate model data
- EXIT if convergence criteria are met
- pretabulate continua if necessary
- reselect lines if necessary
- enter integration loop (> 90% of computation time)
  independent tasks use a magnetic Feautrier solver for all wavelength points and add the results
- calculate temperature corrections
- calculate and apply pressure corrections
- search for noise in the flux distribution and check convergence
- apply temperature corrections if not converged yet
- apply Ng acceleration (optional)
Feautrier solver

- solve radiation transport equation as boundary value problem

\[ \frac{dl^\pm}{d\tau_{5000}} = I^\pm - S \]

- define flux like and intensity like quantities
- combine equations for outward and inward rays
- discretize and add boundary conditions
Feautrier solver

- solve radiation transport equation as boundary value problem
- define flux like and intensity like quantities

\[ H_n = \frac{I^+_n - I^-_n}{2}, \quad J_n = \frac{I^+_n + I^-_n}{2} \]

- combine equations for outward and inward rays
- discretize and add boundary conditions
Feautrier solver

- solve radiation transport equation as boundary value problem
  \[ \frac{dl^\pm}{d\tau_{5000}} = I^\pm - S \]
- define flux like and intensity like quantities
- combine equations for outward and inward rays
  \[ \frac{dJ}{d\tau_{5000}} = H, \quad \frac{dH}{d\tau_{5000}} = J - S \]
  \[ \frac{d^2 J}{d\tau_{5000}^2} = J - S \]
- discretize and add boundary conditions
Feautrier solver

- solve radiation transport equation as boundary value problem
- define flux like and intensity like quantities
- combine equations for outward and inward rays
- discretize and add boundary conditions
  - \( I^{-}(\tau = 0) = 0 \), no incident radiation on the surface

\[
\frac{dJ}{d\tau_{5000}}|_{\tau=0} = J(\tau = 0)
\]

- \( H_{N} = I_{N}^{+} - J_{N} \), diffusive at innermost depth-point

\[
\frac{dJ}{d\tau_{5000}}|_{N} = I_{N}^{+} - J_{N} \text{ with } I_{N}^{+} = S_{N} + \frac{dS}{d\tau_{5000}}|_{N}
\]
Angle dependent opacity

**line absorption terms**

\[
\phi_I = \frac{1}{4} \left( 2 \phi_p \sin^2 \gamma + (\phi_r + \phi_b)(1 + \cos^2 \gamma) \right)
\]

\[
\phi_Q = \frac{1}{4} \left( 2 \phi_p - (\phi_r + \phi_b) \right) \sin^2 \gamma \cos 2\chi
\]

\[
\phi_U = \frac{1}{4} \left( 2 \phi_p - (\phi_r + \phi_b) \right) \sin^2 \gamma \sin 2\chi
\]

\[
\phi_V = \frac{1}{2} \left( \phi_r - \phi_b \right) \cos \gamma
\]

\(\phi_{p, b, r}\) ... line absorption profiles

**Faraday terms**

\[
\phi'_Q = \frac{1}{4} \left( 2 \phi'_p - (\phi'_r + \phi'_b) \right) \sin^2 \gamma \cos 2\chi
\]

\[
\phi'_U = \frac{1}{4} \left( 2 \phi'_p - (\phi'_r + \phi'_b) \right) \sin^2 \gamma \sin 2\chi
\]

\[
\phi'_V = \frac{1}{2} \left( \phi'_r - \phi'_b \right) \cos \gamma
\]

\(\phi'_{p, b, r}\) ... anomalous dispersion profiles
Combining the flux criteria

- **0th moment:** \[ \frac{d\Delta H_I}{d\tau_{5000}} = \frac{\int_0^\infty [K_\nu \Delta \tilde{J}_\nu]_I d\nu}{\kappa_{5000}} - \frac{\int_0^\infty [K_\nu]_I, I \Delta S_\nu d\nu}{\kappa_{5000}} \]

\[ \frac{dH_{eq}}{d\tau_{5000}} = 0, \] 0th moment for \[ \frac{dH_{mod}}{d\tau_{5000}} \]

\[ \int_0^\infty [K_\nu \tilde{X}_{\nu, eq}] d\nu \sim \int_0^\infty [K_\nu \tilde{X}_{\nu, mod}] d\nu \]

\[ \frac{\Delta S}{\Delta T} = \frac{4\sigma T^3}{\pi} \]

\[ \frac{4\sigma T^3}{\pi} \Delta T = \frac{S}{\int_0^\infty [K_\nu]_I, I S_\nu d\nu} - \frac{S}{\int_0^\infty [K_\nu]_I, I S_\nu d\nu} \frac{\int_0^\infty [K_\nu \tilde{J}_\nu]_I d\nu}{J_I} \Delta J_I \]

- **1st moment:** \[ \frac{d\Delta K_I}{d\tau_{5000}} = \frac{\int_0^\infty [K_\nu \Delta \tilde{H}_\nu]_I d\nu}{\kappa_{5000}} \]

integration

variable Eddington factors \[ f_\tau = \frac{K_\tau}{J_\tau} \] and \[ g = \frac{H_0}{J_0} \]

\[ \frac{f_0}{g} \Delta H_I(0) + \int_0^{\tau} \frac{\int_0^\infty [K_\nu \Delta \tilde{H}_\nu]_I d\nu}{\kappa_{5000}} d\tau_{5000} \]