BA: Does Preprocessing Help under Congestion?

Klaus-Tycho Foerster, Janne H. Korhonen (IST Austria), Joel Rybicki (IST Austria), Stefan Schmid
Motivation

• Standard distributed computing models (eg CONGEST with $O(\log n)$ message sizes & IDs):
  ◦ Network topology is unknown
  ◦ Compute from scratch

• But in many networking applications:
  ◦ Communication topology remains unchanged
  ◦ Only the problem input changes
  ◦ Can we leverage preprocessing?

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Idea of the SUPPORTED model (Schmid and Suomela, 2013)
1. Perform any preprocessing on communication graph $H$
2. Solve problem for subgraph $G \subseteq H$ in eg CONGEST model
   • Use preprocessing information
   • Communicate on $H$
**Brief Background**

- **Congested Clique**
  - Introduced at SPAA 2003: Lotker, Pavlov, Patt-Shamir, Peleg
    - Analogy: SUPPORTED CONGEST model if communication graph H is a clique

- **SUPPORTED model**
  - Introduced for LOCAL and CONGEST at HotSDN 2013: Schmid and Suomela
  - CONGEST: Applications to subgraph detection at OPODIS 2017: Korhonen and Rybicki
  - LOCAL: Approximation bounds and connections to SLOCAL at INFOCOM 2019: Foerster, Hirvonen, Suomela, Schmid

- **This BA:** How do CONGEST lower bounds transfer to the SUPPORTED CONGEST model?
Many Communication-Complexity Bounds Transfer

• Common observation:
  ◦ Many CONGEST lower bounds rely on small cuts
  ◦ Topology information needs to be transferred over congested cut

• High-level idea:
  ◦ If small cut is also present on communication graph, then preprocessing does not help
  ◦ Topology information of input/problem graph still needs to get across congested cut

• Adapt proof from Abboud, Censor-Hillel, Khoury, Paz (arXiv 2019)
  ◦ Family of lower bound graphs construction
Transfer of Lower Bounds from **CONGEST** to **SUPPORTED CONGEST**

<table>
<thead>
<tr>
<th>Lower bound</th>
<th>Problem</th>
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<tbody>
<tr>
<td>$\Omega\left(n^{1/2}/\log n\right)$</td>
<td>4-cycle [Drucker, Kuhn, Oshman PODC’14], 2k-cycle [Korhonen, Rybicki OPODIS’17], Girth ($(2 - \varepsilon)$-apx.) [Frischknecht, Holzer, Wattenhofer SODA’12]</td>
</tr>
<tr>
<td>$\Omega(n/\log n)$</td>
<td>(2k + 1)-cycle [DKO PODC’14], APSP, Diameter ($(3/2 - \varepsilon)$-apx.) [FHW SODA’12]</td>
</tr>
<tr>
<td>$\Omega\left(n/(\log n)^2\right)$</td>
<td>Diameter on sparse graphs [Abboud, Censor-Hillel, Khoury DISC’16]</td>
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<tr>
<td>$\Omega\left(n/(\log n)^3\right)$</td>
<td>On sparse graphs: Diameter and radius ($(3/2 - \varepsilon)$-apx.), eccentricities ($(5/3 - \varepsilon)$-apx.) [ACK DISC’16]</td>
</tr>
<tr>
<td>$\Omega\left(n^{2-1/k} / (k \log n)\right)$</td>
<td>Subgraph detection (for any k) [Fischer, Gonen, Kuhn, Oshman SPAA’18]</td>
</tr>
<tr>
<td>$\Omega\left(n^2/(\log n)^2\right)$</td>
<td>Min. vertex cover, max. independent set, chrom. number ($(4/3 - \varepsilon)$-apx.), weighted 8-cycle [Censor-Hillel, Khoury, Paz DISC’17]</td>
</tr>
<tr>
<td>$\Omega\left(n^2\right)$</td>
<td>Identical subgraphs (deterministic only) [CHKP DISC’17]</td>
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Summary and Outlook

- We investigated the power of **preprocessing** in the *CONGEST* model

- Many *CONGEST* lower bounds hold even under arbitrary **preprocessing**
  - Is SUPPORTED *CONGEST* maybe the proper way to look at lower bounds?

- Is there a “proper” **separation** between *CONGEST* and SUPPORTED *CONGEST* for general graphs?
  - “Proper”: Without relying on identifiers and graph size?
    - Note: Easy on restricted graph classes, e.g., if H has small chromatic number
References

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